

CHAPTER 4

Section 4-2

4-1. a) $P(1 < X) = \int_1^{\infty} e^{-2x} dx = (-e^{-2x}) \Big|_1^{\infty} = e^{-2} = 0.1353$

b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-2x} dx = (-e^{-2x}) \Big|_1^{2.5} = e^{-2} - e^{-5} = 0.1286$

c) $P(X = 3) = \int_3^3 e^{-2x} dx = 0$

d) $P(X < 4) = \int_0^4 e^{-2x} dx = (-e^{-2x}) \Big|_0^4 = 1 - e^{-8} = 0.9997$

e) $P(3 \leq X) = \int_3^{\infty} e^{-2x} dx = (-e^{-2x}) \Big|_3^{\infty} = e^{-6} = 0.0025$

f) $P(x < X) = \int_x^{\infty} e^{-2x} dx = (-e^{-2x}) \Big|_x^{\infty} = e^{-2x} = 0.10$

Then, $2x = -\ln(0.10) = 2.3 \Rightarrow x = 1.15$

g) $P(X \leq x) = \int_0^x e^{-2x} dx = (-e^{-2x}) \Big|_0^x = 1 - e^{-2x} = 0.10$

Then, $x = \frac{-\ln(0.9)}{2} = 0.0527$

4-2. a) $P(X < 2) = \int_0^2 \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256} \right) \Big|_0^2 = \left(\frac{3}{16} - \frac{1}{32} \right) - 0 = 0.1563$

b) $P(X < 9) = \int_0^8 \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256} \right) \Big|_0^8 = (3 - 2) - 0 = 1$

c) $P(2 < X < 4) = \int_2^4 \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256} \right) \Big|_2^4 = \left(\frac{3}{4} - \frac{1}{4} \right) - \left(\frac{3}{16} - \frac{1}{32} \right) = 0.3438$

d) $P(X > 6) = \int_6^8 \frac{3(8x - x^2)}{256} dx = \left(\frac{3x^2}{64} - \frac{x^3}{256} \right) \Big|_6^8 = (3 - 2) - \left(\frac{27}{16} - \frac{27}{32} \right) = 0.1563$

e) $P(X < x) = \int_0^x \frac{3(8u - u^2)}{256} du = \left(\frac{3u^2}{64} - \frac{u^3}{256} \right) \Big|_0^x = \left(\frac{3x^2}{64} - \frac{x^3}{256} \right) - 0 = 0.95$

Then, $x^3 - 12x^2 + 243.2 = 0$, and $x = 6.9172$

$$4-3. \text{ a) } P(X < 0) = \int_{-\pi/2}^0 \cos x dx = \sin x \Big|_{-\pi/2}^0 = 0 - (-1) = 1$$

$$\text{b) } P(X < -\pi/4) = \int_{-\pi/2}^{-\pi/4} \cos x dx = \sin x \Big|_{-\pi/2}^{-\pi/4} = -0.7071 - (-1) = 0.2929$$

$$\text{c) } P(-\pi/4 < X < \pi/4) = \int_{-\pi/4}^{\pi/4} \cos x dx = \sin x \Big|_{-\pi/4}^{\pi/4} = 0.7071 - (-0.7071) = 1.4142$$

$$\text{d) } P(X > -\pi/4) = \int_{-\pi/4}^{\pi/2} \cos x dx = \sin x \Big|_{-\pi/4}^{\pi/2} = 1 - (-0.7071) = 1.7071$$

$$\text{e) } P(X < x) = \int_{-\pi/2}^x \cos x dx = \sin x \Big|_{-\pi/2}^x = \sin x - (-1) = 0.95$$

Then, $\sin x = -0.05$, and $x = -0.05$ radians.

$$4-4. \text{ a) } P(X < 2) = \int_1^2 \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_1^2 = \left(\frac{-1}{4} \right) - (-1) = 0.75$$

$$\text{b) } P(X > 3) = \int_3^{\infty} \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_3^{\infty} = 0 - \left(\frac{-1}{9} \right) = 0.11$$

$$\text{c) } P(4 < X < 8) = \int_4^8 \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_4^8 = \left(\frac{-1}{64} \right) - \left(\frac{-1}{16} \right) = 0.0469$$

d) $P(X < 4 \text{ or } X > 8) = 1 - P(4 < X < 8)$. From part (c), $P(4 < X < 8) = 0.0469$. Therefore, $P(X < 4 \text{ or } X > 8) = 1 - 0.0469 = 0.9531$

$$\text{e) } P(X < x) = \int_1^x \frac{2}{x^3} dx = \left(\frac{-1}{x^2} \right) \Big|_1^x = \left(\frac{-1}{x^2} \right) - (-1) = 0.95$$

