PROBLEM: 1.1 Create a list of five real engineering problems or societal challenges that we are able to address with the modeling introduced in this chapter and studied in fluid mechanics?

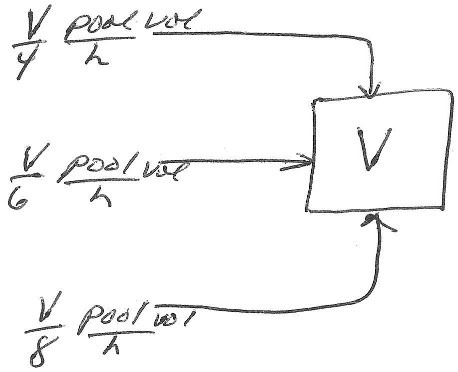
SOLUTION:

1. **energy generation through flow** (hydroelectric)
2. **biomedical problems** (flow through heart; blood pressure and disease; how does blood pressure affect flows in the body; narrowing of the arteries.
3. **water runoff** where does the water go after a rainstorm? flow through porous media
4. **flood control** levees, flash floods
5. **weather** tornado prediction and elimination, hurricanes
6. **airplane flight improvements** and reduction in fuel costs
7. **fuel efficiency** due to dreg; reducing drag saves energy
8. **Insect population** how aqueous habitat works with animal life

PROBLEM: 1.2 The green hose fills a swimming pool in four hours, the red hose fills the same pool in 6 hours, and the yellow hose fills the same pool in 8 hours. With all three hoses running at those rates, how long will it take to fill the pool?

SOLUTION:

1.2 The green hose fills a pool in four hours, the red hose fills the same pool in 6 hours and the yellow hose fills the same pool in 8 hours. With all three hoses running at those rates, how long will it take to fill the pool?



t hours to fill

$$\left(\frac{V}{4}\right)t + \left(\frac{V}{6}\right)t + \frac{V}{8}(t) = V$$

$$\frac{t}{4} + \frac{t}{6} + \frac{t}{8} = 1$$

$$\frac{6t}{24} + \frac{4t}{24} + \frac{3t}{24} = 1$$

$$13t = 24$$

$$t = \frac{24}{13} \text{ hrs}$$

$$\boxed{t = 1.85h}$$

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PROBLEM: 1.3 What is a typical volumetric flow rate (in gpm and lpm (liters per minute) for household plumbing? What is a typical value of average velocity in a pipe? Assume half-inch type-K copper tubing (see Perry's Chemical Engineering Handbook [132] for dimensions).

SOLUTION:

Copper Tube Handbook has the dimensions for copper tubes

$ID = 0.527$ in for $\frac{1}{2}$" type K

$$Q \sim \left(\frac{3\,gal}{min}\right)\left(\frac{1000\,l}{264.17\,gal}\right)$$

$$Q = 11.356\ l/min$$

$$\boxed{Q = 11\,lpm} \quad \boxed{Q = 3\,gpm}$$

$$\langle v \rangle = \frac{Q}{\pi R^2} = \frac{4Q}{\pi D^2}$$

$\longrightarrow$

$$\langle v \rangle = \frac{(4)}{(\pi)} \left(\frac{0.18927\cancel{\ell}}{s}\right)\left(\frac{1}{0.527\cancel{in}}\right)^2\left(\frac{\cancel{in}}{2.54\cancel{cm}}\right)^2$$

$$\times \frac{m}{100\cancel{cm}}\left(\frac{10^3\cancel{m\ell}}{1\cancel{\ell}}\right)\left(\frac{1cm^3}{\cancel{m\ell}}\right)$$

$$\langle v \rangle = 1.345 \ m/s$$

$$\left(\frac{1.345\cancel{m}}{s}\right)\left(\frac{3.2808 \ ft}{\cancel{m}}\right) = 4.413 \ ft/s$$

$$\boxed{\begin{array}{l} 1.3 \ m/s \\ 4.4 \ ft/s \end{array}}$$

PROBLEM: 1.4 Compare typical values of velocity head, pressure head, elevation head, and friction head. What is a good rule of thumb for when velocity differences are significant in the flow of household water? Assume that the relevant piping is half-inch type K copper tubing (see Perry's Chemical Engineering Handbook [132] for dimensions).

