4

4.1 (a) System = Ball (1) + Water (2)

Energy balance: $M_1\hat{U}_1^f + M_2\hat{U}_2^f - M_1\hat{U}_1^i - M_2\hat{U}_2^i = 0$

$\Rightarrow M_1 C_{V,1}\left(T_1^f - T_1^i\right) + M_2 C_{V,2}\left(T_2^f - T_2^i\right) = 0$; also $T_1^f - T_2^f$. Thus

$$T^f = \frac{M_1 C_{V,1} T_1^i + M_2 C_{V,2} T_2^i}{M_1 C_{V,1} + M_2 C_{V,2}} = \frac{5\times10^3\times0.5\times75+12\times10^3\times4.2\times5}{5\times10^3\times0.5+12\times10^3\times4.2}$$

$= 8.31°\text{C}$

[Note: Since only ΔT s are involved, $°\text{C}$ were used instead of K)].

(b) For solids and liquids we have (eqn. 4.4-6). That $\Delta S = M\int C_P \frac{dT}{T} = MC_P \ln\frac{T_2}{T_1}$ for the case in which C_P is a constant. Thus

$$\text{Ball: } \Delta S = 5\times10^3 \text{ g}\times 0.5\frac{\text{J}}{\text{g}\cdot\text{K}}\times\ln\left\{\frac{8.31+273.15}{75+273.15}\right\} = -531.61\frac{\text{J}}{\text{K}}$$

$= -531.61$ s

$$\text{Water: } \Delta S = 12\times10^3 \text{ g}\times 4.2\frac{\text{J}}{\text{g}\cdot\text{K}}\times\ln\left\{\frac{8.31+273.15}{5+273.15}\right\} = +596.22\frac{\text{J}}{\text{K}}$$

and

$$\Delta S(\text{Ball}+\text{Water}) = 596.22 - 531.61\frac{\text{J}}{\text{K}} = 64.61\frac{\text{J}}{\text{K}}$$

Note that the system Ball + Water is isolated. Therefore

$$\Delta S = S_{\text{gen}} = 64.61\frac{\text{J}}{\text{K}}$$

4.2 Energy balance on the combined system of casting and the oil bath

$M_c C_{V,c}\left(T^f - T_c^i\right) + M_o C_{V,o}\left(T^f - T_o^i\right) = 0$ since there is a common final temperature.

$$20 \text{ kg}\times0.5\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left(T^f - 450\right)\text{K} + 150 \text{ kg}\times2.6\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left(T^f - 450\right)\text{K} = 0$$

This has the solution T^f = 60ºC = 313.15 K

Since the final temperature is known, the change in entropy of this system can be calculated from $\Delta S = 20\times0.5\times\ln\left(\frac{273.15+60}{273.15+450}\right) + 150\times2.6\times\ln\left(\frac{273.15+60}{273.15+50}\right) = 4.135\frac{\text{kJ}}{\text{K}}$

4.3 Closed system energy and entropy balances

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_s - P\frac{dV}{dt};\quad \frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}};$$

Thus, in general $\dot{Q} = T\frac{dS}{dt} - T\dot{S}_{\text{gen}}$ and

$$\dot W_s = \frac{dU}{dt} - \dot Q + P\frac{dV}{dt} = \frac{dU}{dt} - T\frac{dS}{dt} + T\dot S_{gen} + P\frac{dV}{dt}$$

Reversible work: $\dot W_s^{\text{Rev}} = \dot W_s^{\text{Rev}}\left(\dot S_{\text{gen}} = 0\right) = \frac{dU}{dt} - T\frac{dS}{dt} + P\frac{dV}{dt}$

(a) System at constant U & $V \Rightarrow \frac{dU}{dt} = 0$ and $\frac{dV}{dt} = 0$

$$\dot W_s\left(\dot S_{gen} = 0\right) = \dot W_S^{\text{Rev}} = -T\frac{dS}{dt}$$

(b) System at constant S & $P \Rightarrow \frac{dS}{dt} = 0$ and $\frac{dP}{dt} = 0 \Rightarrow P\frac{dV}{dt} = \frac{d}{dt}(PV)$

so that

$$\dot W_s\left(\dot S_{gen} = 0\right) = \dot W_S^{\text{rev}} = \frac{dU}{dt} + \frac{d}{dt}(PV) = \frac{d}{dt}(U + PV) = \frac{dH}{dt}$$

4.4

700 bar, 600°C  10 bar, T = ?

