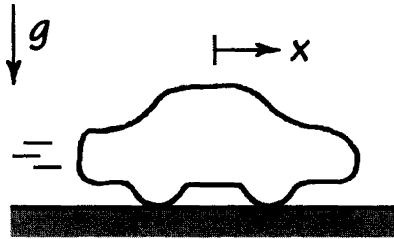


### 2.1.1

**GOAL:** Determine the distance and time needed for the car to reach its maximum speed.

**GIVEN:** The 2007 BMW Z4 Coupe 3.0si can accelerate from 0 to 96 km/hr in 5.6 s, and its maximum speed is  $\dot{x}_{\max} = 249$  km/hr. Assume that it accelerates from 0 to 96 km/hr at a constant rate and that this acceleration is maintained as the vehicle pushes toward its maximum speed.

**DRAW:**



### FORMULATE EQUATIONS:

Since we're assuming that the car's acceleration is constant, we can say that

$$\ddot{x} = \frac{\Delta \dot{x}}{\Delta t} \quad (1)$$

$$\dot{x}^2 - \dot{x}_0^2 = 2\ddot{x}\Delta x \quad (2)$$

### SOLVE:

To go from 0 to 96 km/hr in 5.6 s at a constant rate, the car's acceleration needs to be

$$(1) \Rightarrow \ddot{x} = \frac{(96 \text{ km/hr}) \left(\frac{\text{hr}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)}{5.6 \text{ s}} \\ \ddot{x} = 4.76 \text{ m/s}^2$$

If the car continues to accelerate at this rate, then it will reach its maximum speed after traveling

$$(2) \Rightarrow \Delta x_{\max} = \frac{(\dot{x}_{\max})^2}{2\ddot{x}} \\ \Delta x_{\max} = \frac{\left[(249 \text{ km/hr}) \left(\frac{\text{hr}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)\right]^2}{2(4.76 \text{ m/s}^2)}$$

$$\boxed{\Delta x_{\max} = 502.5 \text{ m} = 0.5025 \text{ km}}$$

The time it takes for the car to attain maximum speed is

(1)  $\Rightarrow$

$$\Delta t_{\max} = \frac{\dot{x}_{\max}}{\ddot{x}}$$

$$\Delta t_{\max} = \frac{(249 \text{ km/hr}) \left(\frac{\text{hr}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)}{4.76 \text{ m/s}^2}$$

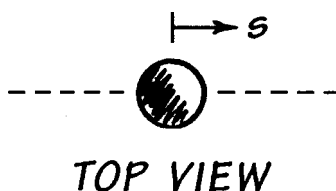
$$\boxed{\Delta t_{\max} = 14.5 \text{ s}}$$

2.1.5

**GOAL:** Determine the ball bearing's speed  $v$  after traveling the given distance.

**GIVEN:** The ball bearing's acceleration is described by  $\ddot{s} = as + be^{cs}$ , where  $a = 3\text{ s}^{-2}$ ,  $b = 0.3\text{ m/s}^2$ , and  $c = 0.06\text{ m}^{-1}$ . The ball bearing travels  $d = 3\text{ m}$ , and it starts from rest.

**DRAW:**



**FORMULATE EQUATIONS:**

Since the ball bearing's acceleration is a function of its position,

$$\begin{aligned}\ddot{s} &= \frac{d\dot{s}}{dt} = \frac{d\dot{s}}{ds} \cdot \frac{ds}{dt} = \frac{d\dot{s}}{ds} \dot{s} \\ \ddot{s} ds &= \dot{s} d\dot{s}\end{aligned}\tag{1}$$

**SOLVE:**

(1)  $\Rightarrow$

$$\begin{aligned}(as + be^{cs}) ds &= \dot{s} d\dot{s} \\ \int_0^d (as + be^{cs}) ds &= \int_0^v \dot{s} d\dot{s} \\ \left(\frac{1}{2}as^2 + \frac{b}{c}e^{cs}\right) \Big|_0^d &= \frac{1}{2}\dot{s}^2 \Big|_0^v \\ \frac{1}{2}ad^2 + \frac{b}{c}(e^{cd} - 1) &= \frac{1}{2}v^2 \\ v &= \sqrt{ad^2 + \frac{2b}{c}(e^{cd} - 1)}\end{aligned}$$

$$v = \sqrt{(3\text{ s}^{-2})(3\text{ m})^2 + \frac{2(0.3\text{ m/s}^2)}{0.06\text{ m}^{-1}}(e^{(0.06\text{ m}^{-1})(3\text{ m})} - 1)}$$

$$v = 5.38\text{ m/s} = 19.37\text{ km/h}$$

### 2.1.6

**GOAL:** Find the constant acceleration  $a_0$  that brings a jet from 274 km/hr to 0 km/h in 73m and the elapsed time  $\Delta t$ .

