CHAPTER 6

6.1 Several factors make it very difficult to accurately estimate the magnitudes of deflections. These include the problems of controlling field work, the varying ages of concretes when the forms are removed, the imperfect estimation of the magnitudes of member end restraints, the difficulties of estimating the effective moments of inertia of the various members, and so on.

6.2 Long term deflections are caused by shrinkage and creep. In detail the factors affecting the magnitudes of shrinkage and creep and thus long term deflections are humidity, temperature, curing conditions, amounts of compression steel used, ratio of stress to strength, and the age of the concrete at the time of loading.

6.3 The deflections of reinforced concrete beams can be limited by increasing member depths, by cambering the members and by using compression reinforcing.

6.4 It is necessary to limit the width of concrete cracks to certain maximum values so the appearances of the structures are not spoiled. In addition limitations on crack widths probably help to reduce the corrosion of the reinforcing. Crack widths can be limited by using smaller bars spaced more closely together. Furthermore the use of smaller yield stresses in reinforcement design will result in smaller crack widths.

PROB # 6.5

$$I_g = \left(\frac{1}{12}\right)(20)(34)^3 = 65{,}507 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(65{,}507)}{17}$$

$$= 1{,}827{,}806 \text{ in.-lbs} = 152.32 \text{ ft-k}$$

$$M_a = (3.5)(14)(7) = 343 \text{ ft-k}$$

Transformed area calculations

$$(20x)\left(\frac{x}{2}\right) = (8)(3.79)(31-x)$$

$$10x^2 = 940 - 30.32x$$

$$10x^2 + 30.32x = 940$$

$$x^2 + 1.516x - 47 = 0$$

$$x = 8.30 \text{ in.}$$

$$I_{cr} = \left(\frac{1}{3}\right)(20)(8.30)^3 + (8)(3.79)(31-8.3)^2 = 19{,}436 \text{ in.}^4$$

Effective moment of inertia

$$I_e = \left(\frac{152.32}{343}\right)^3(65507) + \left[1 - \left(\frac{152.32}{343}\right)^3\right]19{,}436$$

$$= 23{,}470 \text{ in.}^4$$

$$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$$

Instantaneous deflection

$$\delta = \frac{\left(\frac{3500}{12}\right)(12 \times 14)^4}{(8)(3.605 \times 10^6)(23470)} = \boxed{0.343 \text{ in.} \downarrow}$$

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PROB # 6.6

$$I_g = \left(\frac{1}{12}\right)(16)(32)^3 = 43{,}691 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(43{,}691)}{16} = 1{,}295{,}279 \text{ in.-lbs}$$

$$= 107.94 \text{ ft-k}$$

$$M_a = (20)(18) + (2)(18)(9) = 684 \text{ ft-k}$$

Transformed area calculations

$$(16x)\left(\frac{x}{2}\right) = (8)(9.37)(27.5 - x)$$

$$8x^2 = 2061.4 - 74.96x$$

$$8x^2 + 74.96x = 2061.4$$

$$x^2 + 9.37x = 257.68$$

$$(x + 4.685)(x + 4.685) = 257.68 + (4.685)^2 = 279.63$$

$$x + 4.685 = \sqrt{279.63} = 16.722$$

$$x = 12.04 \text{ in.}$$

$$I_{cr} = \left(\frac{1}{3}\right)(16)(12.04)^3 + (8)(9.37)(15.46)^2 = 27{,}225 \text{ in.}^4$$

Effective moment of inertia

$$I_e = \left(\frac{107.94}{684}\right)^3 (43{,}691) + \left[1 - \left(\frac{107.94}{684}\right)^3\right] 27{,}225$$

$$= 27{,}290 \text{ in.}^4$$

$$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$$

Instantaneous deflection

$$\delta = \frac{\left(\frac{2000}{12}\right)(12 \times 18)^4}{(8)(3.605 \times 10^6)(27{,}290)} + \frac{(20{,}000)(12 \times 18)^3}{(3)(3.605 \times 10^6)(27{,}290)}$$

$$= \boxed{1.14 \text{ in.}}$$ vgcm©

PROB 6.7

$$I_g = \left(\frac{1}{12}\right)(24)(34)^3 = 78{,}608 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(78608)}{17} = 2{,}193{,}356 \text{ in.-lbs}$$