Steady-state balance equations

$$\frac{dM}{dt} = 0 = \dot M_1 + \dot M_2$$

$$\frac{dU}{dt} = 0 = \dot M_1 \hat H_1 + \dot M_2 \hat H_2 + \cancelto{0}{\dot Q} + \cancelto{0}{\dot W_s} - P\cancelto{0}{\frac{dV}{dt}} = \dot M_1 \hat H_1 + \dot M_2 \hat H_2$$

or $\hat H_1 = \hat H_2$

Drawing a line of constant enthalpy on Mollier Diagram we find, at $P = 10$ bar, $T \cong 308^\circ\text{C}$

At 700 bar and 600° C	At 10 bar and 308° C
$\hat V = 0.003973\ \text{m}^3/\text{kg}$	$\hat V \approx 0.2618\ \text{m}^3/\text{kg}$
$\hat H = 3063\ \text{kJ/kg}$	$\hat H \approx 3063\ \text{kJ/kg}$
$\hat S = 5.522\ \text{kJ/kg K}$	$\hat S = 7.145\ \text{kJ/kg K}$

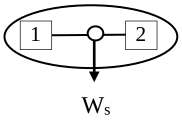
Also

$$\frac{dS}{dt} = 0 = \dot M_1 \hat S_1 + \dot M_2 \hat S_2 + \frac{\cancelto{0}{\dot Q}}{T} + \dot S_{\text{gen}} = 0$$

$$\Rightarrow \dot S_{\text{gen}} = \dot M_1\left(\hat S_2 - \hat S_1\right) \text{ or } \frac{\dot S_{\text{gen}}}{\dot M_1} = \hat S_2 - \hat S_1 = 7.145 - 5.522 = 1.623\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

4.5

System



Energy balance

$$\Delta U = \left(U_2^f - U_2^i\right) + \left(U_1^f - U_1^i\right) = \cancelto{\text{adiabatic}}{\dot Q} + W_S - \cancelto{\text{constant volume}}{\int PdV}$$

$$W_s = MC_p\left(T_2^f - T_2^i\right) + MC_p\left(T_1^f - T_1^i\right) = MC_p\left[\left(T_2^f - T_2^i\right) + \left(T_1^f - T_1^i\right)\right]$$

$$\text{but } T_1^f = T_2^f = T^f \Rightarrow \frac{W_s}{MC_P} = \left[2T^f - T_1^i - T_2^i\right]$$

Entropy balance

$$\Delta S = \left(S_2^f - S_2^i\right) + \left(S_1^f - S_1^i\right) = \overset{\text{adiabatic}}{\cancelto{0}{\int \frac{\dot{Q}}{T}\,dt}} + \underset{\text{0 for maximum work}}{\cancel{S_{\text{gen}}}}$$

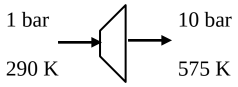
$$\left(S_2^f - S_2^i\right) + \left(S_1^f - S_1^i\right) = 0 = MC_P \ln\frac{T_2^f}{T_2^i} + MC_P \ln\frac{T_1^f}{T_1^i}$$

$$\text{or } \ln\left\{\frac{T_2^f T_1^f}{T_2^i T_1^i}\right\} = 0\,;\ T_1^f T_2^f = T_1^i T_2^i;\ \text{but } T_1^f = T_2^f = T^f$$

$$\Rightarrow \left(T^f\right)^2 = \left(T_1^i T_2^i\right) \text{ or } T^f = \sqrt{T_1^i T_2^i} \text{ and}$$

$$\frac{W_s}{MC_P} = \left[2T^f - T_1^i - T_2^i\right] = \left[2\sqrt{T_1^i T_2^i} - T_1^i - T_2^i\right]$$

4.6



(a) Entropy change per mole of gas

$$\Delta \underline{S} = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad \text{eqn. (4.4-3)}$$

$$\text{Thus } \Delta S = 29.3\frac{\text{J}}{\text{mol K}}\ln\frac{575}{290} - 8.314\frac{\text{J}}{\text{mol K}}\ln\frac{10}{1} = 0.9118\frac{\text{J}}{\text{mol K}}$$

(b) System = contents of turbine (steady-state system)

$$\text{Mass balance } \frac{dN}{dt} = 0 = \dot{N}_1 + \dot{N}_2 \Rightarrow -\dot{N}_2 = \dot{N}_1 = \dot{N}$$

$$\text{Energy balance } \frac{dU}{dt} = 0 = \dot{N}_1\underline{H}_1 + \dot{N}_2\underline{H}_2 + \cancelto{0}{\dot{Q}} + \dot{W}_s - P\cancelto{0}{\frac{dV}{dt}}$$

$$\dot{W}_s = \dot{N}(\underline{H}_2 - \underline{H}_1) = \dot{N}C_P(T_2 - T_1)$$

$$W = \frac{\dot{W}_s}{\dot{N}} = C_P(T_2 - T_1) = 29.3\frac{\text{J}}{\text{mol K}} \times (575 - 290)\text{K}$$

$$= 8350.5\frac{\text{J}}{\text{mol}}$$

(c) In Illustration 4.5-1, $W = 7834.8\ \text{J/mol}$ because of irreversiblitities $(\Delta S \neq 0)$, more work is done on the gas here. What happens to this additional energy input? It appears as an increase of the internal energy (temperature) of the gas.