**GIVEN:** Distance needed to go from landing speed to zero.

**FORMULATE EQUATIONS:** Because the acceleration is constant we can use

$$v_2^2 - v_1^2 = 2a_0(x_2 - x_1) \quad (1)$$

$$v_2 - v_1 = a_0\Delta t \quad (2)$$

where  $a_0$  is the constant acceleration,  $\Delta t$  denotes the elapsed time, and the subscripts 1,2 indicate initial and final conditions, respectively.

**SOLVE:** First we'll convert 274 km/h to m/s:

$$\frac{(274\text{km})}{(1\text{hr})} \times \frac{(10^3\text{m})}{(1\text{km})} \times \frac{(1\text{hr})}{(3600\text{s})} = 76.1\text{m/s}$$

Using this, along with the known distance traveled, in (1) gives us

$$0 - (76.1\text{m/s})^2 = 2a_0(73\text{m})$$

$$\boxed{a_0 = -39.7\text{m/s}^2}$$

Because 1  $g$  is equal to  $9.81 \text{ m/s}^2$  we have

$$\boxed{a_0 = -39.7 \text{ m/s}^2 \frac{(1g)}{(9.81 \text{ m/s}^2)} = -4.05 g}$$

We can then use (2) to find the time  $\Delta t$  taken for the jet to come to a halt:

$$0 - (76.1\text{m/s}) = (-39.7\text{m/s}^2\Delta t)$$

$$\boxed{\Delta t = 1.92 \text{ s}}$$

## 2.1.7

**GOAL:** Find the needed deck space to allow a fighter jet to take off from an aircraft carrier.

**GIVEN:** The jet is brought from 0 to 265 km/h in 2.23 s.

**DRAW:**

$$\begin{array}{c} F \\ \circ \rightarrow \end{array} = \begin{array}{c} ma \\ \bullet \rightarrow \end{array}$$

$FBD = IRD$

**ASSUME:** We'll assume that the acceleration is constant.

**FORMULATE EQUATIONS:** We'll use the equation relating speed and acceleration, when starting from rest under a constant acceleration:

$$v = at$$

and the expression for distance traveled under a constant acceleration:

$$x = \frac{1}{2}at^2$$

**SOLVE:**

First we'll convert 265 km/h to m/s:

$$\frac{(265 \text{ km})}{(1 \text{ hr})} \times \frac{(10^3 \text{ m})}{(1 \text{ km})} \times \frac{(1 \text{ hr})}{(3600 \text{ s})} = 73.6 \text{ m/s}$$

The acceleration is therefore

$$a = \frac{v}{t} = \frac{73.6 \text{ m/s}}{2.23 \text{ s}} = 33 \text{ m/s}^2$$

The needed space is given by

$$x = \frac{1}{2}(33 \text{ m/s}^2)(2.23 \text{ s})^2$$

$$\boxed{x = 82 \text{ m}}$$

### 2.1.9

**GOAL:** Find the speed,  $\dot{x}$ , and the acceleration,  $\ddot{x}$ , at prescribed times.

**GIVEN:**  $x$  as a function of time.

**GOVERNING EQUATIONS:** The position of the car is given by:

$$x(t) = 30 \text{ m} - (27 \text{ m/s})t + (3 \text{ m/s}^2)t^2 \quad (1)$$

Differentiating with respect to time, we get:

$$\dot{x}(t) = -27 \text{ m/s} + (6 \text{ m/s}^2)t \quad (2)$$

and

$$\ddot{x}(t) = 6 \text{ m/s}^2 \quad (3)$$

**SOLVE:**

$$(2) \Rightarrow \boxed{\dot{x}(2) = -27 \text{ m/s} + (6 \text{ m/s}^2)(25) = -15 \text{ m/s}} \quad (4)$$

and

$$(3) \Rightarrow \boxed{\ddot{x}(10) = 6 \text{ m/s}^2} \quad (5)$$

## 2.1.10

**GOAL:** Find the constant acceleration  $a_0$  that brings a car from 0 to 96 km/h in 5 seconds.

**GIVEN:** Time needed to go from zero to 96 km/h.

**FORMULATE EQUATIONS:** Because the acceleration is constant we have

$$v(t) = v(0) + a_0 t \quad (1)$$

where  $a_0$  is a constant acceleration.