SOLUTION:

MEB (head): ①

$$\underbrace{\frac{\Delta P}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{\Delta (V^2)}{2\alpha g}}_{\text{velocity head}} + \underbrace{\Delta z}_{\text{elevation head}} + \frac{F}{g} = -\frac{W_{s,by}}{mg}$$

for a typical household:

$$Q = 3 \text{ gpm}$$

$$\frac{Q}{\pi R^2} = 4.4 \text{ ft/s} \quad \text{for } \underbrace{D = 2R = 0.527 \text{ in}}_{\text{type K copper tubing; see previous problem}}$$

$$\Delta P \sim 60 \text{ psi} = 60 \frac{lb_f}{in^2}$$

$$\Delta z \sim 10 - 30 \text{ ft}$$

(2)

pressure head = $\frac{\Delta P}{\rho g}$

$$= \left(\frac{60\ \cancel{lb_f}}{\cancel{in^2}}\right)\left(\frac{ft^3}{62.25\ \cancel{lb_m}}\right)\left(\frac{s^2}{\cancel{32.174}\ \cancel{ft}}\right)\left(\frac{\cancel{32.174}\ \cancel{ft}\ \cancel{lb_m}}{\cancel{s^2}\ \cancel{lb_f}}\right)$$

$$* \left(\frac{12\ \cancel{in}}{\cancel{ft}}\right)^2$$

$$\boxed{\frac{\Delta P}{\rho g} = 139\ ft}$$ maximum value

velocity head = $\frac{\Delta v^2}{2g}$ $(\alpha = 1)$

$$\frac{\Delta v^2}{2g} = \left(\frac{4.4\ ft}{\cancel{s}}\right)^2\left(\frac{1}{2}\right)\left(\frac{\cancel{s^2}}{32.174\ ft}\right)$$

$$\boxed{\frac{\Delta v^2}{2g} = 0.3\ ft}$$

elevation head = Δz

$$\boxed{\Delta z \sim 10-30\ ft}$$

(3)

As estimated, the pressure head is the most significant, but we used the largest possible ΔP. The effect of elevation is the second most significant with velocity head insignificant. Velocity becomes more important when friction is considered, however, since $\frac{F}{g}$ is proportional to $\frac{V^2}{2g}$.

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PROBLEM: 1.5 What are the viscosity and density of glycerin at room temperature? A useful reference for physical-property data is *Perry's Chemical Engineering Handbook* [132].

SOLUTION:

The densities of organic compounds are available in Perry's [132] (7^{th} edition), section 2, *Chemical and Physical Data*. From Table 2-2 *Physical Properties of Organic Compounds*, page 2-38:

$$\begin{aligned} SG(glycerol) &= 1.260^{50/4} \\ SG &= \frac{\rho(50^oC)}{\rho_{water}(4^oC)} \\ \rho_{water}(4^oC) &= 1.000\ g/cm^3 \\ \rho_{glycerin}(50^oC) &= 1.260\ g/cm^3 \end{aligned}$$

The viscosities of organic compounds are available in the *CRC Handbook of Chemistry and Physics* (88^{th} edition, David R. Lide, editor, CRC Press, New York, 2008). The table on page 6-191 called *Viscosities of Liquids* is organized by chemical formula. Glycerin is $C_3H_8O_3$. The entry for glycerin (or glycerol, which is an alternate name) gives viscosity at four temperatures:

T (^oC)	$\mu_{glycerin}$ $(mPa\ s)$
25	934
50	152
75	39.8
100	14.8

PROBLEM: 1.6 How do the viscosity of sugar-water solutions vary with concentration and temperature? (Find the answer in the literature.) Provide a plot that shows how the data vary; consider carefully how to plot the data so that the trend is displayed meaningfully.

SOLUTION:

The viscosities of aqueous solutions of sucrose at 20^oC are available in the *CRC Handbook of Chemistry and Physics* (88^{th} edition, David R. Lide, editor, CRC Press, New York, 2008). The table on page 8-52 called *Concentrative Properties of Aqueous Solutions: Density, Refractive Index, Freezing Point Depression, and Viscosity* is organized alphabetically. Sugar is Sucrose. The entry for sucrose solutions gives viscosity as a function of mass percent from 0.5 to 80% (see below).