4.7 Heat loss from metal block

$$\frac{dU}{dt} = C_P\frac{dT}{dt} = \dot{Q}$$

$$-\dot{W} = \frac{T-T_2}{T}\dot{Q}(-1)\begin{cases}\dot{Q} = \text{heat out of metal}\\ -\dot{Q} = \text{heat into heat engine}\end{cases}$$

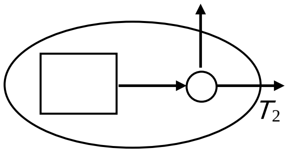
$$C_P\frac{dT}{dt}\frac{(T-T_2)}{T} = -\dot{W} \Rightarrow -\int_0^t \dot{W}dt = C_P\int_{T_1}^{T_2}\left(1-\frac{T_2}{T}\right)dT$$

$$-W = C_P(T_2-T_1) - C_P\cdot T_2\ln\frac{T_2}{T_1} = C_P\left[(T_2-T_1) - T_2\ln\frac{T_2}{T_1}\right]$$

$$-W = C_P T_2\left[\left(1-\frac{T_1}{T_2}\right)-\ln\frac{T_2}{T_1}\right]$$

$$Q = \int_{T_1}^{T_2} C_P dT = C_P(T_2-T_1) = C_P T_2\left(1-\frac{T_1}{T_2}\right)$$

Alternate way to solve the problem



System is the metal block + heat engine (closed)

E.B.: $$\frac{dU}{dt} = C_P\frac{dT}{dt} = \dot{Q}+\dot{W}$$

S.B.: $$\frac{dS}{dt} = \frac{\dot{Q}}{T^2} + \cancel{S_{gen}}$$

0 for maximum work

$$\dot{Q} = T_2\frac{dS}{dt}\ ;\ \frac{dU}{dt} = T_2\frac{dS}{dt}+\dot{W}\ ;\ dU = C_P dT\ ;\ dS = \frac{C_P}{T}dT$$

$$\dot{W} = \frac{dU}{dt} - T_2\frac{dS}{dt} = C_P dT - T_2\frac{C_P}{T}dT = C_P\left(1-\frac{T_2}{T}\right)dT$$

$$W = \int\dot{W}dt = \int_{T_1}^{T_2} C_P\left(1-\frac{T_2}{T}\right)dT = C_P\int_{T_1}^{T_2}\left(1-\frac{T_2}{T}\right)dT$$

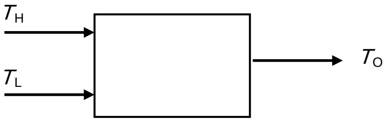
$$W = C_P(T_2-T_1) - T_2 C_P\ln\frac{T_2}{T_1} = C_P T_2\left[\left(1-\frac{T_1}{T_2}\right)-\ln\frac{T_2}{T_1}\right]$$

4.8 This problem is not well posed since we do not know exactly what is happening. There are several possibilities:

(1) Water contact is very short so neither stream changes T very much. In this case we have the Carnot efficiency

$$\eta = \frac{-W}{Q} = \frac{T_{\text{high}} - T_{\text{low}}}{T_{\text{high}}} = \frac{22}{27+273} = \frac{22}{300} = 0.0733 = 7.33\%$$

(2) Both warm surface water (27°C) and cold deep water (5°C) enter work producing device, and they leave at a common temperature.



M.B.: $\frac{dM}{dt} = 0 = \dot{M}_\text{H} + \dot{M}_\text{L} + \dot{M}_0 \Rightarrow \dot{M}_0 = -\left(\dot{M}_\text{H} + \dot{M}_\text{L}\right)$

E.B.: $\frac{dU}{dt} = 0 = \dot{M}_\text{H}\hat{H}_\text{H} + \dot{M}_\text{L}\hat{H}_\text{L} + \dot{M}_0\hat{H}_0 + \dot{W} = 0$

$$\begin{aligned}\dot{W} &= -\dot{M}_\text{H}\hat{H}_\text{H} - \dot{M}_\text{L}\hat{H}_\text{L} + \left(\dot{M}_\text{H} + \dot{M}_\text{L}\right)\hat{H}_0\\ &= \dot{M}_\text{H}\left(\hat{H}_0 - \hat{H}_\text{H}\right) + \dot{M}_\text{L}\left(\hat{H}_0 - \hat{H}_\text{L}\right)\\ &= \dot{M}_\text{H}C_\text{P}(T_0 - T_\text{H}) + \dot{M}_\text{L}C_\text{P}(T_0 - T_\text{L})\end{aligned}$$