$$= 182.8 \text{ ft-k}$$

$$M_a = (20)(10) + (10)(20) + (1.5)(20)(10) = 700 \text{ ft-k}$$

Transformed area calculations

$$(24x)\left(\frac{x}{2}\right) = (8)(6.00)(31.5 - x)$$

$$12x^2 = 1512 - 48x$$

$$12x^2 + 48x = 1512$$

$$x^2 + 4x - 126 = 0$$

$$x = 9.40 \text{ in}$$

$$I_{cr} = \left(\frac{1}{3}\right)(24)(9.40)^3 + (8)(6.00)(31.5 - 9.4)^2$$

$$= 30{,}088 \text{ in.}^4$$

Effective moment of inertia

$$I_e = \left(\frac{182.8}{700}\right)^3 (78608) + \left[1 - \left(\frac{182.8}{700}\right)^3\right](30{,}088)$$

$$= 30{,}952 \text{ in.}^4$$

$$E_c = (57{,}000)(\sqrt{4000}) = 3.605 \times 10^6 \text{ psi}$$

Instantaneous deflection

$$\delta = \frac{\left(\frac{1500}{12}\right)(12\times20)^4}{(8)(3.605\times10^6)(30952)} + \frac{(10{,}000)(12\times20)^3}{(3)(3.605\times10^6)(30952)}$$

$$+ \frac{(20{,}000)(12\times10)^2}{(6)(3.605\times10^6)(30952)}(3\times12\times20 - 12\times10) = \boxed{1.14 \text{ in.}}$$

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PROB #6.8

$A_g = (18)(36) - (6)(10) = 648 - 60 = 588 \text{ in.}^2$

$\bar{y} \text{ from top} = \dfrac{(648)(18) - (60)(13)}{588} = 18.51 \text{ in.}$

$I_g = \left(\frac{1}{3}\right)(18)(18.51^3 + 17.49^3) - \left(\frac{1}{12}\right)(6)(10)^3 - (60)(5.51^2) = 67{,}831 \text{ in.}^4$

$M_{cr} = \dfrac{(7.5\sqrt{4000})(67{,}831)}{(36 - 18.51)} = 1{,}839{,}627 \text{ in.-lbs} = 153.3 \text{ ft-k}$

$M_a = \dfrac{(5)(28)^2}{8} = 490 \text{ ft-k}$

Transformed area calculations

$(18x)\left(\frac{x}{2}\right) - (6)(x-8)\left(\frac{x-8}{2}\right) = (8)(6.33)(33-x)$

$9x^2 - 3x^2 + 48x - 192 = 1671.12 - 50.64x$

$6x^2 + 98.64x = 1863$

$x^2 + 16.44x = 310.52$

$x = 11.22 \text{ in.}$

$I_{cr} = \left(\frac{1}{3}\right)(18)(11.22)^3 - \left(\frac{1}{3}\right)(6)(3.22)^3 + (8)(6.33)(21.78)^2$

$= 32{,}430 \text{ in.}^4$

$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$

$I_e = \left(\frac{153.3}{490}\right)^3 (67{,}831) + \left[1 - \left(\frac{153.3}{490}\right)^3\right] 32430 = 33{,}514 \text{ in.}^4$

Instantaneous deflection

$\delta = \dfrac{(5)\left(\frac{5000}{12}\right)(12 \times 28)^4}{(384)(3.605 \times 10^6)(33{,}514)} = \boxed{0.572 \text{ in.}}$

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PROB # 6.9

$I_g = \left(\frac{1}{12}\right)(14)(18)^3 - \left(\frac{1}{12}\right)(10)(10)^3 = 5971 \text{ in.}^4$

$M_{cr} = \frac{(7.5\sqrt{4000})(5971)}{9.00} = 314,682 \text{ in.-lbs} = 26.22 \text{ ft-k}$

$M_a = (3.5)(12)(6) = 252 \text{ ft-k}$

Transformed area calculations

$(14x)\left(\frac{x}{2}\right) - (10)(x-4)\left(\frac{x-4}{2}\right) = (8)(2.35)(16-x)$

$7x^2 - 5x^2 + 40x - 80 = 300.8 - 18.8x$

$2x^2 + 58.8x = 380.8$

$x = 5.46 \text{ in.}$

$I_{cr} = \left(\frac{1}{3}\right)(14)(5.46)^3 - \left(\frac{1}{3}\right)(10)(1.46)^3 + (8)(2.35)(10.54)^2$

$= 2838 \text{ in.}^4$

$E_c = 57,000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$

$I_e = \left(\frac{26.22}{252}\right)^3(5971) + \left[1 - \left(\frac{26.22}{252}\right)^3\right]2838$

$= 2832 \text{ in.}^4$

Instantaneous deflection

$\delta = \frac{\left(\frac{3500}{12}\right)(12 \times 12)^4}{(8)(3.605 \times 10^6)(2832)} =$ **1.54 in.**

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PROB #6.10

$A_g = (18)(36) - (6)(10) = 648 - 60 = 588 \text{ in.}^2$

$\bar{y}$ from top $= \dfrac{(648)(18) - (60)(13)}{588} = 18.51$ in.