**SOLVE:** First we'll convert 96 km/h to m/s:

$$\frac{(96 \text{ km})}{(1 \text{ hr})} \times \frac{(10^3 \text{ m})}{(1 \text{ km})} \times \frac{(1 \text{ hr})}{(3600 \text{ s})} = 26.7 \text{ m/s}$$

Using this in (1) gives us

$$26.7 \text{ m/s} = 0 + a_0(5 \text{ s})$$

$$\boxed{a_0 = 5.3 \text{ m/s}^2}$$

Because 1  $g$  is equal to  $9.81 \text{ m/s}^2$  we have

$$\boxed{a_0 = 5.3 \text{ m/s}^2 \frac{(1 g)}{(9.81 \text{ m/s}^2)} = 0.54 g}$$

### 2.1.11

**GOAL:** Determine the time to bring a car to a stop from an initial speed along with the distance over which stopping occurs.

**GIVEN:** Initial speed of car and the fact that the car decelerates at  $1 g$ .

**FORMULATE EQUATIONS:**

For a constant acceleration,

$$v(t) = v(0) + at$$

$$x(t) = x(0) + v(0)t + \frac{at^2}{2}$$

**SOLVE:**

$$113 \text{ km/h} = \frac{113 \times 10^3}{8600} \text{ m/s} = 31.4 \text{ m/s}$$

$$v(t^*) = 31.4 \text{ m/s} - (9.81 \text{ m/s}^2)t^* = 0 \Rightarrow \boxed{t^* = 3.2 \text{ s}}$$

And for the distance:

$$x(t^*) = 0 + (31.4 \text{ m/s})t^* - \frac{9.81 \text{ m/s}^2}{2}t^{*2}$$

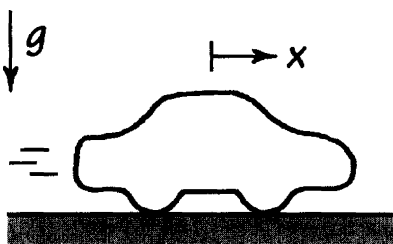
$$\boxed{\text{At } t^* = 3.2 \text{ s, } x = 50.3 \text{ m}}$$

## 2.1.12

**GOAL:** Determine the distance the car needs to reach its maximum speed and the change in its 0-to-60 time such that it and a competitor's car reach their respective maximum speed in the same distance.

**GIVEN:** The 2008 Audi TT Coupe can accelerate from 0 to 96 km/hr in 6.5 s and has a maximum speed of  $\dot{x}_{\max} = 237$  km/hr. Assume that it accelerates from 0 to 96 km/hr at a constant rate and that this acceleration is maintained as the vehicle pushes toward its maximum speed.

**DRAW:**



**FORMULATE EQUATIONS:**

Since we're assuming that the car's acceleration is constant, we can say that

$$\ddot{x} = \frac{\Delta \dot{x}}{\Delta t} \quad (1)$$

$$\dot{x}^2 - \dot{x}_0^2 = 2\ddot{x}\Delta x \quad (2)$$

**SOLVE:**

To go from 0 to 96 km/h in 6.5 s at a constant rate, the car's acceleration needs to be

(1)  $\Rightarrow$

$$\ddot{x} = \frac{(96 \text{ km/hr}) \left(\frac{\text{hr}}{3600 \text{ s}}\right) \left(\frac{10^3 \text{ m}}{\text{km}}\right)}{6.5 \text{ s}}$$

$$\ddot{x} = 4.1 \text{ m/s}^2$$

If the car continues to accelerate at this rate, then it will reach its maximum speed after traveling

(2)  $\Rightarrow$

$$\Delta x_{\max} = \frac{(\dot{x}_{\max})^2}{2\ddot{x}}$$

$$\frac{\left[ (237 \text{ km/hr}) \left( \frac{\text{hr}}{3600 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \right]^2}{2(4.1 \text{ m/s}^2)}$$

$$\boxed{\Delta x_{\text{max}} = 529 \text{ m} = 0.529 \text{ km}}$$

We find that it takes a longer distance for the TT to reach its maximum speed as compared to its competitor. For the car to attain maximum speed in 0.50 km like its competitor, its 0-to-60 time would need to be