S.B.: $\frac{dS}{dt} = 0 = \dot{M}_\text{H}\hat{S}_\text{H} + \dot{M}_\text{L}\hat{S}_\text{L} + M_0\hat{S}_0 + \cancelto{0}{\frac{\dot{Q}}{T}} + \cancelto{0}{S_\text{gen}}$

$$\dot{M}_\text{H}\hat{S}_\text{H} + \dot{M}_\text{L}\hat{S}_\text{L} - \left(\dot{M}_\text{H} + \dot{M}_\text{L}\right)\hat{S}_0 = 0$$

$$\dot{M}_\text{H}\left(\hat{S}_\text{H} - \hat{S}_0\right) + \dot{M}_\text{L}\left(\hat{S}_\text{L} - \hat{S}_0\right) = 0 \Rightarrow \dot{M}_\text{H}C_\text{P}\ln\frac{T_\text{H}}{T_0} + \dot{M}_\text{L}C_\text{P}\ln\frac{T_\text{L}}{T_0} = 0$$

$$\left(\frac{T_\text{H}}{T_0}\right)^{\dot{M}_\text{H}}\left(\frac{T_\text{L}}{T_0}\right)^{\dot{M}_\text{L}} = 1 \text{ or } T_\text{H}^{\dot{M}_\text{H}}T_\text{L}^{\dot{M}_\text{L}} = T_0^{\dot{M}_\text{H}+\dot{M}_\text{L}}$$

$$T_0 = T_\text{H}^{\dot{M}_\text{H}/(\dot{M}_\text{H}+\dot{M}_\text{L})}T_\text{L}^{\dot{M}_\text{L}/(\dot{M}_\text{H}+\dot{M}_\text{L})}$$

From this can calculate T_0. Then

$$\dot{W} = \dot{M}_\text{H}C_\text{P}(T_0 - T_\text{H}) + \dot{M}_\text{L}C_\text{P}(T_0 - T_\text{L})$$

This can be used for any flow rate ratio.

(3) Suppose very large amount of surface water is contacted with a small amount of deep water, i.e., $\dot{M}_\text{H} >> \dot{M}_\text{L}$. Then $T_0 \sim T_\text{H}$

$$\dot{W} = \dot{M}_\text{H}C_\text{P}(T_\text{H} - T_\text{H}) + \dot{M}_\text{L}C_\text{P}(T_\text{H} - T_\text{L}) \sim \dot{M}_\text{L}C_\text{P}(T_\text{H} - T_\text{L})$$

(4) Suppose very large amount of deep water is contacted with a small amount of surface water, i.e., $\dot{M}_\text{H} << \dot{M}_\text{L}$, $T_0 \sim T_\text{L}$.

$$\dot{W} = \dot{M}_\text{H}C_\text{P}(T_\text{L} - T_\text{H}) + \dot{M}_\text{L}C_\text{P}(T_\text{L} - T_\text{L}) \sim \dot{M}_\text{H}C_\text{P}(T_\text{L} - T_\text{H})$$

4.9 System = contents of the turbine. This is a steady-state, adiabatic, constant volume system.

(a) Mass balance $\frac{dM}{dt} = 0 = \dot{M}_1 + \dot{M}_2$ or $\dot{M}_2 = -\dot{M}_1$

Energy balance

$$\frac{dU}{dt} = 0 = \dot{M}_1\hat{H}_1 + \dot{M}_2\hat{H}_2 + \cancelto{\text{adiabatic}}{\dot{Q}} + \dot{W}_s - P\cancelto{\text{constant volume}}{\frac{dV}{dt}}$$

Entropy balance

$$\frac{dS}{dt} = 0 = \dot{M}_1\hat{S}_1 + \dot{M}_2\hat{S}_2 + \cancel{\frac{\dot{Q}}{T}} + \underset{\text{0, by problem statement}}{\cancel{S_\text{gen}}}$$

Thus

$\dot{M}_2 = -\dot{M}_1 = -4500\,\text{kg/h}$ M.B.

$\dot{W}_S = -\dot{M}_1\left(\hat{H}_1 - \hat{H}_2\right)$ E.B.

