Chapter 11

11.1

$$\begin{aligned}(\mathbf{a})\ \ m &= \frac{24\text{ lb}}{5.32\text{ ft/s}^2} = 4.511\text{ slugs} \blacktriangleleft \\ (\mathbf{b})\ \ W &= mg = 4.511(32.2) = 145.3\text{ lb} \ \blacktriangleleft\end{aligned}$$

11.2

$$\begin{aligned}W &= \frac{1}{3}\pi R^2 h\rho g = \frac{\pi}{3}(0.075^2)(0.125)(2700)(9.81) = 19.503\text{ N} \\ &= (19.503\text{ N})\left(\frac{0.2284\text{ lb}}{1.0\text{ N}}\right) = 4.45\text{ lb} \blacktriangleleft\end{aligned}$$

11.3

(a) $100\text{ kN/m}^2 = \dfrac{100\times 10^3\text{ N}}{\text{m}^2}\times\dfrac{0.2248\text{ lb}}{1.0\text{ N}}\times\dfrac{1.0\text{ m}^2}{1550\text{ in.}^2} = 14.50\text{ lb/in.}^2 \blacktriangleleft$

(b) $30\text{ m/s} = \dfrac{30\text{ m}}{\text{s}}\times\dfrac{3.281\text{ ft}}{1.0\text{ m}}\times\dfrac{1.0\text{ mi}}{5280\text{ ft}}\times\dfrac{3600\text{ s}}{1.0\text{ h}} = 67.1\text{ mi/h} \blacktriangleleft$

(c) $800\text{ slugs} = 800\text{ slugs}\times\dfrac{14.593\text{ kg}}{1.0\text{ slug}} = 11.67\times 10^3\text{ kg} = 11.67\text{ Mg} \blacktriangleleft$

(d) $20\text{ lb/ft}^2 = \dfrac{20\text{ lb}}{\text{ft}^2}\times\dfrac{4.448\text{ N}}{1.0\text{ lb}}\times\dfrac{1.0\text{ ft}^2}{0.092\ 903\ 04\text{ m}^2} = 958\text{ N/m}^2 \blacktriangleleft$

11.4

$$I = 20\text{ kg}\cdot\text{m}^2 = 20\text{ kg}\cdot\text{m}^2\times\frac{0.06853\text{ slugs}}{1.0\text{ kg}}\times\frac{10.764\text{ ft}^2}{1.0\text{ m}^2} = 14.75\text{ slugs}\cdot\text{ft}^2$$

But 1.0 slug = 1.0 lb·s^2/ft

$$I = 14.75\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\text{ft}^2 = 14.75\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

11.5

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

Since the dimensions of each term must be the same, we have

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = [M]\left[k^2\right]\left[\frac{1}{T^2}\right]$$

Therefore,

$$[k] = [L]$$

(a) In the SI system

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \blacktriangleleft \qquad [k] = \text{m} \blacktriangleleft$$

(b) In the US system

$$\begin{aligned} [KE] &= [M]\left[\frac{L^2}{T^2}\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L^2}{T^2}\right] = [FL] = \text{lb}\cdot\text{ft} \blacktriangleleft \\ [k] &= \text{ft} \blacktriangleleft \end{aligned}$$

11.6

$$[g]\,[k]\,[x]\left[\frac{1}{W}\right] = \left[\frac{L}{T^2}\right]\left[\frac{F}{L}\right][L]\left[\frac{1}{F}\right] = \left[\frac{L}{T^2}\right] = [a] \quad \text{Q.E.D.}$$

11.7

(a)

$$[\delta] = \left[\frac{PL}{EA}\right] \qquad [L] = \left[\frac{1}{E}\right]\left[\frac{FL}{L^2}\right] \qquad [E] = \left[\frac{F}{L^2}\right] \blacktriangleleft$$

(b) Substituting $[F] = \left[ML/T^2\right]$ into the result of part (a):

$$[E] = \left[\frac{ML}{T^2}\right]\left[\frac{1}{L^2}\right] = \left[\frac{M}{T^2L}\right] \blacktriangleleft$$

11.8

(a) $\left[mv^2\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L^2}{T^2}\right] = [FL] \blacktriangleleft$

(b) $[mv] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T}\right] = [FT] \blacktriangleleft$

(c) $[ma] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F] \blacktriangleleft$

11.9

Rewrite the equation as $y = 1.0\ x^2$

$$[y] = [1.0]\left[x^2\right] \qquad [L] = [1.0]\left[L^2\right] \qquad [1.0] = \left[\frac{1}{L}\right]$$

$y = x^2$ can be dimensionally correct only if the units of the implied constant 1.0 are in.$^{-1}$. ◀

11.10

(a) $[I] = \left[mR^2\right] = \left[\frac{FT^2}{L}\right]\left[L^2\right] = \left[FLT^2\right]$ ◀

(b) $[I] = \left[mR^2\right] = \left[ML^2\right]$ ◀

11.11

(a) $\left[v^3\right] = [A]\left[x^2\right] + [B]\,[v]\left[t^2\right] \qquad \left[\frac{L^3}{T^3}\right] = [A]\left[L^2\right] + [B]\left[\frac{L}{T}\right]\left[T^2\right]$

$$[A] = \left[\frac{L}{T^3}\right] \blacktriangleleft \qquad [B] = \left[\frac{L^2}{T^4}\right] \blacktriangleleft$$

(b) $\left[x^2\right] = [A]\left[t^2\right]\left[e^{[B]\left[t^2\right]}\right] \qquad \left[L^2\right] = [A]\left[T^2\right][1] \qquad [B]\left[T^2\right] = [1]$

$$[A] = \left[\frac{L^2}{T^2}\right] \blacktriangleleft \qquad [B] = \left[\frac{1}{T^2}\right] \blacktriangleleft$$

11.12

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P_0 \sin\omega t$$

$$[m]\left[\frac{d^2x}{dt^2}\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F]$$

Therefore, the dimension of each term in the expression is $[F]$.

$$[c]\left[\frac{dx}{dt}\right] = [c]\left[\frac{L}{T}\right] = [F] \qquad [c] = \left[\frac{FT}{L}\right] \blacktriangleleft$$

$$[k]\,[x] = [k]\,[L] = [F] \qquad [k] = \left[\frac{F}{L}\right] \blacktriangleleft$$

$$[P_0]\,[\sin\omega t] = [P_0]\,[1] = [F] \qquad [P_0] = [F] \blacktriangleleft$$

$$[\omega]\,[t] = [\omega]\,[T] = [1] \qquad [\omega] = \left[\frac{1}{T}\right] \blacktriangleleft$$

11.13

$$F = G\frac{m_A m_B}{R^2} \qquad G = \frac{FR^2}{m_A m_b} \qquad [G] = \frac{[F]\left[L^2\right]}{[M^2]}$$

(a) $[G] = \dfrac{[F]\left[L^2\right]}{[FT^2/L]^2} = \left[\dfrac{L^4}{FT^4}\right]$ ◄

(b) $[G] = \dfrac{\left[ML/T^2\right]\left[L^2\right]}{[M^2]} = \left[\dfrac{L^3}{MT^2}\right]$ ◄