$I_g = \left(\frac{1}{3}\right)(18)(18.51^3 + 17.49^3) - \left(\frac{1}{12}\right)(6)(10)^3 - (60)(5.51^2) = 67{,}831 \text{ in.}^4$

$M_{cr} = \dfrac{(7.5\sqrt{4000})(67{,}831)}{(36 - 18.51)} = 1{,}839{,}627 \text{ in.-lbs} = 153.3 \text{ ft-k}$

$M_a = \dfrac{(5)(28)^2}{8} + \dfrac{25(28)}{4} = 665 \text{ ft-k}$

Transformed area calculations

$(18x)\left(\frac{x}{2}\right) - (6)(x-8)\left(\frac{x-8}{2}\right) = (8)(6.33)(33-x)$

$9x^2 - 3x^2 + 48x - 192 = 1671.12 - 50.64x$

$6x^2 + 98.64x = 1863$

$x^2 + 16.44x = 310.52$

$x = 11.22$ in.

$I_{cr} = \left(\frac{1}{3}\right)(18)(11.22)^3 - \left(\frac{1}{3}\right)(6)(3.22)^3 + (8)(6.33)(21.78)^2$
$= 32{,}430 \text{ in.}^4$

$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6$ psi

$I_e = \left(\frac{153.3}{665}\right)^3(67{,}831) + \left[1 - \left(\frac{153.3}{665}\right)^3\right]32430 = 32864 \text{ in.}^4$

Instantaneous deflection

$\delta = \dfrac{(5)\left(\frac{5000}{12}\right)(12 \times 28)^4}{(384)(3.605 \times 10^6)(32864)} + \dfrac{25(28)^3(12)^3(1000)}{48(3.605 \times 10^6)(32864)}$

$= 0.584 + 0.168 = \boxed{0.750 \text{ in.}}$

PROB #6.11 (1)

(a) Instantaneous deflection for dead load (δ_D)

$$I_g = \left(\frac{1}{12}\right)(12)(20)^3 = 8000 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(8000)}{10} = 379{,}474 \text{ in.-lbs}$$

$$= 31.62 \text{ ft-k}$$

$$M_a = \frac{(1)(20)^2}{8} = 50 \text{ ft-k}$$

Transformed area calculations

$$(12x)\left(\frac{x}{2}\right) = (8)(4.0)(17.5 - x)$$

$$6x^2 = 560 - 32x$$

$$x^2 + 5.33x = 93.33$$

$$(x + 2.67)(x + 2.67) = 93.33 + (2.67)^2 = 100.46$$

$$x + 2.67 = \sqrt{100.46} = 10.02$$

$$x = 7.35 \text{ in.}$$

$$I_{cr} = \left(\frac{1}{3}\right)(12)(7.35)^3 + (8)(4.0)(10.15)^2 = 4885 \text{ in.}^4$$

$$I_e = \left(\frac{31.62}{50}\right)^3(8000) + \left[1 - \left(\frac{31.62}{50}\right)^3\right]4885 = 5673 \text{ in.}^4$$

$$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$$

$$\delta_D = \frac{(5)\left(\frac{1000}{12}\right)(12 \times 20)^4}{(384)(3.605 \times 10^6)(5673)} = \boxed{0.176 \text{ in.}}$$

(b) Instantaneous deflection for dead + full live load (δ_{D+L})

$$M_a = \frac{(3)(20)^2}{8} = 150 \text{ ft-k}$$

$$I_e = \left(\frac{31.62}{150}\right)^3(8000) + \left[1 - \left(\frac{31.62}{150}\right)^3\right]4885 = 4914 \text{ in.}^4$$

$$\delta_{DL} = \frac{(5)\left(\frac{3000}{12}\right)(12 \times 20)^4}{(384)(3.605 \times 10^6)(4914)} = \boxed{0.610 \text{ in.}}$$