$\hat{S}_2 = \hat{S}_1$ S.B.

$$\begin{array}{lll} \text{State } 1 & T_1 = 500^\circ\text{C} & \xrightarrow[\text{Tables}]{\text{Steam}} & \hat{H}_1 = 3422.2 \ \text{kJ/kg} \\ & P_1 = 60 \ \text{bar} & & \hat{S}_1 = 6.8803 \ \text{kJ/kg} \end{array}$$

$$\begin{array}{lll} \text{State } 2 & P_2 = 10 \ \text{bar} & \xrightarrow[\text{Tables}]{\text{Steam}} & T_2 \cong 240.4^\circ\text{C} \\ & \hat{S}_2 = \hat{S}_1 = 6.8803 \dfrac{\text{kJ}}{\text{kgK}} & & \hat{H}_2 \approx 2920.5 \ \text{kJ/kg} \end{array}$$

$$\dot{W}_s = 4500 \frac{\text{kg}}{\text{h}} \times (2920.5 - 3422.2) \frac{\text{kJ}}{\text{kg}} = -2257650 \frac{\text{kJ}}{\text{h}} = -627.1 \ \text{kW}$$

(b) Same exit pressure $(P_2 = 10 \ \text{bar})$, and still adiabatic

$\Rightarrow \dot{W}_s = -\dot{M}_1 \left(\hat{H}_1 - \hat{H}_2 \right)$.

Here, however,

$$\dot{W}_s = 0.8 \dot{W}_s (\text{Part a}) = 0.8 \left(-2.258 \times 10^6 \right) \frac{\text{kJ}}{\text{h}} = 4500 \left(\hat{H}_2 - 3422.2 \right) \frac{\text{kJ}}{\text{h}}$$

$$\begin{array}{lll} \Rightarrow \quad \hat{H}_2 = 3020.8 \ \text{kJ/kg} & \xrightarrow[\text{Tables}]{\text{Steam}} & T_2 \cong 286.7 \ \text{K} \\ \quad\quad P = 10 \ \text{bar} & & \hat{S}_2 \approx 7.0677 \ \text{kJ/kg K} \end{array}$$

Thus

$$\dot{S}_{\text{gen}} = -\dot{M}_1 \left(\hat{S}_1 - \hat{S}_2 \right) = -4500 \frac{\text{kg}}{\text{h}} \times (6.8803 - 7.0677) \frac{\text{kJ}}{\text{kg K}} = 843.3 \frac{\text{kJ}}{\text{K} \cdot \text{h}}$$

(c) Flow across valve is a Joule-Thompson (isenthalpic expansion) ... See Illustration 3.4-1. Thus, $\hat{H}_{\text{into valve}} = \hat{H}_{\text{out of valve}}$, and the inlet conditions to the turbine are

$$\begin{array}{l} \hat{H}_1 = \hat{H}_{\text{out of valve}} = \hat{H}_{\text{into valve}} = 3422.2 \ \text{kJ/kg} \\ P_1 = 30 \ \text{bar} \end{array}$$

$$\begin{array}{ll} \xrightarrow[\text{Tables}]{\text{Steam}} & \begin{array}{l} T_1 \approx 484.8^\circ\text{C} \\ S_1 \approx 7.1874 \ \text{kJ/kg K} \end{array} \end{array}$$

Flow across turbine is isentropic, as in part (a)

$$\begin{array}{lll} \hat{S}_2 = \hat{S}_1 = 7.1874 \ \text{kJ/kg K} & \xrightarrow[\text{Tables}]{\text{Steam}} & T_2 \cong 318.1^\circ\text{C} \\ P_2 = 10 \ \text{bar} & & \hat{H}_2 \approx 3090.4 \ \text{kJ/kg} \end{array}$$

$$\dot{W}_s = 4500 \frac{\text{kg}}{\text{h}} \times (3090.4 - 3422.2) \frac{\text{kJ}}{\text{kg}} = -1.493 \times 10^6 \frac{\text{kJ}}{\text{h}} = -414.8 \ \text{kW}$$

4.10 Since compression is isentropic, and gas is ideal with constant heat capacity, we have

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{P_2}{P_1} \right)^{R/C_\text{P}}$$

So that $T_2 = T_1 \left(\dfrac{P_2}{P_1} \right)^{R/C_\text{P}} = 298.15 \left(\dfrac{3 \times 10^6}{2 \times 10^6} \right)^{8.314/36.8} = 326.75 \ \text{K}$. Now using, from solution to Problem 3.31, that $\dot{W}_s = \dot{N} C_\text{P} (T_2 - T_1)$

$$\begin{aligned}\dot{W}_s &= 125\frac{\text{kg}}{\text{s}}\times\frac{1\text{ mol}}{16\text{ g}}\times 36.8\frac{\text{J}}{\text{mol K}}\times(326.75-298.15)\text{K}\times\frac{1000\text{ g}}{\text{kg}}\\ &= 8.23\times10^{6}\ \text{J/s}\end{aligned}$$