11.14

Using the base dimensions of an absolute [MLT] system:

$$[F] = [C][\rho][v^2][A] \qquad \left[\frac{ML}{T^2}\right] = [C]\left[\frac{M}{L^3}\right]\left[\frac{L^2}{T^2}\right][L^2] \qquad [C] = [1] \text{ Q.E.D}$$ ◄

11.15

$$\begin{aligned}
F &= G\frac{m^2}{R^2} = (6.67 \times 10^{-11})\frac{8^2}{0.4^2} = 2.668 \times 10^{-8} \text{ N} \\
W &= mg = 8(9.81) = 78.48 \text{ N} \\
\frac{F}{W} \times 100\% &= \frac{2.668 \times 10^{-8}}{78.48} \times 100\% = 3.40 \times 10^{-8}\ \%
\end{aligned}$$ ◄

11.16

$$F = G\frac{m^2}{R^2} = \left(3.44 \times 10^{-8}\right)\frac{(2/32.2)^2}{(16/12)^2} = 7.46 \times 10^{-11} \text{ lb}$$ ◄

11.17

$$m = \frac{WR^2}{GM_e} = \frac{(3000)\,(6378 + 1600)^2 \times 10^6}{(6.67 \times 10^{-11})\,(5.9742 \times 10^{24})} = 479 \text{ kg}$$ ◄

11.18

$$g_m = \frac{GM_m}{R_m^2} \qquad g_e = \frac{GM_e}{R_e^2}$$

$$\frac{g_m}{g_e} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{0.073483}{5.9742}\left(\frac{6378}{1738}\right)^2 = 0.1656 \approx \frac{1}{6} \text{ Q.E.D}$$

11.19

$$M_e = 5.9742 \times 10^{24}\ \text{kg}\ \times \frac{0.06853\ \text{slugs}}{1.0\ \text{kg}} = 0.4094 \times 10^{24}\ \text{slugs}$$

$$R_e = 6378 \times 10^3\ \text{m}\ \times \frac{3.281\ \text{ft}}{1.0\ \text{m}} = 20.93 \times 10^6\ \text{ft}$$

$$W = G\frac{M_e m}{(2R_e)^2} = 3.44 \times 10^{-8}\frac{(0.4094 \times 10^{24})(150/32.2)}{(2 \times 20.93 \times 10^6)^2} = 37.4\ \text{lb}\ \blacktriangleleft$$

11.20

$$F = G\frac{M_s m}{R^2} = 6.67 \times 10^{-11}\frac{\left(1.9891 \times 10^{30}\right)(1.0)}{(149.6 \times 10^9)^2} = 0.00593\ \text{N}\ \blacktriangleleft$$

11.21

$$\begin{aligned}
G\frac{M_e m}{r^2} &= G\frac{M_s m}{(R-r)^2} \qquad \frac{M_e}{r^2} = \frac{M_s}{(R-r)^2} \\
\frac{M_e}{M_s} &= \frac{r^2}{R^2 - 2Rr + r^2} \\
\frac{5.9742 \times 10^{24}}{1.9891 \times 10^{30}} &= \frac{r^2}{(149.6 \times 10^9)^2 - 2(149.6 \times 10^9)r + r^2} \\
0 &= 2.238 \times 10^{22} - 2.992 \times 10^{11} r - 3.329\,4 \times 10^5 r^2
\end{aligned}$$

$$r = 259 \times 10^6\ \text{m} = 259 \times 10^3\ \text{km}\ \blacktriangleleft$$

Chapter 12

12.1

$$
\begin{aligned}
y &= -0.16t^4 + 4.9t^3 + 0.14t^2 \text{ ft} \\
v &= \dot{y} = -0.64t^3 + 14.7t^2 + 0.28t \text{ ft/s} \\
a &= \dot{v} = -1.92t^2 + 29.4t + 0.28 \text{ ft/s}^2
\end{aligned}
$$

At maximum velocity ($a = 0$):

$$-1.92t^2 + 29.4t + 0.28 = 0 \qquad t = 15.322 \text{ s}$$

$$
\begin{aligned}
v_{\max} &= -0.64(15.322^3) + 14.7(15.322^2) + 0.28(15.322) \\
&= 1153 \text{ ft/s} \blacktriangleleft \\
y &= -0.16(15.322^4) + 4.9(15.322^3) + 0.14(15.322^2) \\
&= 8840 \text{ ft} \blacktriangleleft
\end{aligned}
$$

12.2

(a) $x = -\frac{1}{2}gt^2 + v_0 t \quad \therefore v = \dot{x} = -gt + v_0 \blacklozenge \quad \therefore a = \ddot{x} = -g \blacklozenge$

When $t = 0$, then $x = 0$ and $v = v_0$. Hence v_0 is the <u>initial velocity</u>.

Since gravity is the only source of acceleration in this problem, g must be the <u>gravitational acceleration</u>.

(b) **When $x = x_{max}$, then $v = 0$.** $\therefore -gt + v_0 = 0 \quad \therefore t = \frac{v_0}{g}$

$$\therefore x_{max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g} \blacklozenge$$

At the end of flight $x = 0$. $\therefore -\frac{1}{2}gt^2 + v_0 t = 0 \quad \therefore t = \frac{2v_0}{g} \blacklozenge$

(c) $v_0 = \frac{(60 \text{ mi/h})(5280 \text{ ft/mi})}{3600 \text{ s/h}} = 88 \text{ ft/s}$

$$\therefore x_{max} = \frac{(88)^2}{2(32.2)} = 120.2 \text{ ft} \blacklozenge \quad \therefore t = \frac{2(88)}{32.2} = 5.47 \text{ s} \blacklozenge$$

12.65

$$x = 20\cos\frac{\pi}{2}t \text{ ft} \qquad y = 64.4(4-t^2) \text{ ft}$$
$$\dot{x} = -10\pi\sin\frac{\pi}{2}t \text{ ft/s} \qquad \dot{y} = -128.8t \text{ ft/s}$$
$$\ddot{x} = -5\pi^2\cos\frac{\pi}{2}t \text{ ft/s}^2 \qquad \ddot{y} = -128.8 \text{ ft/s}^2$$

$$F_x = m\ddot{x} = \frac{4}{32.2}\left(-5\pi^2\cos\frac{\pi}{2}t\right) = -6.130\cos\frac{\pi}{2}t \text{ lb}$$
$$F_y = m\ddot{y} = \frac{4}{32.2}(-128.8) = -16.00 \text{ lb}$$

t (s)	F_x (lb)	F_y (lb)
0	−6.13	−16.0
1	0	−16.0 ◄
2	6.13	−16.0

12.66

(a) $x = v_0 t - r\sin\frac{v_0 t}{R} \qquad \therefore v_x = \dot{x} = v_0 - \frac{v_0 r}{R}\cos\frac{v_0 t}{R} \qquad \therefore a_x = \ddot{x} = \frac{v_0^2 r}{R^2}\sin\frac{v_0 t}{R}$

$y = R - r\cos\frac{v_0 t}{R} \qquad \therefore v_y = \dot{y} = \frac{v_0 r}{R}\sin\frac{v_0 t}{R} \qquad \therefore a_y = \ddot{y} = \frac{v_0^2 r}{R^2}\cos\frac{v_0 t}{R}$

$a = \sqrt{a_x^2 + a_y^2} = \frac{v_0^2 r}{R^2} = \text{constant}$ Q.E.D.