PROB # 6.11 (2)

(c) Initial deflection for full live load (δ_L)

$$\delta_L = \delta_{D+L} - \delta_D = 0.610 - 0.176 = 0.434 \text{ in.}$$

(d) Initial deflection due to dead load + 30% live load ($\delta_D + \delta_{SL}$)

$$M_a = \frac{[1.0 + (0.30)(2.0)](20)^2}{8} = 80 \text{ ft-k}$$

$$I_e = \left(\frac{31.6}{80}\right)^3 (8000) + \left[1 - \left(\frac{31.6}{80}\right)^3\right] 4855 = 5077 \text{ in.}^4$$

$$\delta_D + \delta_{SL} = \frac{(5)\frac{(1000 + 0.30 \times 2000)(12 \times 20)^4}{12}}{(384)(3.605 \times 10^6)(5077)} = \boxed{0.315 \text{ in.}}$$

(e) Initial deflection due to 30% live load (δ_{SL})

$$\delta_{SL} = (\delta_D + \delta_{SL}) - \delta_D = 0.315 - 0.176 = \boxed{0.139 \text{ in.}}$$

(f) Long term deflection for dead load plus four years of 30% sustained live load (δ_{LT})

$$\lambda = \frac{2.0}{1 + 50\rho'} = \frac{2.0}{1 + 0} = 2.0$$

$$\lambda_{4yrs} = \frac{1.85}{1 + 0} = 1.85$$

$$\delta_{LT} = \delta_L + \lambda_\infty \delta_D + \lambda_{4yrs} \delta_{SL}$$

$$= 0.434 + (2.0)(0.176) + (1.85)(0.139)$$

$$= \boxed{1.043 \text{ in.}}$$

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PROB# 6,12 (1)

(a) Instantaneous or short term dead load deflec. (δ_D)

$I_g = \left(\frac{1}{12}\right)(16)(24)^3 = 18{,}432 \text{ in.}^4$

$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(18{,}432)}{12} = 728{,}589 \text{ in.-k}$

$= 60.72 \text{ ft-k}$

$M_a = \frac{(1.6)(30)^2}{8} = 180 \text{ ft-k}$

By transformed area calculations determine X and I

$X = 7.20 \text{ in.}$

$I_{cr} = 10{,}362 \text{ in.}^4$

$I_e = \left(\frac{60.72}{180}\right)^3(18{,}432) + \left[1 - \left(\frac{60.72}{180}\right)^3\right] 10{,}362$

$= 10{,}672 \text{ in.}^4$

$E_c = (57{,}000)(\sqrt{4000}) = 3.605 \times 10^6 \text{ psi}$

$\delta_D = \frac{5w\ell^4}{384EI} = \frac{(5)\left(\frac{1600}{12}\right)(12 \times 30)^4}{(384)(3.605 \times 10^6)(10{,}672)} = \boxed{0.758 \text{ in.}}$

(b) Instantaneous or short-term deflection for full dead and live load (δ_{D+L})

$M_a = \frac{(2.8)(30)^2}{8} = 315 \text{ ft-k}$

$I_e = \left(\frac{60.72}{315}\right)^3(18{,}432) + \left[1 - \left(\frac{60.72}{315}\right)^3\right] 10{,}362$

$= 10{,}420 \text{ in.}^4$

$\delta_{D+L} = \frac{(5)\left(\frac{2800}{12}\right)(12 \times 30)^4}{(384)(3.605 \times 10^6)(10420)} = \boxed{1.358 \text{ in.}}$

PROB # 6.12 (2)

(c) Initial deflection for full live load (δ_L)

$$\delta_L = \delta_{D+L} - \delta_D = 1.358 - 0.758 = \boxed{0.60 \text{ in.}}$$

(d) Initial deflection due to dead load + 30% live load ($\delta_D + \delta_{SL}$)

$$M_a = \frac{[1.6 + (0.3)(1.2)](30)^2}{8} = 220.5 \text{ ft-k}$$

$$I_e = \left(\frac{60.72}{220.5}\right)^3 (18,432) + \left[1 - \left(\frac{60.72}{220.5}\right)^3\right] 10,362 = 10,531 \text{ in.}^4$$

$$\delta_D + \delta_{SL} = \frac{(5)\left(\frac{1600 + 0.30 \times 1200}{12}\right)(12 \times 30)^4}{(384)(3.605 \times 10^6)(10,531)} = \boxed{0.941 \text{ in.}}$$