Then, $x^2 = 20$, and $x = 4.4721$

$$4-5. \text{ a) } P(X < 4) = \int_3^4 \frac{x}{7} dx = \frac{x^2}{14} \Big|_3^4 = \frac{4^2 - 3^2}{14} = 0.5 \text{ because } f_x(x) = 0 \text{ for } x < 3.$$

$$\text{b) } P(X > 3.5) = \int_{3.5}^5 \frac{x}{7} dx = \frac{x^2}{14} \Big|_{3.5}^5 = \frac{5^2 - 3.5^2}{14} = 0.9107 \text{ because } f_x(x) = 0 \text{ for } x > 5.$$

$$\text{c) } P(4 < X < 5) = \int_4^5 \frac{x}{7} dx = \frac{x^2}{14} \Big|_4^5 = \frac{5^2 - 4^2}{14} = 0.6429$$

$$\text{d) } P(X < 4.5) = \int_3^{4.5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_3^{4.5} = \frac{4.5^2 - 3^2}{14} = 0.8036$$

$$\text{e) } P(X > 4.5) + P(X < 3.5) = \int_{4.5}^5 \frac{x}{7} dx + \int_3^{3.5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{4.5}^5 + \frac{x^2}{14} \Big|_3^{3.5} = \frac{5^2 - 4.5^2}{14} + \frac{3.5^2 - 3^2}{14} = 0.5714$$

4-6. a) $P(1 < X) = \int_5^{\infty} e^{-(x-5)} dx = -e^{-(x-5)} \Big|_5^{\infty} = 1$, because $f_X(x) = 0$ for $x < 5$. This can also be

obtained from the fact that $f_X(x)$ is a probability density function for $5 < x$.

b) $P(2 \leq X \leq 5) = \int_5^5 e^{-(x-5)} dx = -e^{-(x-5)} \Big|_5^5 = 0$

c) $P(5 < X) = 1 - P(X \leq 5)$. From part b, $P(X \leq 5) = 0$. Therefore, $P(5 < X) = 1$.

d) $P(8 < X < 12) = \int_8^{12} e^{-(x-5)} dx = -e^{-(x-5)} \Big|_8^{12} = e^{-3} - e^{-7} = 0.0489$

e) $P(X < x) = \int_5^x e^{-(x-5)} dx = -e^{-(x-5)} \Big|_5^x = 1 - e^{-(x-5)} = 0.85$

Then, $x = 5 - \ln(0.15) = 6.897$

4-7. a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^1 2x^2 dx = \frac{2}{3} x^3 \Big|_{0.5}^1 = 0.6667 - 0.0833 = 0.5834$

c) $P(-0.5 \leq X \leq 0.5) = \int_{-0.5}^{0.5} 2x^2 dx = \frac{2}{3} x^3 \Big|_{-0.5}^{0.5} = 0.167$

d) $P(X < -2) = 0$

e) $P(X < 0 \text{ or } X > -0.5) = 1$

f) $P(x < X) = \int_x^1 2x^2 dx = \frac{2}{3} x^3 \Big|_x^1 = 0.667 - 0.667x^3 = 0.05$

Then, $x = 0.925$

4-8. a) $P(X > 5000) = \int_{5000}^{\infty} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_{5000}^{\infty} = e^{-5} = 0.0067$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_0^{1000} \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_0^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_0^x \frac{e^{-\frac{x}{1000}}}{1000} dx = -e^{-\frac{x}{1000}} \Big|_0^x = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and $x = -1000 \ln 0.9 = 105.36$.

4-9. a) $P(X > 25) = \int_{25}^{25.25} 2.5dx = 2.5x \Big|_{25}^{25.25} = 0.625$

b) $P(X > x) = 0.85 = \int_x^{25.25} 2.5dx = 2.5x \Big|_x^{25.25} = 63.125 - 2.5x$

Then, $2.5x = 62.275$ and $x = 31.14$.