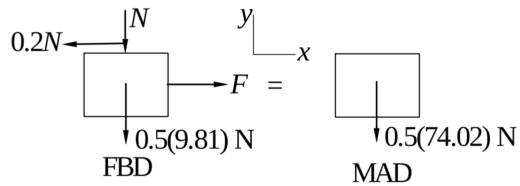
(b) $F = ma = \frac{m v_0^2 r}{R^2} = \frac{\left(\frac{2}{16}\frac{1}{32.2}\right)\left(60\,\frac{5280}{3600}\right)^2(0.8)}{1.25^2} = 15.39 \text{ lb}$ ◆

12.67

$$x = b\sin\frac{2\pi t}{t_0} \qquad y = \frac{b}{4}\left(1+\cos\frac{4\pi t}{t_0}\right)$$
$$v_x = \frac{2\pi b}{t_0}\cos\frac{2\pi t}{t_0} \qquad v_y = -\frac{\pi b}{t_0}\sin\frac{4\pi t}{t_0}$$
$$a_x = -\frac{4\pi^2 b}{t_0^2}\sin\frac{2\pi t}{t_0} \qquad a_y = -\frac{4\pi^2 b}{t_0^2}\cos\frac{4\pi t}{t_0}$$

At point B: $x = 0 \therefore t = 0$

$$\therefore a_x = 0 \qquad \therefore a_y = -\frac{4\pi^2 b}{t_0^2} = -\frac{4\pi^2(1.2)}{0.8^2} = -74.02 \text{ m/s}^2$$



$$\Sigma F_y = ma_y \quad +\uparrow \qquad -N - 0.5(9.81) = -0.5(74.02)$$

$$N = 32.11 \text{ N}$$

$$\Sigma F_x = 0 \quad \overset{+}{\longrightarrow} \qquad F - 0.2N = 0$$

$$F = 0.2N = 0.2(32.11) = 6.42 \text{ N} \quad \blacktriangleleft$$

12.68

$$x = \frac{b}{2}\left(\sin\frac{\pi t}{4t_0} + \sin\frac{3\pi t}{4t_0}\right) \quad \therefore\ v_x = \dot{x} = \frac{b}{2}\left(\frac{\pi}{4t_0}\cos\frac{\pi t}{4t_0} + \frac{3\pi}{4t_0}\cos\frac{3\pi t}{4t_0}\right)$$

$$\therefore\ a_x = \ddot{x} = -\frac{b}{2}\left(\frac{\pi^2}{16t_0^2}\sin\frac{\pi t}{4t_0} + \frac{9\pi^2}{16t_0^2}\sin\frac{3\pi t}{4t_0}\right) = -\frac{\pi^2 b}{32t_0^2}\left(\sin\frac{\pi t}{4t_0} + 9\sin\frac{3\pi t}{4t_0}\right)$$

$$y = \frac{b}{2}\left(\cos\frac{\pi t}{4t_0} - \cos\frac{3\pi t}{4t_0}\right) \quad \therefore\ v_y = \dot{y} = \frac{b}{2}\left(-\frac{\pi}{4t_0}\sin\frac{\pi t}{4t_0} + \frac{3\pi}{4t_0}\sin\frac{3\pi t}{4t_0}\right)$$

$$\therefore\ a_y = \ddot{y} = \frac{b}{2}\left(-\frac{\pi^2}{16t_0^2}\cos\frac{\pi t}{4t_0} + \frac{9\pi^2}{16t_0^2}\cos\frac{3\pi t}{4t_0}\right) = \frac{\pi^2 b}{32t_0^2}\left(-\cos\frac{\pi t}{4t_0} + 9\cos\frac{3\pi t}{4t_0}\right)$$

At point A: $t = t_0$

$$\therefore a_x = -\frac{\pi^2 b}{32t_0^2}\left(\sin\frac{\pi}{4} + 9\sin\frac{3\pi}{4}\right) = -\frac{\pi^2(240)}{32(12)^2}(7.071) = -3.635 \text{ m/s}^2$$

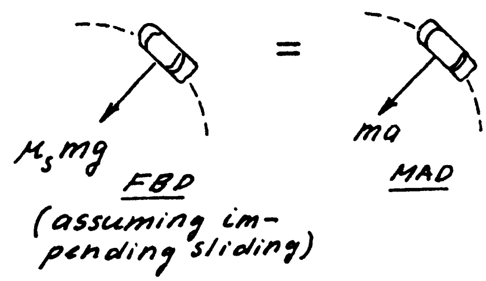
$$\therefore a_y = \frac{\pi^2 b}{32t_0^2}\left(-\cos\frac{\pi}{4} + 9\cos\frac{3\pi}{4}\right) = -\frac{\pi^2(240)}{32(12)^2}(7.071) = -3.635 \text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = 5.141 \text{ m/s}^2$$

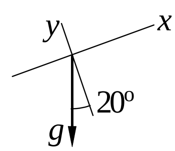
The FBD and MAD show only forces acting in the plane of the motion.

$\Sigma F = ma$: $\mu_s mg = ma$

$$\therefore \mu_s = \frac{a}{g} = \frac{5.141}{9.81} = 0.524 \; \blacklozenge$$



12.69



From the acceleration diagram of a water droplet:

$$a_x = -g\sin 20^\circ = -32.2\sin 20^\circ = -11.013 \text{ ft/s}^2$$
$$a_y = -g\cos 20^\circ = -32.2\cos 20^\circ = -30.26 \text{ ft/s}^2$$

Initial conditions at $t = 0$:

$$x = y = 0$$
$$v_x = 22\cos 30^\circ = 19.053 \text{ ft/s} \qquad v_y = 22\sin 30^\circ = 11.0 \text{ ft/s}$$

Integrating and using initial conditions:

$$v_x = -11.013t + 19.053 \text{ ft/s} \qquad v_y = -30.26t + 11.0 \text{ ft/s}$$
$$x = -5.507t^2 + 19.053t \text{ ft} \qquad y = -15.13t^2 + 11.0t \text{ ft}$$

Droplet lands when $y = 0$:

$$y = -15.13t^2 + 11.0t = 0 \qquad t = 0.7270 \text{ s}$$
$$R = x|_{t=0.7270\text{s}} = -5.507(0.7270^2) + 19.053(0.7270) = 10.94 \text{ ft} \;\blacktriangleleft$$

12.70

From Eqs. (e) of Sample Problem 2.11:

$$\begin{aligned} x &= (v_0 \cos\theta)t = (65\cos 55^\circ)\, t = 37.28t \text{ ft} \\ y &= -\frac{1}{2}gt^2 + (v_0 \sin\theta)t = -\frac{32.2}{2}t^2 + (65\sin 55^\circ)t \\ &= -16.1t^2 + 53.24t \text{ ft} \end{aligned}$$

At point B:

$$\begin{aligned} x &= 60 \text{ ft} \qquad 60 = 37.28t \qquad t = 1.6094 \text{ s} \\ h &= y|_{t=1.6094\text{s}} = -16.1(1.6094^2) + 53.24(1.6094) = 44.0 \text{ ft} \ \blacktriangleleft \end{aligned}$$