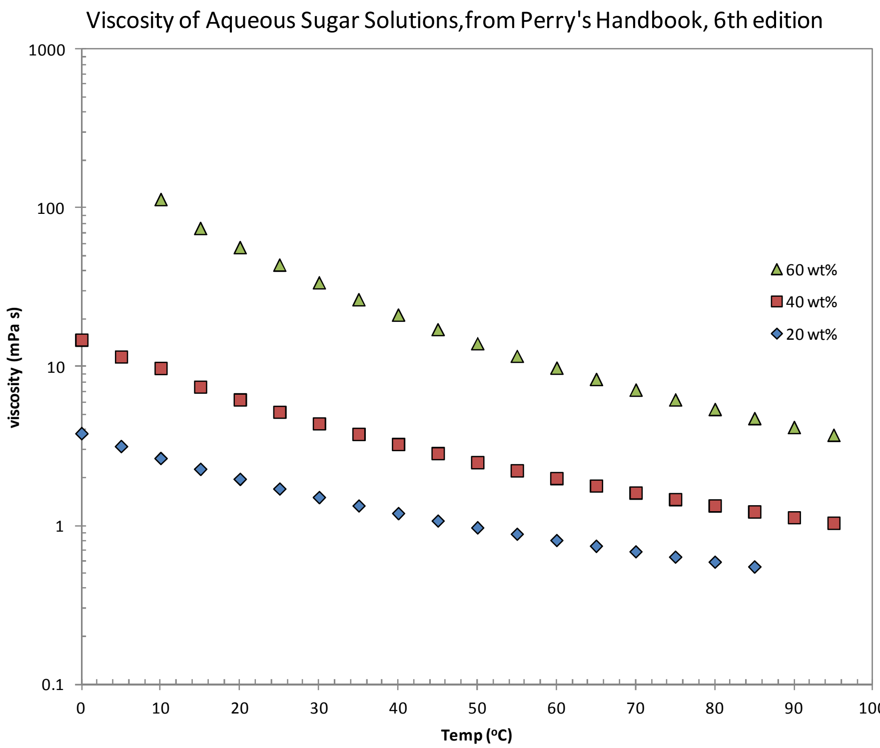
To find the viscosity at a variety of temperatures we must look elsewhere. These can be found in Perry's [132] in the 6^{th} edition in section 3 (see below)

(from the CRC handbook)

$massfrac$ %	$\mu(20^oC)$ $(mPa\ s)$	$massfrac$ %	$\mu(20^oC)$ $(mPa\ s)$
0.5	1.015	22	2.124
1	1.028	24	2.331
2	1.055	26	2.573
3	1.084	28	2.855
4	1.114	30	3.187
5	1.146	32	3.762
6	1.179	34	4.052
7	1.215	36	4.621
8	1.254	38	5.315
9	1.294	40	6.162
10	1.336	42	7.234
12	1.429	44	8.596
14	1.534	46	10.301
16	1.653	48	12.515
18	1.79	50	15.431
20	1.945	60	58.487
		70	481.561

(from Perry's)

T (^oC)	$\mu(20\% sugar)$ ($mPa\ s$)	$\mu(40\% sugar)$ ($mPa\ s$)	$\mu(60\% sugar)$ ($mPa\ s$)
0	3.818	14.82	
5	3.166	11.6	
10	2.662	9.83	113.9
15	2.275	7.496	74.9
20	1.967	6.223	56.7
25	1.71	5.206	44.02
30	1.51	4.398	34.01
35	1.336	3.776	26.62
40	1.197	3.261	21.3
45	1.074	2.858	17.24
50	0.974	2.506	14.06
55	0.887	2.227	11.71
60	0.811	1.989	9.87
65	0.745	1.785	8.37
70	0.688	1.614	7.18
75	0.637	1.467	6.22
80	0.592	1.339	5.42
85	0.552	1.226	4.75
90		1.127	4.17
95		1.041	3.73


Viscosity of Aqueous Sugar Solutions, from Perry's Handbook, 6th edition
1000
100
10
1
0.1
viscosity (mPa s)
0
10
20
30
40
50
60
70
80
90
100
Temp (°C)
60 wt%
40 wt%
20 wt%

PROBLEM: 1.7 Examine the friction-factor/Reynolds-number relationship for turbulent flow in pipes (see Figure 1.21). Calculate the pressure-drop versus the flow-rate for turbulent flow in a rough pipe in an existing apparatus at a chemical plant. List the information needed about the pipe to make the calculation. Which factors are the most critical?