(1)  $\rightarrow$  (2)  $\Rightarrow$

$$(\dot{x}_{\text{max}})^2 = \frac{2(\Delta \dot{x})(\Delta x)}{t^*}$$

$$t^* = \frac{2(\Delta \dot{x})(\Delta x)}{(\dot{x}_{\text{max}})^2}$$

$$t^* = \frac{2(96 \text{ km/hr}) \left( \frac{\text{hr}}{3600 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{\text{km}} \right) (0.5 \text{ km}) \left( \frac{10^3 \text{ m}}{\text{km}} \right)}{\left[ (237 \text{ km/hr}) \left( \frac{\text{hr}}{3600 \text{ s}} \right) \left( \frac{10^3 \text{ m}}{\text{km}} \right) \right]^2}$$

$$t^* = 6.15 \text{ s}$$

Thus, the car's 0-to-60 time would need to decrease by

$$\Delta t = 6.5 \text{ s} - 6.15 \text{ s}$$

$$\boxed{\Delta t = 0.35 \text{ s}}$$

2.1.13

**GOAL:** Determine a piston's maximum speed and acceleration.

**GIVEN:** Position of piston as a function of time.

**FORMULATE EQUATIONS:** The governing equation of motion is given as

$$x(t) = \frac{8.97}{2} \sin \omega t$$

where  $x$  is given in cm and  $\omega = 7000 \text{ rpm} = 733 \text{ rad/s}$ .

**SOLVE:**

$$v(t) = \frac{d}{dt}x(t) = \left(\frac{8.97 \text{ cm}}{2}\right) \omega \cos \omega t$$

$$v_{max} = \left(\frac{8.97 \text{ cm}}{2}\right) (733 \text{ rad/s}) = 3.29 \times 10^3 \text{ cm/s}$$

$$a(t) = \frac{d}{dt}v(t) = -\left(\frac{8.97 \text{ cm}}{2}\right) \omega^2 \sin \omega t$$

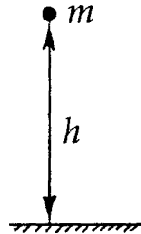
$$a_{max} = \left(\frac{8.97 \text{ cm}}{2}\right) (733 \text{ rad/s})^2 = 2.4 \times 10^6 \text{ cm/s}^2$$

2.1.14

**GOAL:** Find the height  $h$  for which a falling body will contact the ground at 56 km/hr.

**GIVEN:** Speed of contact.

**DRAW:**



**ASSUME:**

$$\begin{aligned}v_i &= 0 \\v_f &= 56 \text{ km/hr} = 15.6 \text{ m/s} \\a &= 9.81 \text{ m/s}^2\end{aligned}\tag{1}$$

**FORMULATE EQUATIONS:** We'll use the formula for the difference in speed due to a constant acceleration  $a$  over a distance  $h$ :

$$v_f^2 - v_i^2 = 2ah\tag{2}$$

**SOLVE:**

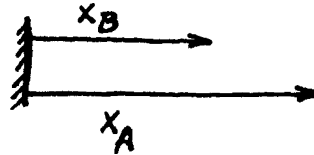
$$\begin{aligned}v_f^2 - v_i^2 &= 2gh \\(1) \rightarrow (2) \Rightarrow h &= \frac{v_f^2 - v_i^2}{2g} = \frac{(15.6 \text{ m/s})^2 - 0}{2(9.81 \text{ m/s}^2)} \\h &= \boxed{12.4 \text{ m}}\end{aligned}$$

2.1.16

**GOAL:** Find the constant speed needed for a pursuing cyclist to catch another cyclist.

**GIVEN:** Initial positions of the cyclists and the lead cyclist's speed.

**DRAW:**



**FORMULATE EQUATIONS:** We'll need to use the formula for position as a function of time due to a constant speed  $v$ :

$$x(t) = x(0) + vt$$

**SOLVE:** Bicyclist  $A$  has to travel 1610 m at the speed of 29 km/h. Thus we have

$$1610 \text{ m} = \left[ \frac{(29 \times 10^3)}{3600} \text{ m/s} \right] t \Rightarrow t = 200 \text{ s}$$

Bicyclist  $B$  has to travel an additional 229 m, for a total of 1839 m and has 200 s to do so. Thus we have

$$1839 \text{ m} = v_B(200 \text{ s})$$

$$v_B = 9.2 \text{ m/s} = 33.12 \text{ km/hr}$$

2.1.17

**GOAL:** Find the terminal speed of an object with a given acceleration and determine at what time it reaches 95 percent of terminal speed.