*12.71

From Sample Problem 12.12:

$$\begin{aligned} x &= C_1 e^{-ct/m} + C_2 \qquad y = C_3 e^{-ct/m} - \frac{mgt}{c} + C_4 \\ v_x &= -C_1 \frac{c}{m} e^{-ct/m} \qquad v_y = -C_3 \frac{c}{m} e^{-ct/m} - \frac{mg}{c} \end{aligned}$$

$$\begin{aligned} \frac{c}{m} &= \frac{0.0025}{1.2/32.2} = 0.06708 \text{ s}^{-1} \qquad \frac{mg}{c} = \frac{1.2}{0.0025} = 480.0 \text{ ft/s} \\ x &= C_1 e^{-0.06708t} + C_2 \qquad y = C_3 e^{-0.06708t} - 480t + C_4 \\ v_x &= -C_1 \left(0.06708 e^{-0.06708t}\right) \\ v_y &= -C_3 \left(0.06708 e^{-0.06708t}\right) - 480.0 \end{aligned}$$

$$\begin{aligned} v_0 \sin\theta &= 70\sin 65^\circ = 63.44 \text{ ft/s} \\ v_0 \cos\theta &= 70\cos 65^\circ = 29.58 \text{ ft/s} \end{aligned}$$

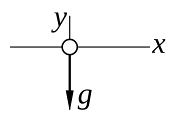
Initial conditions at $t = 0$:

$$\begin{aligned} x &= 0 \quad \therefore C_2 = -C_1 \\ y &= 0 \quad \therefore C_4 = -C_3 \\ v_x &= v_0 \cos\theta \quad \therefore -C_1(0.06708) = 29.58 \qquad C_1 = -441.0 \text{ ft} \\ v_y &= v_0 \sin\theta \quad \therefore -C_3(0.06708) - 480.0 = 63.44 \qquad C_3 = -8101 \text{ ft} \end{aligned}$$

When $x = 60$ ft:

$$\begin{aligned} 60 &= -441.0e^{-0.06708t} + 441.0 \qquad t = 2.180 \text{ s} \\ h &= y|_{t=2.180} = -8101e^{-(0.06708)(2.180)} - 480(2.180) + (8101) = 55.7 \text{ ft} \ \blacktriangleleft \end{aligned}$$

12.72



Acceleration diagram

At $t = 0$ (initial conditions):

$$x = 0 \qquad v_x = 200 \sin 30^\circ = 100 \text{ m/s}$$
$$y = 1200 \text{ m} \qquad v_y = -200 \cos 30^\circ = -173.21 \text{ m/s}$$

Integrating acceleration and applying initial conditions:

$$a_x = 0 \qquad a_y = -9.81 \text{ m/s}^2$$
$$v_x = 100 \text{ m/s} \qquad v_y = -9.81t - 173.21 \text{ m/s}$$
$$x = 100t \text{ m} \qquad y = -4.905t^2 - 173.21t + 1200 \text{ m}$$

When $y = 0$:

$$-4.905t^2 - 173.21t + 1200 = 0 \qquad t = 5.932 \text{ s}$$

$$x = 100(5.932) = 593.2 \text{ m}$$

$$d = 1200 \tan 30^\circ - 593.2 = 99.6 \text{ m} \blacktriangleleft$$

12.73

Eqs. (d) and (e) of Sample Problem 12.11:

$$x = v_0 t \cos\theta = 2500t \cos\theta \text{ ft}$$
$$y = v_0 t \sin\theta - \frac{1}{2} g t^2 = 2500t \sin\theta - 16.1t^2 \text{ ft}$$

Setting $x = R = 5280$ ft and solving for t:

$$5280 = 2500t \cos\theta \qquad t = \frac{5280}{2500 \cos\theta} = \frac{2.112}{\cos\theta} \text{ s}$$

Setting $y = 0$, we get after dividing by t:

$$2500 \sin\theta - 16.1t = 0 \qquad 2500 \sin\theta - 16.1\left(\frac{2.112}{\cos\theta}\right) = 0$$

$$\sin\theta\cos\theta = 16.1\left(\frac{2.112}{2500}\right) = 0.013\ 601$$

$$\frac{1}{2}\sin 2\theta = 0.013\ 601 \qquad \sin 2\theta = 2(0.013\ 601) = 0.02720$$

$$\theta = \frac{1}{2}\sin^{-1}(0.02720) = 0.779^\circ \blacktriangleleft$$

12.74

From Eqs. (e) of Sample Problem 12.11:

$$x = (v_0 \cos\theta)\, t \qquad y = -\frac{1}{2} g t^2 + (v_0 \sin\theta)\, t$$

(a)

$$\begin{aligned} x &= (42\cos 28^\circ)\, t = 37.08t \text{ ft} \qquad \therefore t = \frac{x}{37.08} \text{ s} \\ y &= -\frac{1}{2}(32.2)t^2 + (42\sin 28^\circ)\, t = -16.1t^2 + 19.718t \end{aligned}$$

Substituting for t:

$$\begin{aligned} y &= -16.1\left(\frac{x}{37.08}\right)^2 + 19.718\left(\frac{x}{37.08}\right) \\ &= -0.01171x^2 + 0.5318x \text{ ft} \quad \blacktriangleleft \end{aligned}$$

(b) Check if ball hits the ceiling.

$$\begin{aligned} \frac{dy}{dx} &= -0.02342x + 0.5318 = 0 \qquad x = 22.71 \text{ ft} \\ y_{\max} &= -0.01171(22.71)^2 + 0.5318(22.71) = 6.04 \text{ ft} \end{aligned}$$

Since $y_{\max} < 25$ ft, the ball will not hit the ceiling.
Check if ball clears the net. When $x = 22$ ft:

$$y = -0.01171(22)^2 + 0.5318(22) = 6.03 \text{ ft}$$

Since $y > 5$ ft, the ball clears the net. ◀
When $x = 42$ ft:

$$y = -0.01171(42)^2 + 0.5318(42) = 1.679 \text{ ft}$$

Since $y > 0$, the ball lands behind the baseline. ◀

12.75

From Eqs. (e) of Sample Problem 12.11:

$$x = (v_0 \cos\theta)\, t \qquad y = -\frac{1}{2} g t^2 + (v_0 \sin\theta)\, t$$

$$\begin{aligned} y &= -\frac{1}{2}(32.2)t^2 + (v_0 \sin 70^\circ)t = -16.1t^2 + 0.9397v_0 t \text{ ft} \\ v_y &= \dot{y} = -32.2t + 0.9397v_0 \text{ ft/s} \end{aligned}$$

When $y = y_{\max}$

$$v_y = 0 \qquad -32.2t + 0.9397v_0 = 0 \qquad t = 0.02918v_0 \text{ s}$$

$$y_{\max} = 27 \text{ ft} \qquad -16.1(0.02918v_0)^2 + 0.9397v_0(0.02918v_0) = 27$$

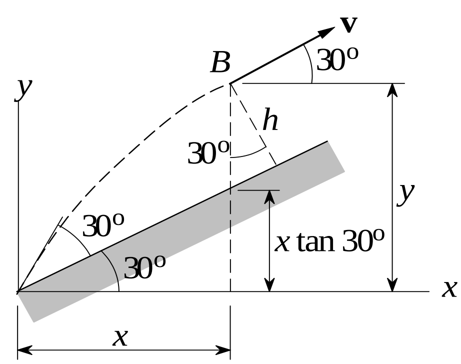
$$0.013712v_0^2 = 27 \qquad v_0 = 44.4 \text{ ft/s} \quad \blacktriangleleft$$