SOLUTION: Pressure-drop versus flow rate for rough pipes is given by the Colebrook correlation (Equation 1.95) and the Moody plot (Figure 1.21).

$$
\begin{aligned}
\frac{1}{\sqrt{f}} &= -4.0 \log\left(\frac{\varepsilon}{D} + \frac{4.67}{\mathrm{Re}\sqrt{f}}\right) + 2.28 \\
f &= \frac{\Delta p D}{2L\rho\langle v\rangle^2} \\
Q &= \frac{\pi D^2 \langle v\rangle}{4} \\
\mathrm{Re} &= \frac{\rho\langle v\rangle D}{\mu}
\end{aligned}
$$

Thus, we need D, L, fluid properties (ρ, μ), and the pipe roughness ε.

In turbulent flow, the friction factor does not vary much with Reynolds number (see Figure 1.21). Thus, although we need ρ, $\langle v\rangle$, D, and μ to calculate the Reynolds number and pipe length L to calculate pressure drop from the friction factor, the roughness factor ε is the factor that has the most significant effect at high Reynolds numbers. The Moody chart (Figure 1.21) shows that the value of the roughness parameter changes the value of the friction factor significantly for rough pipes. The value of Reynolds number is less significant as long as we know we are in turbulent flow. The curves of friction factor as a function of Reynolds number are flat in the turbulent regime for rough pipes.

PROBLEM: 1.8 For household water in steady flow in a 1/2 *in* Schedule 40 horizontal pipe at 3.0 *gpm* (see Figure 1.20), what are the frictional losses over a 100 *ft* run of pipe? The flow may be laminar or turbulent. (This problem was proposed originally as Example 1.8; on completion of this chapter we now can solve it.)

SOLUTION:

1.8 For water in steady flow in $\frac{1}{2}$" Schedule 40 horizontal smooth pipe at 3.0 gpm what are the frictional losses over a 100 ft run of pipe?

$\frac{1}{2}$" Schedule 40 ID = 0.622 in (Perry's)

$$F = \frac{2fL}{D}\langle v\rangle^2$$

To determine if flow is laminar or turbulent use Re (see below)

$$Re = \frac{\rho\langle v\rangle D}{\mu} = \frac{\left(\frac{62.25\ \cancel{lbm}}{\cancel{ft^3}}\right)\left(\frac{3.1676\ \cancel{ft}}{s}\right)\times\left(\frac{0.622\ \cancel{in}\ \cancel{ft}}{12\ \cancel{in}}\right)}{\left(6.005\times10^{-4}\ \frac{\cancel{lbm}}{\cancel{ft}\ \cancel{s}}\right)}$$

$Re = 1.702\times10^4$
= turbulent

$$\langle v \rangle = \frac{Q}{\pi R^2} = \frac{4Q}{\pi D^2}$$

$$= \left(\frac{4}{\pi}\right)\left(\frac{3.0\,\cancel{gpm}}{(0.622\,\cancel{in})^2}\right)\left(\frac{2.22802 \times 10^{-3}\,ft^3/s}{1\,\cancel{gpm}}\right)\left(\frac{12\,\cancel{in}}{\cancel{ft}}\right)^2$$

$$\boxed{\langle v \rangle = 3.1676 \; ft/s}$$

To calculate f use Colebrook equation

$$\frac{1}{\sqrt{f}} = -4 \log\left\{\frac{\varepsilon}{D} + \frac{4.67}{Re\sqrt{f}}\right\} + 2.28$$

We know $Re = 17{,}020$. From Fig 1.21 we estimate $f \sim 0.006$ and do an iterative solution in a spreadsheet software program.

Row	H	I	J	K	L	M	N
1							
2							
3							
4	Re	17,020					
5	f1	0.006					
6	f2	0.0068495	=1/(LOG(4.67/(I4*SQRT(I5)))*(-4)+2.28)^2				
	f3	0.0067209					
	f4	0.0067391					
	f5	0.0067365					
	f6	0.0067369					

guess

RHS Colebrook

f

$$F = \frac{2fL}{D}\langle v\rangle^2$$

$$= \frac{(2)(0.00674)(100\,\cancel{ft})}{\left(\frac{0.622\,\cancel{in}}{12\,\cancel{in}/\cancel{ft}}\right)}\left(3.1676\,\frac{ft}{s}\right)^2$$

$$F = 260.9\ \frac{ft^2}{s^2}$$

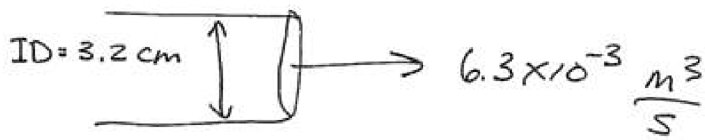
$$\frac{F}{g} = \frac{260.9\,ft^{\cancel{2}}}{\cancel{s^2}}\ \frac{\cancel{s^2}}{32.174\,\cancel{ft}} = \boxed{8.1\ ft\ \text{of head loss}}$$