The load on the gas cooler is, from Problem 3.31,

$$\begin{aligned}\dot{Q} &= \dot{N}C_{\text{p}}(T_3-T_2)\\ &= \frac{125\ \text{kg/s}\times1000\ \text{g/kg}}{16\ \text{g/mol}}\times36.8\frac{\text{J}}{\text{mol K}}\times(298.15-326.75)K\\ &= -8.23\times10^{+6}\,\text{J/s}\end{aligned}$$

4.11 (a) This is a Joule-Thomson expansion

$$\begin{aligned}&\Rightarrow \hat{H}(70\text{ bar, } T=?) = \hat{H}(1.0133\text{ bar, } T=400^\circ\text{C})\approx\hat{H}(1\text{ bar, } T=400^\circ\text{C})\\ &= 3278.2\ \text{kJ/kg}\\ &\text{and } T=447^\circ\text{C},\ \hat{S}=6.619\ \text{kJ/kg K}\end{aligned}$$

(b) If turbine is adiabatic and reversible $\left(\dot{S}_{\text{gen}}=0\right)$, then $\hat{S}_{\text{out}}=\hat{S}_{\text{in}}=6.619\ \text{kJ/kg K}$ and $P=1.013$ bar. This suggests that a two-phase mixture is leaving the turbine

Let x = fraction vapor $\quad \hat{S}^{\text{V}}=7.3594\ \text{kJ/kg K}$

$\hat{S}^{\text{L}}=1.3026\ \text{kJ/kg K}$

Then $x(7.3594)+(1-x)(1.3026)=6.619\ \text{kJ/kg K}$ or $x=0.8778$. Therefore the enthalpy of fluid leaving turbine is

$$\hat{H}=0.8788\times\underbrace{2675.5}_{\hat{H}^{\text{V}}\ (\text{sat'd, 1 bar})}+(1-0.8778)\times\underbrace{417.46}_{\hat{H}^{\text{L}}\ (\text{sat'd, 1 bar})}=2399.6\frac{\text{kJ}}{\text{kg}}$$

Energy balance

$$0=\dot{M}_{\text{in}}\hat{H}_{\text{in}}+\dot{M}_{\text{out}}\hat{H}_{\text{out}}+\cancelto{0}{\dot{Q}}+\dot{W}_{\text{s}}-P\cancelto{0}{\frac{dV}{dt}}$$

but $\dot{M}_{\text{in}}=-\dot{M}_{\text{out}}$

$$\Rightarrow -\frac{\dot{W}_s}{\dot{M}_{\text{in}}}=3278.2-2399.6=878.6\frac{\text{kJ}}{\text{kg}}$$

(c) Saturated vapor at 1 bar

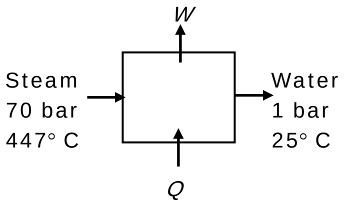
$$\hat{S}=7.3594\ \text{kJ/kg K};\ \hat{H}=2675.5\ \text{kJ/kg}$$

$$-\left.\frac{\dot{W}_s}{\dot{M}_{\text{in}}}\right|_{\text{Actual}}=3278.2-2675.5=602.7\ \text{kJ/kg}$$

$$\text{Efficiency }(\%)=\frac{602.7\times100}{878.6}=68.6\%$$

$$\frac{\dot{S}_{\text{gen}}}{\dot{M}_{\text{in}}}=7.3594-6.619=0.740\ \text{kJ/Kh}$$

(d)



$$0 = \dot{M}_1 + \dot{M}_2 \Rightarrow \dot{M}_2 = -\dot{M}_1$$

$$0 = \dot{M}_1\left(\hat{H}_1 - \hat{H}_2\right) + \dot{W}_s + \dot{Q} - P\frac{dV}{dt}$$

$$0 = \dot{M}_1\left(\hat{S}_1 - \hat{S}_2\right) + \frac{\dot{Q}}{T} + \dot{S}_{\text{gen}}$$

Simplifications to balance equations

$\dot{S}_{\text{gen}} = 0$ (for maximum work); $P\dfrac{dV}{dt} = 0$ (constant volume)

$\dfrac{\dot{Q}}{T} = \dfrac{\dot{Q}}{T_0}$ where $T_0 = 25°\text{C}$ (all heat transfer at ambient temperature)