12.76

Equations (e) of Sample Problem 12.10:

$$\begin{aligned} x &= (v_0 \cos\theta)t = (30\cos 60^\circ)t = 15t \text{ ft} \\ y &= -\frac{1}{2}gt^2 + (v_0 \sin\theta)t = -\frac{1}{2}(32.2)t^2 + (30\sin 60^\circ)t \\ &= -16.1t^2 + 25.98t \text{ ft} \end{aligned}$$

$$v_x = \dot{x} = 15 \text{ ft/s} \qquad v_y = \dot{y} = -32.2t + 25.98 \text{ ft/s}$$



At point B:

$$\mathbf{v} \text{ is parallel to the inclined surface} \quad \therefore \frac{v_y}{v_x} = \tan 30^\circ$$

$$\frac{-32.2t + 25.98}{15} = \tan 30^\circ \qquad t = 0.5379 \text{ s}$$

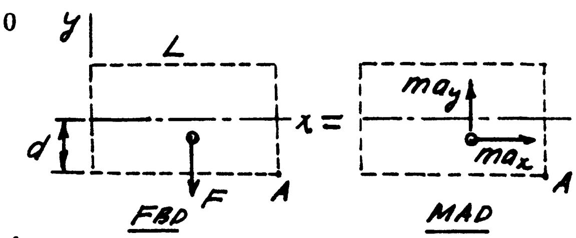
$$\begin{aligned} x &= 15(0.5379) = 8.069 \text{ ft} \\ y &= -16.1(0.5379^2) + 25.98(0.5379) = 9.316 \text{ ft} \\ h &= (y - x\tan 30^\circ)\cos 30^\circ = (9.316 - 8.069\tan 30^\circ)\cos 30^\circ \\ &= 4.03 \text{ ft} \blacktriangleleft \end{aligned}$$

12.77

$\Sigma F_x = ma_x$: $\overset{+}{\rightarrow}$ $0 = ma_x$ $\therefore a_x = 0$

$\Sigma F_y = ma_y$: $+\uparrow$ $-F = ma_y$

$\therefore a_y = -\dfrac{F}{m} = -\dfrac{q\,\Delta V}{2md}$



Horizontal motion

$v_x = \int a_x\,dt + C_1 = 0 + C_1$ $\quad x = \int v_x\,dt + C_2 = C_1 t + C_2$

Initial conditions: $x = 0$, $v_x = v_0$ when $t = 0$. $\therefore C_1 = v_0$ $\quad \therefore C_2 = 0$ $\quad \therefore x = v_0 t$

Vertical motion

$v_y = \int a_y\,dt + C_3 = -\dfrac{q\,\Delta V}{2md}t + C_3$ $\quad y = \int v_y\,dt + C_4 = -\dfrac{q\,\Delta V}{4md}t^2 + C_3 t + C_4$

Initial conditions: $y = 0$, $v_y = 0$ when $t = 0$. $\therefore C_3 = C_4 = 0$ $\quad \therefore y = -\dfrac{q\,\Delta V}{4md}t^2$

When particle is at A:

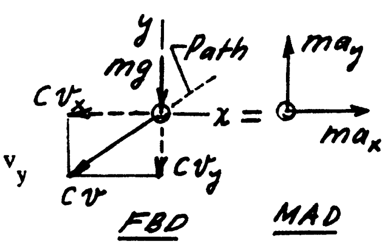
$x = L$ $\quad \therefore v_0 t = L$ $\quad \therefore t = L/v_0$

$y = -d$ $\quad \therefore -\dfrac{q\,\Delta V}{4md}(L/v_0)^2 = -d$ $\quad \therefore \Delta V = \dfrac{4m}{q}\left(v_0\dfrac{d}{L}\right)^2$ ◆

*12.78

$\Sigma F_x = ma_x$: $\overset{+}{\rightarrow}$ $-cv_x = ma_x$ $\quad \therefore a_x = -\dfrac{c}{m}v_x$

$\Sigma F_y = ma_y$: $+\uparrow$ $-mg - cv_y = ma_y$ $\quad \therefore a_y = -g - \dfrac{c}{m}v_y$



(a) Vertical motion

$\dfrac{dv_y}{dt} = a_y = -g - \dfrac{c}{m}v_y$ $\quad \therefore -dt = \dfrac{dv_y}{g + (c/m)v_y}$ $\quad \therefore -t = \dfrac{m}{c}\ln\left(g + \dfrac{c}{m}v_y\right) + C_1$

Initial condition: $v_y = v_0 \sin\alpha$ when $t = 0$. $\therefore C_1 = -\frac{m}{c}\ln\left(g + \frac{c}{m}v_0 \sin\alpha\right)$

$$\therefore t = \frac{m}{c}\ln\frac{g + (c/m)v_0 \sin\alpha}{g + (c/m)v_y}$$

At point A: $v_y = 0$ $\quad \therefore t = \frac{m}{c}\ln\left(1 + \frac{c}{mg}v_0 \sin\alpha\right)$ ◆

(b) Horizontal motion

$$\frac{dv_x}{dt} = a_x = -\frac{c}{m}v_x \qquad \therefore \frac{1}{v_x}dv_x = -\frac{c}{m}dt \qquad \therefore -\frac{c}{m}t = \ln v_x + C_2$$

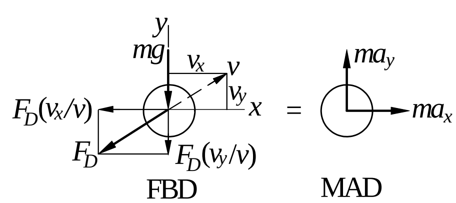
Initial condition: $v_x = v_0 \cos\alpha$ when $t = 0$. $\therefore C_2 = -\ln(v_0 \cos\alpha)$

$$\therefore -\frac{c}{m}t = \ln\frac{v_x}{v_0 \cos\alpha} \qquad \therefore v_x = (v_0 \cos\alpha)e^{-(c/m)t}$$

At point A: $v = v_x = (v_0 \cos\alpha)\left(1 + \frac{c}{mg}v_0 \sin\alpha\right)^{-1} = \dfrac{v_0 \cos\alpha}{1 + \frac{c}{mg}v_0 \sin\alpha}$ ◆

■12.79

(a)



$$\Sigma F_x = ma_x \quad \xrightarrow{+} \qquad -F_D\frac{v_x}{v} = ma_x$$

$$\begin{aligned} a_x &= -\frac{F_D}{m}\frac{v_x}{v} = -\frac{0.0005v^2}{0.1}\frac{v_x}{v} = -0.005vv_x \\ &= -0.005v_x\sqrt{v_x^2 + v_y^2} \text{ m/s}^2 \end{aligned}$$ ◄

$$\Sigma F_y = ma_y \quad +\uparrow \qquad -F_D\frac{v_y}{v} - mg = ma_y$$

$$\begin{aligned} a_y &= -\frac{F_D}{m}\frac{v_y}{v} - g = -\frac{0.0005v^2}{0.1}\frac{v_y}{v} - 9.81 = -0.005vv_y - 9.81 \\ &= -0.005v_y\sqrt{v_x^2 + v_y^2} - 9.81 \text{ m/s}^2 \end{aligned}$$ ◄