PROBLEM: 1.9 What is the range of friction factor for turbulent flow in smooth and rough pipes? What is the range of friction factor for laminar flow?

SOLUTION: Looking at the Moody plot (Figure 1.21), the range of friction factor for turbulent flow is from 0.01 at Re=4,000 to 0.002 at Re=10^7 for smooth pipes. For rough pipes it can be as high as 0.02 at all Reynolds numbers. For laminar flow, the friction factor can go arbitrarily high at low Reynolds number ($f = 16/\mathrm{Re}$), and reaches a low value of 16/2100=0.008 at Re=2100, the highest value of Reynolds number for laminar flow.

PROBLEM: 1.10 Water at 25^oC flows at $6.3 \times 10^{-3}\ m^3/s$ through the irregularly shaped container in Figure 1.48. What is the average fluid velocity at the exit? The apparatus is open to the atmosphere at entrance and exit.

SOLUTION



$$\langle v \rangle = \frac{4Q}{\pi D^2}$$

$$= \frac{4\left(6.3\times10^{-3}\ \frac{m^3}{s}\right)}{\pi\left(\frac{3.2\,cm}{100\,cm/m}\right)^2}$$

$$= 0.07833\ m/s$$

$$= \boxed{0.078\ m/s}$$

PROBLEM: 1.11 At a Reynolds number of 10,000 flow in a pipe is turbulent, and it is not possible to produce a laminar flow. What is the friction factor for a flow in smooth pipe at this Reynolds number? If somehow we could produce a laminar flow at this Reynolds number, what would the friction factor be? Repeat for Re=10^5. Compare the two answers and discuss.

SOLUTION: According to the Moody chart (Figure 1.21), when Re=10,000, the friction factor is between 0.007 and 0.008. If a laminar flow could be produced at this Reynolds number, the friction factor would be $16/10,000 = 1.6 \times 10^{-3}$. The friction is much higher (by a factor of 5) at this Reynolds number. The higher friction factor of turbulent flow versus laminar flow is due to the energy losses of the tumbling, twisting flow that is turbulent flow.

If we consider flow at $\mathrm{Re} = 10^5$, the friction factor is 4.5×10^{-3} in turbulent flow (from the Colebrook correlation, Equation 1.95) and in laminar flow it is $\mathrm{Re} = 16/10^5 = 1.6 \times 10^{-4}$. The ratio of these two friction factors is 28.

We see that at high Reynolds number the difference in friction factor between hypothetical laminar and actual turbulent flow becomes ever more significant. Assuming laminar flow in the turbulent regime would be a significant error.

PROBLEM: 1.12 *Piping* and *tubing* are names for conduits of fluids, but the two terms differ in that the outer diameter of piping is standardized to allow pipe fitters to mount pipes into standard-size holders. The tubing OD is not standardized. What are the ID and OD of nominal 1/2 *in*, 3/4 *in*,and 1 *in* Schedule 40 pipe? Give dimensions in both *inches* and *mm*. What are the closest metric standard pipe sizes to these three sizes? Search for these answers in the literature.

SOLUTION: The dimensions of piping are available in Perry's [132], in Geakoplis [55], and elsewhere. The dimensions are summarized in the table below.

Pipe	ID	OD	wall thickness
1/2 *in* NPS	0.622 *in* 15.80 *mm*	0.840 *in* 21.34 *mm*	0.109 *in* 2.769 *mm*
15 DN	(see above)		
3/4 *in* NPS	0.824 *in* 20.93 *mm*	1.050 *in* 26.67 *mm*	0.113 *in* 2.870 *mm*
20 DN	(see above)		
1 *in* NPS	1.049 *in* 26.64 *mm*	1.315 *in* 33.40 *mm*	0.133 *in* 3.378 *mm*
25 DN	(see above)		