(e) Initial deflection due to 30% live load (δ_{SL})

$$\delta_{SL} = (\delta_D + \delta_{SL}) - \delta_D = 0.941 - 0.758 = \boxed{0.183 \text{ in.}}$$

(f) Long term deflection for dead load plus four years of 30% sustained live load (δ_{LT})

$$\rho' = \frac{2.0}{(16)(21)} = 0.005952$$

$$\lambda_\infty = \frac{2.0}{1 + (50)(0.005952)} = 1.54$$

$$\lambda_{4yrs} = \frac{1.85}{1 + (50)(0.005952)} = 1.43$$

$$\delta_{LT} = \delta_L + \lambda_\infty \delta_D + \lambda_{4yrs} \delta_{SL}$$
$$= 0.60 + (1.54)(0.758) + (1.43)(0.183)$$
$$= \boxed{2.03 \text{ in.}}$$

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PROB# 6.13(1)

(a) Instantaneous deflection for dead load (δ_D)

$I_g = \left(\frac{1}{12}\right)(16)(24)^3 = 18{,}432 \text{ in.}^4$

$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{4000})(18{,}432)}{12} = 728{,}589 \text{ in.-lbs.}$

$= 60.72 \text{ ft-k}$

$M_a = \frac{(1.6)(30)^2}{8} = 180 \text{ ft-k}$

Transformed area calculations

$(16x)\left(\frac{x}{2}\right) = (8)(5.06)(21-x)$

$8x^2 = 850.08 - 40.48x$

$8x^2 + 40.48x = 850.08$

$x^2 + 5.06x = 106.26$

$(x+2.53)(x+2.53) = 106.26 + (2.53)^2 = 112.66$

$x + 2.53 = \sqrt{112.66} = 10.61$

$x = 8.08 \text{ in.}$

$I_{cr} = \left(\frac{1}{3}\right)(16)(8.08)^3 + (8)(5.06)(12.92)^2 = 9570.6 \text{ in.}^4$

$I_e = \left(\frac{60.72}{180}\right)^3(18{,}432) + \left[1 - \left(\frac{60.72}{180}\right)^3\right]9570.6 = 9911 \text{ in.}^4$

$E_c = 57{,}000\sqrt{4000} = 3.605 \times 10^6 \text{ psi}$

$\delta_D = \frac{(5)\left(\frac{1600}{12}\right)(12 \times 30)^4}{(384)(3.605 \times 10^6)(9911)} = \boxed{0.816 \text{ in.}}$ ✓

(b) Instantaneous deflection for dead + full live load (δ_{D+L}).

$M_a = \frac{(2.8)(30)^2}{8} = 315 \text{ ft-k}$

$I_e = \left(\frac{60.72}{315}\right)^3(18{,}432) + \left[1 - \left(\frac{60.72}{315}\right)^3\right]9570.6 = 9634 \text{ in.}^4$

$\delta_{D+L} = \frac{(5)\left(\frac{2800}{12}\right)(12 \times 30)^4}{(384)(3.605 \times 10^6)(9634)} = \boxed{1.469 \text{ in.}}$ ✓

PROB # 6.13 (2)

(c) Initial deflection for full live load (δ_L)

$$\delta_L = \delta_{D+L} - \delta_D = 1.469 - 0.816 = \boxed{0.653 \text{ in.}}$$ ✓

(d) Initial deflection due to dead load plus 30% live load ($\delta_D + \delta_{SL}$)

$$M_a = \left[\frac{[1.6 + (0.30)(1.2)](30)^2}{8}\right] = 220.5 \text{ ft-k}$$

$$I_e = \left(\frac{60.72}{220.5}\right)^3 (18{,}432) + \left[1 - \left(\frac{60.72}{220.5}\right)^3\right] 9570.6 = 9756 \text{ in}^4$$

$$\delta_{D+SL} = \frac{(5)\left(\frac{1600 + 0.30 \times 1200}{12}\right)(12 \times 30)^4}{(384)(3{,}605 \times 10^6)(9756)} = \boxed{1.016 \text{ in.}}$$ ✓

(e) Initial deflection due to 30% live load (δ_{SL})

$$\delta_{SL} = (\delta_D + \delta_{SL}) - \delta_D = 1.016 - 0.816 = \boxed{0.200 \text{ in.}}$$ ✓