4-10. a) $P(X < 74.7) = \int_{74.6}^{74.7} 1.3dx = 1.3x \Big|_{74.6}^{74.7} = 0.13$

b) $P(X < 74.8 \text{ or } X > 75.2) = P(X < 74.8) + P(X > 75.2)$ because the two events are mutually exclusive.

$$\begin{aligned} P(X < 74.8) &= \int_{74.6}^{74.8} 1.3dx & P(X < 75.2) &= \int_{75.2}^{75.4} 1.3dx \\ &= 1.3x \Big|_{74.6}^{74.8} & &= 1.3x \Big|_{75.2}^{75.4} \\ &= 1.3 \times 0.2 & &= 1.3 \times 0.2 \\ &= 0.26 & &= 0.26 \end{aligned}$$

The result is $0.26 + 0.26 = 0.52$.

c) $P(74.7 < X < 75.3) = \int_{74.7}^{75.3} 1.3dx = 1.3x \Big|_{74.7}^{75.3} = 1.3(0.6) = 0.780$

4-11. a) $P(X < 2.25 \text{ or } X > 2.75) = P(X < 2.25) + P(X > 2.75)$ because the two events are mutually exclusive. Then, $P(X < 2.25) = 0$ and

$$P(X > 2.75) = \int_{2.75}^{2.9} 2dx = 2(0.15) = 0.30.$$

b) If the probability density function is centered at 2.60 meters, then $f_X(x) = 2$ for $2.3 < x < 2.9$ and all rods will meet specifications.

4-12. a) $P(X < 90) = 0$ because the pdf is not defined in the range $(-\infty, 90)$.

b)

$$\begin{aligned} P(100 < X \leq 300) &= \int_{100}^{300} (-5.56 \times 10^{-4} + 5.56 \times 10^{-6} x) dx = (-5.56 \times 10^{-4} x + 2.78 \times 10^{-6} x^2) \Big|_{100}^{300} \\ &= (-5.56 \times 10^{-4} \times 300 + 2.78 \times 10^{-6} \times 300^2) - (-5.56 \times 10^{-4} \times 100 + 2.78 \times 10^{-6} \times 100^2) \\ &= 0.111 \end{aligned}$$

c)

$$\begin{aligned} P(X > 800) &= \int_{800}^{1000} (4.44 \times 10^{-3} - 4.44 \times 10^{-6} x) dx = (4.44 \times 10^{-3} x - 2.22 \times 10^{-6} x^2) \Big|_{800}^{1000} \\ &= (4.44 \times 10^{-3} \times 1000 - 2.22 \times 10^{-6} \times 1000^2) - (4.44 \times 10^{-3} \times 800 - 2.22 \times 10^{-6} \times 800^2) \\ &= 0.0888 \cong 0.09 \end{aligned}$$

d) Find a such that $P(X > a) = 0.1$

$$P(X > a) = \int_a^{1000} (4.44 \times 10^{-3} - 4.44 \times 10^{-6} x) dx = (4.44 \times 10^{-3} x - 2.22 \times 10^{-6} x^2) \Big|_a^{1000} = 0.1$$

$$(4.44 \times 10^{-3} \times 10^3 - 2.22 \times 10^{-6} \times 10^6) - (4.44 \times 10^{-3} \times a - 2.22 \times 10^{-6} \times a^2) = 0.1$$

$$(2.22) - (4.44 \times 10^{-3} \times a - 2.22 \times 10^{-6} \times a^2) = 0.1$$

Then, $a \cong 787.76$

Section 4-3

4-13. a) $P(X < 2.8) = P(X \leq 2.8)$ because X is a continuous random variable.

Then, $P(X < 2.8) = F(2.8) = 0.3(2.8) = 0.84$.

b) $P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - 0.3(1.5) = 0.55$

c) $P(X < -2) = F_X(-2) = 0$

d) $P(X > 6) = 1 - F_X(6) = 0$

4-14. a) $P(X < 1.7) = P(X \leq 1.7) = F_X(1.7)$ because X is a continuous random variable. Then,

$$F_X(1.7) = 0.2(1.7) + 0.5 = 0.84$$

b) $P(X > -1.5) = 1 - P(X \leq -1.5) = 1 - 0.2 = 0.8$

c) $P(X < -2) = 0.1$

d) $P(-1 < X < 1) = P(-1 < X \leq 1) = F_X(1) - F_X(-1) = 0.7 - 0.3 = 0.4$

4-15. Now, $f(x) = e^{-2x}$ for $0 < x$ and $F_X(x) = \int_0^x e^{-2x} dx = -e^{-2x} \Big|_0^x = 1 - e^{-2x}$

$$\text{for } 0 < x. \text{ Then, } F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-2x}, & x > 0 \end{cases}$$