(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4$$

$$\dot{x}_3 = -0.005x_3\sqrt{x_3^2 + x_4^2} \qquad \dot{x}_4 = -0.005x_4\sqrt{x_3^2 + x_4^2} - 9.81$$

The initial conditions are

$$x_1(0) = 0 \qquad x_2(0) = 2 \text{ m/s}$$

$$x_3(0) = 30\cos 50^\circ = 19.284 \text{ m/s} \qquad x_4(0) = 30\sin 50^\circ = 22.981 \text{ m/s}$$

The following MATLAB program was used to integrate the equations:

```
function problem12_79
[t,x] = ode45(@f,(0:0.05:2),[0 2 19.284 22.981]);
printSol(t,x)
    function dxdt = f(t,x)
    v = sqrt(x(3)^2 + x(4)^2);
    dxdt = [x(3)
            x(4)
            -0.005*x(3)*v
            -0.005*x(4)*v-9.81];
    end
end
```

The two lines of output that span $x = 30$ m are

t	x1	x2	x3	x4
1.7000e+000	2.9607e+001	2.3944e+001	1.5990e+001	3.6997e+000
1.7500e+000	3.0405e+001	2.4117e+001	1.5925e+001	3.1952e+000

Linear interpolation for h:

$$\frac{30.405 - 29.607}{24.117 - 23.944} = \frac{30 - 29.607}{h - 23.944} \qquad h = 24.0 \text{ m} \blacktriangleleft$$

Linear interpolation for v_x and v_y:

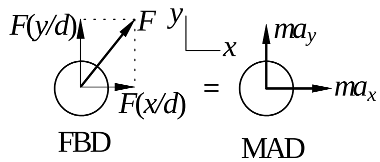
$$\frac{30.405 - 29.607}{15.925 - 15.990} = \frac{30 - 29.607}{v_x - 15.990} \qquad v_x = 15.958 \text{ m/s}$$

$$\frac{30.405 - 29.607}{3.1952 - 3.6997} = \frac{30 - 29.607}{v_y - 3.6997} \qquad v_y = 3.451 \text{ m/s}$$

$$\therefore v = \sqrt{15.958^2 + 3.451^2} = 16.33 \text{ m/s} \blacktriangleleft$$

■**12.80**

(a)



$$\Sigma F_x = ma_x \quad \xrightarrow{+} \qquad F\frac{x}{d} = ma_x$$

$$a_x = \frac{1}{m}F\frac{x}{d} = \frac{1}{0.01}\frac{0.005}{d^2}\frac{x}{d} = 0.5\frac{x}{d^3} = \frac{0.5x}{(x^2+y^2)^{3/2}} \text{ m/s}^2 \quad \blacktriangleleft$$

$$\Sigma F_y = ma_y \quad +\uparrow \qquad F\frac{y}{d} = ma_y$$

$$a_y = \frac{1}{m}F\frac{y}{d} = \frac{1}{0.01}\frac{0.005}{d^2}\frac{y}{d} = 0.5\frac{y}{d^3} = \frac{0.5y}{(x^2+y^2)^{3/2}} \text{ m/s}^2 \quad \blacktriangleleft$$

The initial conditions are:

$$x = 0.3 \text{ m} \quad y = 0.4 \text{ m} \quad v_x = 0 \quad v_y = -2 \text{ m/s} \quad \text{at } t = 0 \quad \blacktriangleleft$$

(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4 \qquad \dot{x}_3 = \frac{0.5x_1}{(x_1^2+x_2^2)^{3/2}} \qquad \dot{x}_4 = \frac{0.5x_2}{(x_1^2+x_2^2)^{3/2}}$$

The MATLAB program for solving the equations is

```
function problem12_80
[t,x] = ode45(@f,(0:0.005:0.25),[0.3 0.4 0 -2]);
printSol(t,x)
    function dxdt = f(t,x)
    d3 = (sqrt(x(1)^2 + x(2)^2))^3;
    dxdt = [x(3)
            x(4)
            0.5*x(1)/d3
            0.5*x(2)/d3];
    end
end
```

The two output lines spanning $y = 0$ are shown below.

```
  t            x1            x2            x3            x4
2.2000e-001  3.5879e-001  3.0450e-003  6.5959e-001 -1.6667e+000
2.2500e-001  3.6213e-001 -5.2884e-003  6.7883e-001 -1.6668e+000
```

Linear interpolation for x at $y = 0$:

$$\frac{0.36213 - 0.35879}{-0.0052884 - 0.0030450} = \frac{x - 0.35879}{0 - 0.0030450} \qquad x = 0.360 \text{ m} \blacktriangleleft$$

Linear interpolation for v_x:

$$v_x : \frac{0.67883 - 0.65959}{-0.0052884 - 0.0030450} = \frac{v_x - 0.65959}{0 - 0.0030450} \qquad v_x = 0.6666 \text{ m/s}$$

By inspection $v_y = -1.6667$ m/s.

$$\therefore v = \sqrt{0.6666^2 + 1.6667^2} = 1.795 \text{ m/s} \blacktriangleleft$$

■12.81

(a) The signs of a_x and a_y in the solution of Prob. 12.80 must be reversed.

$$a_x = -\frac{0.5x}{(x^2 + y^2)^{3/2}} \text{ m/s}^2 \blacktriangleleft \qquad a_y = -\frac{0.5y}{(x^2 + y^2)^{3/2}} \text{ m/s}^2 \blacktriangleleft$$

The initial conditions are the same as in Problem 12.80:

$$x = 0.3 \text{ m} \quad y = 0.4 \text{ m} \quad v_x = 0 \quad v_y = -2 \text{ m/s} \quad \text{at } t = 0. \blacktriangleleft$$

(b) MATLAB program:

```
function problem12_81
[t,x] = ode45(@f,(0:0.005:0.2),[0.3 0.4 0 -2]);
printSol(t,x)
    function dxdt = f(t,x)
    d3 = (sqrt(x(1)^2 + x(2)^2))^3;
    dxdt = [x(3)
            x(4)
            -0.5*x(1)/d3
            -0.5*x(2)/d3];
    end
end
```

The two lines of output spanning $y = 0$ are:

```
  t            x1            x2            x3            x4
1.8000e-001  2.5952e-001  8.5117e-003 -6.3935e-001 -2.3329e+000
1.8500e-001  2.5623e-001 -3.1544e-003 -6.7692e-001 -2.3333e+000
```

Linear interpolation for x at $y = 0$:

$$\frac{0.25623 - 0.25952}{-0.0031544 - 0.0085117} = \frac{x - 0.25952}{0 - 0.0085177} \qquad x = 0.257 \text{ m} \blacktriangleleft$$

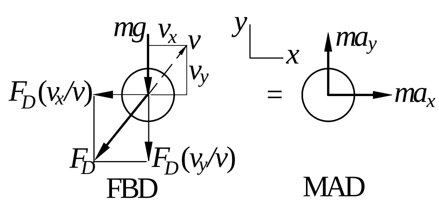
Linear interpolation for v_x:

$$\frac{-0.67692-(-0.63935)}{-0.0031544-0.0085117}=\frac{v_x-(-0.63935)}{0-0.0085177} \qquad v_x=-0.6668 \text{ m/s}$$