(f) Long term deflection for dead load plus 48 months of 30% sustained live load (δ_{LT})

$$\lambda_\infty = \frac{2.0}{1 + 50\rho'} = \frac{2.0}{1 + 0} = 2.0$$

$$\lambda_{48 \text{ months}} = \frac{1.85}{1 + 0} = 1.85$$

$$\delta_{LT} = \delta_L + \lambda_\infty \delta_D + \lambda_{48 \text{months}} \delta_{SL}$$

$$= 0.653 + (2.0)(0.816) + (1.85)(0.200)$$

$$= \boxed{2.655 \text{ in.}}$$

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PROB# 6.14(1)

$\lambda = 0.85$ for semi-lightweight

(a) Instantaneous or short term dead load deflec. (δ_D)

$I_g = \left(\frac{1}{12}\right)(16)(24)^3 = 18{,}432 \text{ in.}^4$

$M_{cr} = \frac{f_r I_g}{y_t} = \frac{7.5(.85)\sqrt{4000}(18{,}432)}{12} = 619{,}300 \text{ in.-k} = 51.61 \text{ ft-k}$

$M_a = \frac{(1.6)(30)^2}{8} = 180 \text{ ft-k}$

By transformed area calculations determine X and I

$x = 7.91$ in. (See Ex. 2.5 in Textbook)

$I_{cr} = 12{,}413 \text{ in.}^4$

$I_e = \left(\frac{51.61}{180}\right)^3(18{,}432) + \left[1 - \left(\frac{51.61}{180}\right)^3\right]12{,}413$

$= 12{,}555 \text{ in.}^4$

$E_c = 33(125)^{1.5}\left(\sqrt{4000}\right) = 2.917 \times 10^6 \text{ psi}$, $n = \frac{29}{2.917} = 9.94$ use 10

$\delta_D = \frac{5 w \ell^4}{384 EI} = \frac{(5)\left(\frac{1600}{12}\right)(12 \times 30)^4}{(384)(2.917 \times 10^6)(12555)} = \boxed{0.796 \text{ in.}}$

(b) Instantaneous or short-term deflection for full dead and live load (δ_{D+L})

$M_a = \frac{(2.8)(30)^2}{8} = 315 \text{ ft-k}$

$I_e = \left(\frac{51.61}{315}\right)^3(18{,}432) + \left[1 - \left(\frac{51.61}{315}\right)^3\right]12{,}413$

$= 12{,}440 \text{ in.}^4$

$\delta_{D+L} = \frac{(5)\left(\frac{2800}{12}\right)(12 \times 30)^4}{(384)(2.917 \times 10^6)(12{,}440)} = \boxed{1.406 \text{ in.}}$

PROB # 6.14(2)

(c) Initial deflection for full live load (δ_L)

$$\delta_L = \delta_{D+L} - \delta_D = 1.406 - 0.796 = \boxed{0.610 \text{ in.}}$$

(d) Initial deflection due to dead load + 30% live load ($\delta_D + \delta_{SL}$)

$$M_a = \frac{[1.6 + (0.3)(1.2)](30)^2}{8} = 220.5 \text{ ft-k}$$

$$I_e = \left(\frac{51.61}{220.5}\right)^3 (18,432) + \left[1 - \left(\frac{51.61}{220.5}\right)^3\right] 12,413 = 12,490 \text{ in.}^4$$

$$\delta_D + \delta_{SL} = \frac{(5)\left(\frac{1600 + 0.30 \times 1200}{12}\right)(12 \times 30)^4}{(384)(2.917 \times 10^6)(12,490)} = \boxed{0.980 \text{ in.}}$$

(e) Initial deflection due to 30% live load (δ_{SL})

$$\delta_{SL} = (\delta_D + \delta_{SL}) - \delta_D = 0.980 - 0.796 = \boxed{0.184 \text{ in.}}$$

(f) Long term deflection for dead load plus four years of 30% sustained live load (δ_{LT})

$$\rho' = \frac{2.0}{(16)(21)} = 0.005952$$

$$\lambda_\infty = \frac{2.0}{1 + (50)(0.005952)} = 1.54$$

$$\lambda_{4yrs} = \frac{1.85}{1 + (50)(0.005952)} = 1.43$$

$$\delta_{LT} = \delta_L + \lambda_\infty \delta_D + \lambda_{4yrs} \delta_{SL}$$

$$= 0.610 + (1.54)(0.796) + (1.43)(0.184)$$

$$= \boxed{2.10 \text{ in.}}$$

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