4-16. Now, $f(x) = \frac{3(8x - x^2)}{256}$ for $0 < x < 8$ and

$$F_X(x) = \int_0^x \frac{3(8u - u^2)}{256} du = \left(\frac{3u^2}{64} - \frac{u^3}{256} \right) \Big|_0^x = \frac{3x^2}{64} - \frac{x^3}{256} \text{ for } 0 < x.$$

$$\text{Then, } F_X(x) = \begin{cases} 0, & x \leq 0 \\ \frac{3x^2}{64} - \frac{x^3}{256}, & 0 \leq x < 8 \\ 1, & x \geq 8 \end{cases}$$

4-17. Now, $f(x) = \cos x$ for $-\pi/2 < x < \pi/2$ and

$$F_X(x) = \int_{-\pi/2}^x \cos u du = \sin x \Big|_{-\pi/2}^x = \sin x + 1$$

$$\text{Then, } F_X(x) = \begin{cases} 0, & x \leq -\pi/2 \\ \sin x + 1, & -\pi/2 \leq x < \pi/2 \\ 1, & x \geq \pi/2 \end{cases}$$

4-18. Now, $f(x) = \frac{2}{x^3}$ for $x > 1$ and

$$F_X(x) = \int_1^x \frac{2}{u^3} du = \left. \left(\frac{-1}{u^2} \right) \right|_1^x = \left(\frac{-1}{x^2} \right) + 1$$

$$\text{Then, } F_X(x) = \begin{cases} 0, & x \leq 1 \\ 1 - \frac{1}{x^2}, & x > 1 \end{cases}$$

4-19. Now, $f(x) = x/7$ for $3 < x < 5$ and $F_X(x) = \int_3^x \frac{u}{7} du = \left. \frac{u^2}{14} \right|_3^x = \frac{x^2 - 9}{14}$

$$\text{for } 0 < x. \text{ Then, } F_X(x) = \begin{cases} 0, & x < 3 \\ \frac{x^2 - 9}{14}, & 3 \leq x < 5 \\ 1, & x \geq 5 \end{cases}$$

4.20. Now, $f(x) = \frac{e^{-x/1000}}{1000}$ for $0 < x$ and

$$F_X(x) = \frac{1}{1000} \int_0^x e^{-y/1000} dy = \left. -e^{-y/1000} \right|_0^x = 1 - e^{-x/1000} \text{ for } 0 < x.$$

$$\text{Then, } F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$$

$$P(X > 3000) = 1 - P(X \leq 3000) = 1 - F(3000) = e^{-3000/1000} = 0.5$$

4-21. Now, $f(x) = 2$ for $2.3 < x < 2.9$ and $F(x) = \int_{2.3}^x 2 dy = 2x - 4.6$

for $2.3 < x < 2.9$. Then,

$$F(x) = \begin{cases} 0, & x < 2.3 \\ 2x - 4.6, & 2.3 \leq x < 2.9 \\ 1, & 2.9 \leq x \end{cases}$$

$P(X > 2.7) = 1 - P(X \leq 2.7) = 1 - F(2.7) = 1 - 0.8 = 0.2$ because X is a continuous random variable.

4-22. Now, $f(x) = \frac{e^{-x/10}}{10}$ for $0 < x$ and

$$F_X(x) = 1/10 \int_0^x e^{-x/10} dx = -e^{-x/10} \Big|_0^x = 1 - e^{-x/10} \text{ for } 0 < x.$$

$$\text{Then, } F_X(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x/10}, & x > 0 \end{cases}$$

a) $P(X < 30) = F(30) = 1 - e^{-3} = 1 - 0.0498 = 0.9502$

b) $1/10 \int_{15}^{30} e^{-x/10} dx = e^{-1.5} - e^{-3} = 0.173343$

c) $P(X_1 > 40) + P(X_1 < 40 \text{ and } X_2 > 40) = e^{-4} + (1 - e^{-4}) e^{-4} = 0.0363$

d) $P(20 < X < 40) = F(40) - F(20) = e^{-2} - e^{-4} = 0.117$

4-23. $F(x) = \int_0^x 1.5x dx = \frac{1.5x^2}{2} \Big|_0^x = 0.75x^2$ for $0 < x < 2$. Then,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.75x^2, & 0 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