$\hat{H}$ (sat'd liq, $25°\text{C}) = 104.89\dfrac{\text{kJ}}{\text{kg}}$; $\hat{S}$ (sat'd liq, $T = 25°\text{C}) = 0.3674\dfrac{\text{kJ}}{\text{kg K}}$

$$\frac{\dot{Q}}{\dot{M}} = T_0\left(\hat{S}_2 - \hat{S}_1\right);\ \left.\frac{-\dot{W}_s}{\dot{M}}\right|_{\max} = \hat{H}_1 - \hat{H}_2 + T_0\left(\hat{S}_2 - \hat{S}_1\right) = \left(\hat{H}_1 - T_0\hat{S}_1\right) - \left(\hat{H}_2 - T_0\hat{S}_2\right)$$

$$\left.\frac{-\dot{W}_s}{\dot{M}}\right|_{\max} = [3278.2 - 298.15 \times 6.619] - [104.89 - 298.15 \times 0.3674]$$

$$= 1304.75 + 4.65 = 1309.4 \text{ kJ/kg}$$

4.12 Take that portion of the methane initially in the tank that is also in the tank finally to be in the system. This system is isentropic $S_f = S_i$.

(a) The ideal gas solution

$$\underline{S}_f = \underline{S}_i \Rightarrow T_f = T_i\left(\frac{P_f}{P_i}\right)^{R/C_p} = 300\left(\frac{3.5}{70}\right)^{8.314/36} = 150.2 \text{ K}$$

$$N = \frac{PV}{RT} \Rightarrow N_i = \frac{P_iV}{RT_i} = 1964.6 \text{ mol};\ N_f = \frac{P_fV}{RT_f} = 196.2 \text{ mol}$$

$$\Delta N = N_f - N_i = -1768.4 \text{ mol}$$

(b) Using Figure 2.4-2.

70 bar ≈ 7 MPa, T = 300 K $\qquad \hat{S}_i = 5.05 \text{ kJ/kg K} = \hat{S}_f$

$$\hat{V}_i = 0.0195\frac{\text{m}^3}{\text{kg}},\ \text{so that } m_i = \frac{0.7\text{m}^3}{0.0195\dfrac{\text{m}^3}{\text{kg}}} = 35.90 \text{ kg.}$$

$$N_i = \frac{35.90 \text{ kg} \times 1000\dfrac{\text{g}}{\text{kg}}}{28\dfrac{\text{g}}{\text{mol}}} = 1282 \text{ mol}$$

At 3.5 bar = 0.35 MPa and $\hat{S}_f = 5.05 \text{ kJ/kg K} \Rightarrow T \approx 138 \text{ K}$. Also,

$$\hat{V}_f = 0.192 \frac{\text{m}^3}{\text{kg}}, \text{ so that } m_f = \frac{0.7\text{m}^3}{0.192 \frac{\text{m}^3}{\text{kg}}} = 3.646 \text{ kg.}$$

$$N_f = \frac{3.646 \text{ kg} \times 1000 \frac{\text{g}}{\text{kg}}}{28 \frac{\text{g}}{\text{mol}}} = 130.2 \text{ mol}$$

$$\Delta N = N_f - N_i = 130.2 - 1282 = -1151.8 \text{ mol}$$

4.13 $d\underline{S} = C \dfrac{dT}{T} + R\dfrac{d\underline{V}}{\underline{V}}$ eqn. (4.4-1)

$$\Delta S = \int \left[(a-R) + bT + cT^2 + dT^3 + \frac{e}{T^2} \right] \frac{dT}{T} + R\int \frac{d\underline{V}}{\underline{V}}$$

so that

$$\underline{S}(T_2, \underline{V}_2) - \underline{S}(T_1, \underline{V}_1) = (a-R)\ln\frac{T_2}{T_1} + b(T_2 - T_1) + \frac{c}{2}\left(T_2^2 - T_1^2\right)$$
$$+\frac{d}{3}\left(T_2^3 - T_1^3\right) - \frac{e}{2}\left(T_2^{-2} - T_1^{-2}\right) + R\ln\frac{\underline{V}_2}{\underline{V}_1}$$

Now using

$$P\underline{V} = RT \Rightarrow \frac{\underline{V}_2}{\underline{V}_1} = \frac{T_2}{T_1} \cdot \frac{P_1}{P_2} \Rightarrow$$

$$\underline{S}(T_2, P_2) - S(T_1, P_1) = a\ln\frac{T_2}{T_1} + b(T_2 - T_1) + \frac{c}{2}\left(T_2^2 - T_1^2\right)$$
$$+\frac{d}{3}\left(T_2^3 - T_1^3\right) - \frac{e}{2}\left(T_2^{-2} - T_1^{-2}\right) - R\ln\frac{P_2}{P_1}$$