By inspection, $v_y=-2.3332$ m/s.

$$v=\sqrt{0.6668^2+2.3332^2}=2.43 \text{ m/s} \blacktriangleleft$$

■12.82



$$\Sigma F_x=ma_x \quad \xrightarrow{+} \qquad -F_D\frac{v_x}{v}=ma_x \qquad -c_Dv^{1.5}\frac{v_x}{v}=ma_x$$

$$\therefore a_x=-\frac{c_D}{m}v_x\sqrt{v}$$

$$\frac{c_D}{m}=\frac{0.0012}{(9/16)(32.2)}=0.06869 \text{ (ft}\cdot\text{s)}^{-0.5}$$

$$\therefore a_x=-0.06869v_x\left(v_x^2+v_y^2\right)^{0.25} \text{ ft/s}^2 \blacktriangleleft$$

$$\Sigma F_y=ma_y \quad +\uparrow \qquad -F_D\frac{v_y}{y}-mg=ma_y \qquad -c_Dv^{1.5}\frac{v_y}{y}-mg=ma_y$$

$$\therefore a_y=-\frac{c_D}{m}v_y\sqrt{v}-g=-0.06869v_y\left(v_x^2+v_y^2\right)^{0.25}-32.2 \text{ ft/s}^2 \blacktriangleleft$$

The initial conditions are:

$$x=0 \quad y=6 \text{ ft} \quad v_x=120 \text{ ft/s} \quad v_y=0 \quad \text{at } t=0 \blacktriangleleft$$

(b) Letting $x_1=x$, $x_2=y$, $x_3=v_x$ and $x_4=v_y$, the equivalent first-order equations are

$$\begin{aligned}\dot{x}_1 &= x_3 \qquad \dot{x}_2=x_4\\ \dot{x}_3 &= -0.06869x_3\left(x_3^2+x_4^2\right)^{0.25}\\ x_4 &= -0.06869x_4\left(x_3^2+x_4^2\right)^{0.25}-32.2\end{aligned}$$

The MATLAB program is

```
function problem12_82
[t,x] = ode45(@f,(0:0.02:0.7),[0 6 120 0]);
printSol(t,x)
    function dxdt = f(t,x)
    v25 = sqrt((sqrt(x(3)^2 + x(4)^2)));
    dxdt = [x(3)
            x(4)
            -0.06869*x(3)*v25
            -0.06869*x(4)*v25-32.2];
    end
end
```

The two lines of output that span $y = 0$ are:

```
  t           x1           x2           x3           x4
6.4000e-001  6.1878e+001  2.6073e-001  7.7840e+001 -1.6851e+001
6.6000e-001  6.3426e+001 -8.0650e-002  7.6894e+001 -1.7286e+001
```

Linear interpolation for x at $y = 0$:

$$\frac{63.426 - 61.878}{-0.080650 - 0.26073} = \frac{R - 61.878}{0 - 0.26072} \qquad R = 63.1 \text{ ft} \quad \blacktriangleleft$$

Linear interpolation for t at $y = 0$:

$$\frac{0.66 - 0.64}{-0.080650 - 0.26073} = \frac{t - 0.64}{0 - 0.26072} \qquad t = 0.655 \text{ s} \quad \blacktriangleleft$$

■12.83

$$a_x = -10 - 0.5v_x \text{ m/s}^2 \qquad a_y = -9.81 - 0.5v_y \text{ m/s}^2$$

(a) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4 \qquad \dot{x}_3 = -10 - 0.5x_3 \qquad \dot{x}_4 = -9.81 - 0.5x_4$$

At $t = 0$ (initial conditions):

$$x_1 = x_2 = 0 \qquad x_3 = 30\cos 40^\circ = 22.98 \text{ m/s} \qquad x_4 = 30\sin 40^\circ = 19.284 \text{ m/s}$$

MATLAB program:

```
function problem12_83
[t,x] = ode45(@f,(0:0.05:3.5),[0 0 22.98 19.284]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
```

```
grid on
xlabel('x (ft)'); ylabel('y (ft)')
    function dxdt = f(t,x)
    dxdt = [x(3)
            x(4)
            -10-0.5*x(3)
            -9.81-0.5*x(4)];
    end
end
```

Two lines of output that span $y = 0$:

```
 t           x1           x2           x3           x4
3.1000e+000  5.7152e+000  4.7141e-001 -1.0878e+001 -1.1363e+001
3.1500e+000  5.1656e+000 -1.0184e-001 -1.1103e+001 -1.1567e+001
```

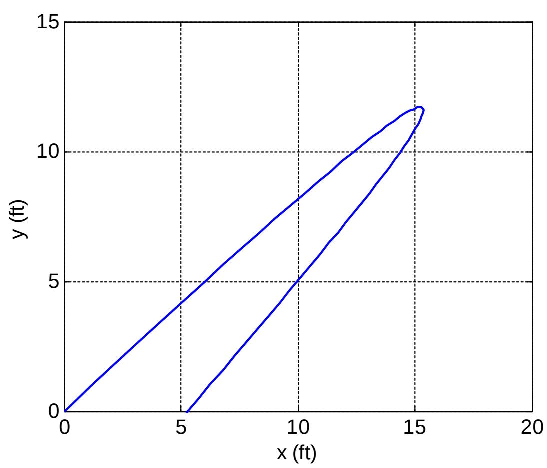
Linear interpolation for x at $y = 0$:

$$\frac{5.1656 - 5.7152}{-0.10184 - 0.47141} = \frac{b - 5.7152}{0 - 0.47141} \qquad b = 5.26 \text{ m} \blacktriangleleft$$

Linear interpolation for t at $y = 0$:

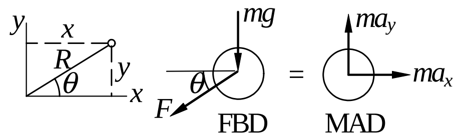
$$\frac{3.15 - 3.10}{-0.10184 - 0.47141} = \frac{t - 3.10}{0 - 0.47141} \qquad t = 3.14 \text{ s} \blacktriangleleft$$

(b)



■**12.84**

(a)



Spring force is $F = k(R - L_0)$, where $R = \sqrt{x^2 + y^2}$.

$$\Sigma F_x = ma_x \quad \xrightarrow{+} \qquad -F\cos\theta = ma_x$$

$$\begin{aligned} a_x &= -\frac{F}{m}\cos\theta = -\frac{k(R-L_0)}{m}\frac{x}{R} = -\frac{k}{m}\left(1 - \frac{L_0}{R}\right)x \\ &= -\frac{10}{0.25}\left(1 - \frac{0.5}{R}\right)x = -40\left(1 - \frac{0.5}{R}\right)x \text{ m/s}^2 \quad \blacktriangleleft \end{aligned}$$

$$\Sigma F_y = ma_y \quad +\uparrow \qquad -F\sin\theta - mg = ma_y$$

$$\begin{aligned} a_y &= -\frac{F}{m}\sin\theta - g = -\frac{k(R-L_0)}{m}\frac{y}{R} - g = -\frac{k}{m}\left(1 - \frac{L_0}{R}\right)y - g \\ &= -40\left(1 - \frac{0.5}{R}\right)y - 9.81 \text{ m/s}^2 \quad \blacktriangleleft \end{aligned}$$