4-24. $f_X(x) = 2e^{-3x}$, $x > 0$

$$4-25. \quad f(x) = \begin{cases} 0.25, & 0 < x < 4 \\ 0.04, & 4 \leq x < 9 \end{cases}$$

$$4-26. \quad f_X(x) = \begin{cases} 0.5, & -2 < x < 1 \\ 0.75, & 1 \leq x < 1.5 \end{cases}$$

Section 4-4

$$4-27. \quad E(X) = \int_0^4 0.3x dx = 0.3 \frac{x^2}{2} \Big|_0^4 = 2.4$$

$$V(X) = \int_0^4 0.3(x-2.4)^2 dx = 0.3 \frac{(x-2.4)^3}{3} \Big|_0^4 = 0.4096 + 1.3824 = 1.792$$

$$4-28. \quad E(X) = \int_0^4 0.13x^2 dx = 0.13 \frac{x^3}{3} \Big|_0^4 = 2.7733$$

$$V(X) = \int_0^4 0.13x(x - \frac{8}{3})^2 dx = 0.13 \int_0^4 (x^3 - \frac{16}{3}x^2 + \frac{64}{9}x) dx$$

$$= 0.13 \left(\frac{x^4}{4} - \frac{16}{3} \frac{x^3}{3} + \frac{64}{9} \cdot \frac{1}{2} x^2 \right) \Big|_0^4 = 0.92444$$

$$4-29. \quad E(X) = \int_{-1}^1 2x^3 dx = 2 \frac{x^4}{4} \Big|_{-1}^1 = 0$$

$$V(X) = \int_{-1}^1 2x^2(x-0)^2 dx = 2 \int_{-1}^1 x^4 dx$$

$$= 2 \frac{x^5}{5} \Big|_{-1}^1 = 0.8$$

$$4-30. \quad E(X) = \int_3^5 x \frac{x}{4} dx = \frac{x^3}{12} \Big|_3^5 = \frac{5^3 - 3^3}{12} = 8.167$$

$$V(X) = \int_3^5 (x - 8.167)^2 \frac{x}{4} dx = \int_3^5 \left(\frac{x^3}{4} - \frac{16.334x^2}{4} + \frac{66.6999x}{4} \right) dx$$

$$= \frac{1}{4} \left(\frac{x^4}{4} - \frac{18.334x^3}{3} + \frac{66.6999x^2}{2} \right) \Big|_3^5 = 34.0055$$

$$4-31. \quad E(X) = \int_0^8 x \frac{3(8x - x^2)}{256} dx = \left(\frac{x^3}{32} - \frac{3x^4}{1024} \right) \Big|_0^8 = (16 - 12) - 0 = 4$$

$$V(X) = \int_0^8 (x-4)^2 \frac{3(8x - x^2)}{256} dx = \int_0^8 \left(\frac{-3x^4}{256} + \frac{3x^3}{16} - \frac{15x^2}{16} + \frac{3x}{2} \right) dx$$

$$V(X) = \left(\frac{-3x^5}{1280} + \frac{3x^4}{64} - \frac{5x^3}{16} + \frac{3x^2}{4} \right) \Big|_0^8 = \left(\frac{-384}{5} + 192 - 160 + 48 \right) = 3.2$$

$$4-32. \quad E(X) = \int_{2.3}^{2.9} 2x dx = x^2 \Big|_{2.3}^{2.9} = 2.9^2 - 2.3^2 = 3.12$$

$$V(X) = \int_{2.3}^{2.9} 2(x - 3.12)^2 dx = \int_{2.3}^{2.9} (2x^2 - 12.48x + 19.4688) dx = \left(\frac{2}{3}x^3 - 6.24x^2 + 19.4688x \right) \Big|_{2.3}^{2.9} = 0.36$$

$$4-33. \quad E(X) = \int_1^{\infty} x * \frac{3}{2} x^{-3} dx = -\frac{3}{2} x^{-1} \Big|_1^{\infty} = \frac{3}{2}$$

4-34. a)

$$E(X) = \int_{1710}^{1720} x \cdot 0.1 dx = 0.05x^2 \Big|_{1710}^{1720} = 1715$$

$$V(X) = \int_{1710}^{1720} (x-1715)^2 \cdot 0.1 dx = 0.1 \frac{(x-1715)^3}{3} \Big|_{1710}^{1720} = 8.333$$

$$\text{Therefore, } \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1705 < X < 1715) = P(1710 < X < 1715) = \int_{1710}^{1715} 0.1 dx = 0.1x \Big|_{1710}^{1715} = 0.5$$