Finally, eliminating T_2 using $T_2 = T_1 P_2 \underline{V}_2 / P_1 \underline{V}_1$ yields

$$\underline{S}(P_2, \underline{V}_2) - \underline{S}(P_1, \underline{V}_1) = a\ln\left(\frac{P_2\underline{V}_2}{P_1\underline{V}_1}\right) + \frac{b}{R}(P_2\underline{V}_2 - P_1\underline{V}_1)$$
$$+\frac{c}{2R^2}\left[(P_2\underline{V}_2)^2 - (P_1\underline{V}_1)^2\right]$$
$$+\frac{d}{3R^3}\left[(P_2\underline{V}_2)^3 - (P_1\underline{V}_1)^3\right]$$
$$-\frac{eR^2}{2}\left[\left(P_2\underline{V}_2^{-2}\right) - \left(P_1\underline{V}_1^{-2}\right)\right] - R\ln\frac{P_2}{P_1}$$

4.14 System: contents of valve (steady-state, adiabatic, constant volume system)

Mass balance $0 = \dot{N}_1 + \dot{N}_2$

Energy balance $0 = \dot{N}_1 \underline{H}_1 + \dot{N}_2 \underline{H}_2 + \cancelto{0}{\dot{Q}} + \cancelto{0}{\dot{W}_s} - P \cancelto{0}{\frac{dV}{dt}}$

$$\Rightarrow \underline{H}_1 = \underline{H}_2$$

Entropy balance $0 = \dot{N}_1 \underline{S}_1 + \dot{N}_2 \underline{S}_2 + \dot{S}_{\text{gen}} + \cancelto{0}{\frac{\dot{Q}}{T}}$

$$\Rightarrow \Delta \underline{S} = \underline{S}_2 - \underline{S}_1 = \frac{\dot{S}_{\text{gen}}}{\dot{N}}$$

(a) Using the Mollier Diagram for steam (Fig. 3.3-1a) or the Steam Tables

$$\begin{array}{ll} T_1 = 600\text{ K} & P_2 = 7\text{ bar} \\ P_1 = 35\text{ bar} & \hat{H}_2 = 3045.3\text{ J/g} \end{array} \Rightarrow \begin{array}{l} T_2 \approx 293^\circ\text{C} \\ \hat{S}_2 = 7.277\text{ J/g K} \end{array}$$

$\hat{H}_1 = \hat{H}_2 = 3045.3$ J/g . Thus $\hat{S}_1 = 6.5598$ J/g K ; $T_{\text{exit}} = 293^\circ$C

$\Delta\hat{S} = \hat{S}_2 - \hat{S}_1 = 0.717$ J/g K

(b) For the ideal gas, $\underline{H}_1 = \underline{H}_2 \Rightarrow T_1 = T_2 = 600$ K

$$\Delta\underline{S} = \underline{S}(T_2, P_2) - \underline{S}(T_1, P_1) = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= -R\ln\frac{P_2}{P_1} = 13.38\text{ J/mol K} \Rightarrow$$

$\Delta\hat{S} = 0.743$ J/mol K

4.15 From the Steam Tables

At 200ºC,

$P = 1.5538$ MPa

$\hat{V}^L = 0.001157\ \text{m}^3/\text{kg}$ $\hat{V}^V = 0.12736\ \text{m}^3/\text{kg}$

$\hat{U}^L = 850.65$ kJ / kg $\hat{U}^V = 2595.3$ kJ / kg

$\hat{H}^L = 852.45$ kJ / kg $\hat{H}^L = 2793.2$ kJ / kg

$\hat{S}^L = 2.3309\ \text{kJ/kg}\cdot\text{K}$ $\hat{S}^V = 6.4323\ \text{kJ/kg}\cdot\text{K}$

$\Delta\hat{H}^{\text{vap}} = 1940.7$ kJ / kg $\Delta\hat{S}^{\text{vap}} = 4.1014\ \text{kJ/kg}\cdot\text{K}$

(a) Now assuming that there will be a vapor-liquid mixture in the tank at the end, the properties of the steam and water will be

At 150ºC,

$P = 0.4578$ MPa

$\hat{V}^L = 0.001091\ \text{m}^3/\text{kg}$ $\hat{V}^V = 0.3928\ \text{m}^3/\text{kg}$

$\hat{U}^L = 631.68$ kJ / kg $\hat{U}^V = 2559.5$ kJ / kg

$\hat{H}^L = 632.20$ kJ / kg $\hat{H}^V = 2746.5$ kJ / kg

$\hat{S}^L = 1.8418\ \text{kJ/kg}\cdot\text{K}$ $\hat{S}^V = 6.8379\ \text{kJ/kg}\cdot\text{K}$

$\Delta\hat{H}^{\text{vap}} = 2114.3$ kJ / kg $\Delta\hat{S}^{\text{vap}} = 4.9960\ \text{kJ/kg}\cdot\text{K}$

(b) For simplicity of calculations, assume 1 m^3 volume of tank.