The initial conditions are:

$$x = 0.5 \text{ m} \quad y = -0.5 \text{ m} \quad v_x = v_y = 0 \quad \text{at } t = 0 \quad \blacktriangleleft$$

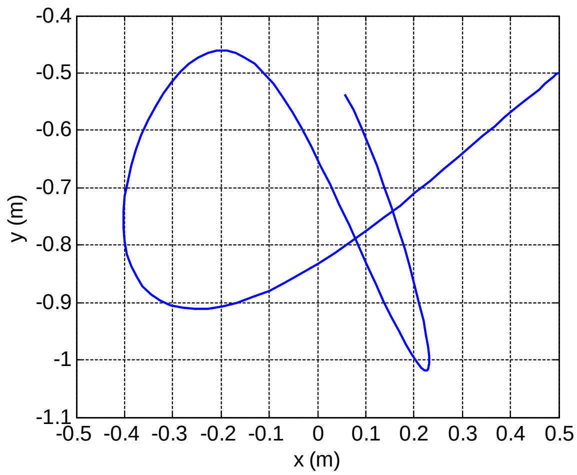
(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\begin{aligned} \dot{x}_1 &= x_3 \qquad \dot{x}_2 = x_4 \\ \dot{x}_3 &= -40\left(1 - \frac{0.5}{R}\right)x_1 \qquad \dot{x}_4 = -40\left(1 - \frac{0.5}{R}\right)x_2 - 9.81 \end{aligned}$$

where $R = \sqrt{x_1^2 + x_2^2}$.
The MATLAB program is:

```
function problem12_84
[t,x] = ode45(@f,(0:0.02:2),[0.5 -0.5 0 0]);
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('y (m)')
    function dxdt = f(t,x)
```

```
    rr = 1-0.5/sqrt(x(1)^2 + x(2)^2);
    dxdt = [x(3)
            x(4)
            -40*rr*x(1)
            -40*rr*x(2)-9.81];
    end
end
```



■12.85

(a) The expressions for the accelerations in Prob. 12.84 are now valid only when the spring is in tension. If the spring is not in tension, the spring force is zero. Therefore, we have

$$a_x = \begin{cases} -40\left(1 - \dfrac{0.5}{R}\right) x \text{ m/s}^2 & \text{if } R > 0.5 \text{ m} \\ 0 & \text{if } R \le 0.5 \text{ m} \end{cases} \blacktriangleleft$$

$$a_y = \begin{cases} -40\left(1 - \dfrac{0.5}{R}\right) y - 9.81 \text{ m/s}^2 & \text{if } R > 0.5 \text{ m} \\ -9.81 \text{ m/s}^2 & \text{if } R \le 0.5 \text{ m} \end{cases} \blacktriangleleft$$

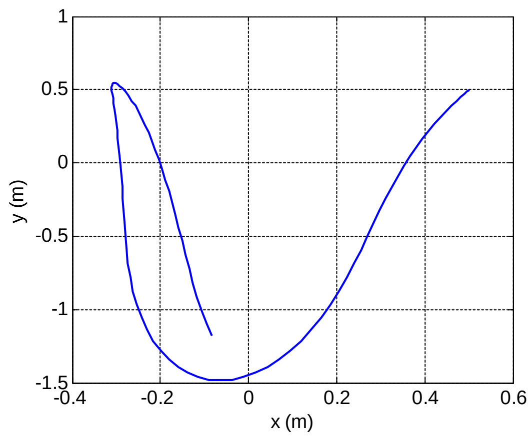
The initial conditions are:

$$x = y = 0.5\text{m} \quad v_y = v_y = 0 \quad \text{at } t = 0 \blacktriangleleft$$

(b) MATLAB program:

```
function problem12_85
[t,x] = ode45(@f,(0:0.02:2),[0.5 0.5 0 0]);
```

```
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (m)'); ylabel('y (m)')
    function dxdt = f(t,x)
    rr = 1-0.5/sqrt(x(1)^2 + x(2)^2);
    if rr < 0; rr = 0; end
    dxdt = [x(3)
            x(4)
            -40*rr*x(1)
            -40*rr*x(2)-9.81];
    end
end
```



■12.86

(a)

$$a_x = -a_D \cos\theta + a_L \sin\theta \qquad a_y = -a_D \sin\theta - a_L \cos\theta - g$$

Substitute

$$\sin\theta = \frac{v_y}{v} \qquad \cos\theta = \frac{v_x}{v} \qquad a_D = 0.05v^2$$
$$a_L = 0.16\omega v = 0.16(10)v = 1.6v$$

where $v = v_x^2 + v_y^2$.

$$a_x = -0.05v^2\frac{v_x}{v} + 1.6v\frac{v_y}{v} = -0.05vv_x + 1.6v_y \text{ ft/s}^2 \blacktriangleleft$$

$$a_y = -0.05v^2\frac{v_y}{v} - 1.6v\frac{v_x}{v} - 32.2 = -0.05vv_y - 1.6v_x - 32.2 \text{ ft/s}^2 \blacktriangleleft$$

The initial conditions at $t = 0$ are:

$$x = y = 0 \quad v_x = 60\cos 60^\circ = 30 \text{ ft/s} \quad v_y = 60\sin 60^\circ = 51.96 \text{ ft/s}$$

(b) Letting $x_1 = x$, $x_2 = y$, $x_3 = v_x$ and $x_4 = v_y$, the equivalent first-order equations are

$$\begin{aligned} \dot{x}_1 &= x_3 \qquad \dot{x}_2 = x_4 \\ \dot{x}_3 &= -0.05vx_3 + 16x_4 \qquad \dot{x}_4 = -0.05vx_4 - 16x_3 - 32.2 \end{aligned}$$

MATLAB program:

```
function problem12_86
[t,x] = ode45(@f,(0:0.02:1.2),[0 0 30 51.96]);
printSol(t,x)
axes('fontsize',14)
plot(x(:,1),x(:,2),'linewidth',1.5)
grid on
xlabel('x (ft)'); ylabel('y (ft)')
    function dxdt = f(t,x)
    v = sqrt(x(3)^2 + x(4)^2);
    dxdt = [x(3)
            x(4)
            -0.05*v*x(3)+1.6*x(4)
            -0.05*v*x(4)-1.6*x(3)-32.2];
    end
end
```

The two lines of output that span $y = 0$ are

```
  t           x1            x2            x3            x4
1.0400e+000  1.9377e+001  2.5868e-001  9.6888e-001 -2.2523e+001
1.0600e+000  1.9388e+001 -1.9335e-001  2.3210e-001 -2.2675e+001
```

Linear interpolation for t at $y = 0$:

$$\frac{1.06 - 1.04}{-0.19335 - 0.25868} = \frac{t - 1.04}{0 - 0.25868} \qquad t = 1.051 \text{ s} \blacktriangleleft$$

Linear interpolation for x at $y = 0$:

$$\frac{19.388 - 19.377}{-0.19335 - 0.25868} = \frac{x - 19.377}{0 - 0.25868} \qquad x = 19.38 \text{ ft} \blacktriangleleft$$