**GIVEN:**  $a_0 = 24,384 \text{ m/s}^2$ ,  $a_1 = 0.108 \text{ s/m}^2$

**FORMULATE EQUATIONS:** The acceleration is given by

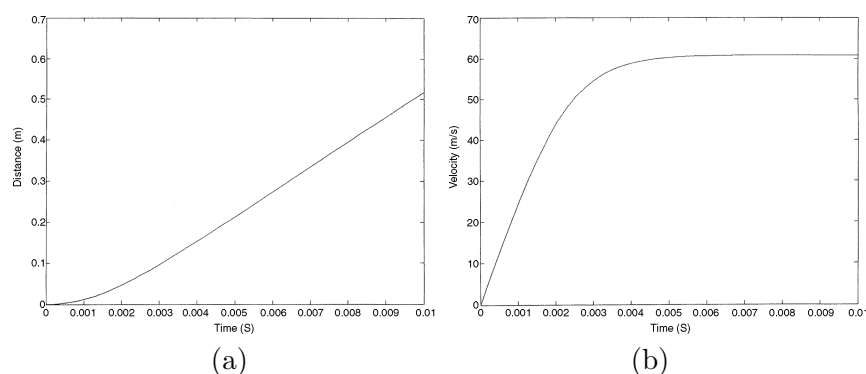
$$\ddot{s} = a_0 - a_1 \dot{s}^3$$

**SOLVE:**

We can numerically integrate using MATLAB with initial conditions of  $s(0) = 0$ ,  $\dot{s}(0) = 0$ . Using a time interval from  $t = 0$  to  $t = 0.01 \text{ s}$  yields the following plot:

It can be seen from the plot that the terminal speed is

$$v_{term} = 60.9 \text{ m/s}$$



This result can be seen analytically as well. When  $v_{term}$  is reached, the acceleration  $\ddot{s}$  is zero. Using this in our acceleration equation gives

$$0 = a_0 - a_1 v_{term}^3$$

which, when solved, returns the result  $v_{term} = 60.9 \text{ m/s}$ .

An examination of the output data allows the time at which the speed reaches 95 percent of its terminal value ( $0.95(60.9 \text{ m/s}) = 57.9 \text{ m/s}$ ) to be determined as

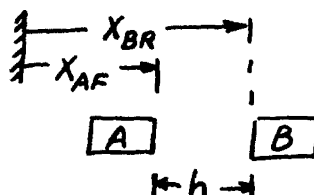
$$t_{term} = 0.00367 \text{ s}$$

2.1.22

**GOAL:** Find the time of collision between Car *A* to hit Car *B* and their relative speed at collision.

**GIVEN:** At  $t = 0$  s Car *A* is traveling at a constant speed of 30 m/s and Car *B* is 6.9 m in front of Car *A*, traveling at 24 m/s and decelerating at  $6 \text{ m/s}^2$ . At  $t = 0.5$  s Car *A* decelerates at a constant  $9 \text{ m/s}^2$ . The separation of the two cars is given by  $h$ .

**DRAW:**



**FORMULATE EQUATIONS:** We'll use the general expressions for position and speed, given a constant acceleration and initial position and speed  $x_0, v_0$ :

$$x(t) = x_0 + v_0 t + \frac{at^2}{2}$$

$$v(t) = v_0 + at$$

**SOLVE:**

At  $t = 0$  s let the position of Car *A* be  $x_{AF} = 0$  m and the position of Car *B* be  $x_{BR} = 6.9$  m. After 0.5 s have elapsed the positions and speeds are found from

$$x_{AF}(0.5 \text{ s}) = (30 \text{ m/s})(0.5 \text{ s}) = 15 \text{ m}$$

$$x_{BR}(0.5 \text{ s}) = 6.9 \text{ m} + (24 \text{ m/s})(0.5 \text{ s}) - 0.5(6 \text{ m/s}^2)(0.5 \text{ s})^2 = 18.9 \text{ m} - 0.75 \text{ m} = 18.15 \text{ m}$$

$$v_{AF}(0.5 \text{ s}) = 30 \text{ m/s}$$

$$v_{BR}(0.5 \text{ s}) = 24 \text{ m/s} - (6 \text{ m/s}^2)(0.5 \text{ s}) = 21 \text{ m/s}$$

At this point Car *A* begins to decelerate at  $-9 \text{ m/s}^2$ . For convenience we'll reset time to zero ( $t$  indicates time beyond the 0.5 s needed for Car *A*'s braking to begin.)

$$x_{AF}(t) = 15 \text{ m} + (30 \text{ m/s})t + 0.5(-9 \text{ m/s}^2)t^2$$

$$x_{BR}(t) = 18.15 \text{ m} + (21 \text{ m/s})t + 0.5(-6 \text{ m/s}^2)t^2$$

The separation  $h$  is given by

$$h = x_{BR} - x_{AF} = 3.15 \text{ m} - (9 \text{ m/s})t + (1.5 \text{ m/s}^2)t^2$$