4-35. a) $E(X) = \int_{100}^{120} x \frac{500}{x^2} dx = 500 \ln x \Big|_{100}^{120} = 91.16$

$$V(X) = \int_{100}^{120} (x-91.16)^2 \frac{500}{x^2} dx = 500 \int_{100}^{120} \left(1 - \frac{2(91.16)}{x} + \frac{(91.16)^2}{x^2} \right) dx$$

$$= 500(x - 182.32 \ln x - 91.16^2 x^{-1}) \Big|_{100}^{120} = 304.688$$

b.) Average cost per part = \$0.50*91.16 = \$45.58.

4-36. (a) $E(X) = \int_{0.5}^{35} x f(x) dx = \int_{0.5}^{35} \frac{70}{69x} dx = \frac{70}{69} \ln x \Big|_{0.5}^{35} = 4.3101$

$$E(X^2) = \int_{0.5}^{35} x^2 f(x) dx = \int_{0.5}^{35} \frac{70}{69} dx = 35$$

$$\text{Var}(X) = E(X^2) - (EX)^2 = 35 - 18.5770 = 16.423$$

(b) $3 * 4.3101 = 12.9303$

(c) $P(X > 25) = \int_{25}^{35} f(x) dx = 0.0116$

4-37. a) $E(X) = \int_5^{\infty} x 10e^{-10(x-5)} dx.$

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)} dx$, we obtain

$$E(X) = -xe^{-10(x-5)} \Big|_5^{\infty} + \int_5^{\infty} e^{-10(x-5)} dx = 5 - \frac{e^{-10(x-5)}}{10} \Big|_5^{\infty} = 5.1$$

Now, $V(X) = \int_5^{\infty} (x-5.1)^2 10e^{-10(x-5)} dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$$dv = 10e^{-10(x-5)}, \text{ we obtain } V(X) = -(x-5.1)^2 e^{-10(x-5)} \Big|_5^{\infty} + 2 \int_5^{\infty} (x-5.1)e^{-10(x-5)} dx.$$

From the definition of $E(X)$ the integral above is recognized to equal 0. Therefore,
 $V(X) = (5 - 5.1)^2 = 0.01$.

$$b) P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)} dx = -e^{-10(x-5)} \Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$$

Section 4-5

4-38. a) $E(X) = (5.5 + 2.5)/2 = 4$

$$V(X) = \frac{(5.5 - 2.5)^2}{12} = 0.75, \text{ and } \sigma_x = \sqrt{0.75} = 0.866$$

$$b) P(X < 2.5) = \int_{2.5}^{2.5} 0.25 dx = 0.25x \Big|_{2.5}^{2.5} = 0$$

$$c) F(x) = \begin{cases} 0, & x < 2.5 \\ 0.25x - 0.375, & 2.5 \leq x < 5.5 \\ 1, & 5.5 \leq x \end{cases}$$

4-39. a) $E(X) = (-2+3)/2 = 0.5$,

$$V(X) = \frac{(3 - (-2))^2}{12} = 2.083, \text{ and } \sigma_x = 1.443$$

$$b) P(-x < X < x) = \int_{-x}^x \frac{1}{5} dt = 0.2t \Big|_{-x}^x = 0.2(2x) = 0.4x,$$

$0.4x = 0.9$. Therefore, x should equal 2.25.

$$c) F(x) = \begin{cases} 0, & x < -2 \\ 0.2x + 0.4, & -2 \leq x < 3 \\ 1, & 3 \leq x \end{cases}$$

4.40. a) $f(x) = 2.0$ for $49.75 < x < 50.25$.

$$E(X) = (50.25 + 49.75)/2 = 50.0,$$

$$V(X) = \frac{(50.25 - 49.75)^2}{12} = 0.0208, \text{ and } \sigma_x = 0.144.$$

$$b) F(x) = \int_{49.75}^x 2.0 dy \text{ for } 49.75 < x < 50.25. \text{ Therefore,}$$

$$F(x) = \begin{cases} 0, & x < 49.75 \\ 2x - 99.5, & 49.75 \leq x < 50.25 \\ 1, & 50.25 \leq x \end{cases}$$

$$c) P(X < 50.1) = F(50.1) = 2(50.1) - 99.5 = 0.7$$