CHAPTER 1

Kinematics of Particles

Solutions to Tutorial Questions

Q1.1 a=0.3 m/ s^2, u=0, $v = 22$ m/s

The distance s in the acceleration phase is:

$$v^2 = u^2 + 2as \Rightarrow (22)^2 = 0 + 2 \times 0.3 \times s \Rightarrow s = 806.67 \text{ m=0.807 km}$$

The time in the acceleration phase is:

$$v = u + at \Rightarrow 22 = 0 + 0.3 \times t \Rightarrow t = 73.33 \text{ s = 1 min 13.33 s}$$

b) t= 1 min, v=0, $u = 22.347$ m/s

The acceleration a is:

$$v = u + at \Rightarrow 0 = 22 + a \times 60 \Rightarrow a = -0.367 \text{ m/s}^2$$

To calculate the distance s in the deceleration phase:

$$v^2 = u^2 + 2as \Rightarrow 0 = (22)^2 - 2 \times 0.367 \times s \Rightarrow s = 660 = 0.66 \text{ km}$$

c) Distance during the constant velocity=11-0.807-0.66=9.533 km

Time during the constant velocity is:

$$t = \frac{s}{v_{av}} = \frac{9.533 \times 1000}{22} = 433.318 \text{ s=7 min 13.318 s}$$

Total time of the journey: 1 min 13.33 s + 7 min 13.318 s + 1 min = 9 min 27 s

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Q1.2.a) u=0, a =4m/s^2:

$$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times 4 \times 200 \Rightarrow v = 40 \text{ m/s}$$

b) $v = u + at \Rightarrow 40 = 0 + 4 \times t \Rightarrow t = 10 \text{ s}$

Q1.3. a) For the acceleration phase:

Acceleration phase, u=0, v=v_1=80 m/s, a=a_1=4 m/s^2:

$v^2 = u^2 + 2as \Rightarrow (60)^2 = 0 + 2\times 3\times s \Rightarrow s = s_1 = 600$ m

Deceleration phase, with $u = v_1 = 60$ m/s, $a = a_2 = -6$ m/s^2, $t = t_2 = 5$ s:

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 60\times 5 + \frac{1}{2}\times(-6)\times(5)^2 \Rightarrow s = s_2 = 225\text{ m}$$

Total distance= $s_1 + s_2 = 600 + 225 = 825$ m

b) Acceleration phase $u=0$, $v = v_1 = 60$ m/s, $a = a_1 = 3$ m/s^2, thus

$v = u + at \Rightarrow 60 = 0 + 3\times t \Rightarrow t = t_1 = 20$s

Deceleration phase, $t_2 = 5$ s thus

Total time= $t_1 + t_2 = 20 + 5 = 25$ s

c) Deceleration phase $u = v_1 = 60$ m/s, $v = v_2 = ?$, $a = a_2 = -6$ m/s^2, $t = t_2 = 5$ s:

$v = u + at \Rightarrow v = 60 - 6\times 5 \Rightarrow v = v_2 = 30$ m/s

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Q1.4. a) $u=60$ km/hr, $a = 6000$ km/hr^2, $v=110$ km/hr

$v = u + at \Rightarrow 110 = 60 + 6000\times t \Rightarrow t = 8.33\times 10^{-3}\text{hr} = 30\text{s}$

b) $v^2 = u^2 + 2as \Rightarrow 110^2 = 60^2 + 2\times 6000\times s \Rightarrow s = 0.708$km=708 m

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Q1.5 a) During period of constant velocity $v=40$ m/s, $t=3$ s:

$s = vt = 40\times 3 = 120$ m

b) Turning point at $u=40$ m/s , $v=0$, $a=-10$ m/s^2:

$v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2\times(-10)\times s_{max} \Rightarrow s_{max} = 80$ m

Time to turning point C:

$v = u + at \Rightarrow 0 = 40 - 10\times t \Rightarrow t = 4$ s

c) The distance from A to C=120+80=200 m

Time to travel back to A, $u=0$, $a=-10$m/s^2 and $s=-200$ m,

$$s = ut + \frac{1}{2}at^2 \Rightarrow -200 = 0 + \frac{1}{2}\times(-10)\times(t)^2 = 6.325\text{s}$$=6.3 s

Its velocity when it returns point A:

$v = u + at \Rightarrow v = 0 - 10\times 6.325 \Rightarrow v = 63.25$ m/s

d) Total distance $s = 2\times(120 + 80) = 400$ m

Total time:

$t = 3 + 4 + 6.3 = 13.3$ s

Q1.6. a) $u=20$ km/hr, $t = 12$ s, $v=100$ km/hr

$$v = u + at \Rightarrow 100 = 20 + a\times\frac{12}{60\times 60} \Rightarrow a = 24000\text{ km/hr}^2$$

b) $v^2 = u^2 + 2as \Rightarrow 100^2 = 20^2 + 2\times 24000\times s \Rightarrow s = 0.2$ km=200 m

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Q1.7 $s = t^3 - \frac{15}{2}t^2 + 18t + 5$

$$v = \frac{ds}{dt} = 3t^2 - 15t + 18$$

And acceleration $a = \frac{dv}{dt} = 6t - 15$

a) At v=0, $v = 3t^2 - 15t + 18 = 0 \Rightarrow t^2 - 5t + 6 = 0 \Rightarrow (t-2)(t-3) = 0$, thus t=2 s or 3 s

b) $t = 2 \Rightarrow s = 2^3 - \frac{15}{2} \times 2^2 + 18 \times 2 + 5 = 19$ m

$t = 3 \Rightarrow s = 3^3 - \frac{15}{2} \times 3^2 + 18 \times 3 + 5 = 18.5$ m

c) $t = 2 \Rightarrow a = 6 \times 2 - 15 = -3$ m/s^2

$t = 3 \Rightarrow a = 6 \times 3 - 15 = 3$ m/s^2

At $t = 4 \Rightarrow s = 4^3 - \frac{15}{2} \times 4^2 + 18 \times 4 + 5 = 21$ m

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Q1.8 $a = 5.8 + 0.021s$

$$a = v\frac{dv}{ds} = (5.8 + 0.021s) \Rightarrow v\,dv = (5.8 + 0.021s)ds$$

Integrating: $\int v\,dv = \int (5.8 + 0.021s)ds$

$$\frac{v^2}{2} = 5.8s + 0.021\frac{s^2}{2} \Rightarrow v^2 = 11.6s + 0.021s^2$$

For s=3000 m

$v^2 = 11.6 \times 3000 + 0.021 \times 3000^2 \Rightarrow v = 473.1$ m/s

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Q1.9 a) $v = 4s^2 + 3$

$$a = v\frac{dv}{ds} = (4s^2 + 3)(8s) = 32s^3 + 24s$$

b) For s=3 m

$v = 4s^2 + 3 = 4 \times 3^2 + 3 = 39$ m/s

$a = v\frac{dv}{ds} = (4s^2 + 3)(8s) = 39 \times 8 \times 3 = 936$ m/s^2

Q1.10 $a = -12t$

Integrating over time

$v = -6t^2 + 20$

Integrating again

$s = -2t^3 + 20t$

at v=0, $0 = -6t^2 + 20 \Rightarrow t = 1.8257\text{s}$

$s = -2 \times 1.8257^3 + 20 \times 1.8257 = 24.35\,\text{m}$

Q1.11 $v = t^2 + t/2$

Integrating over time

$$s = \frac{t^3}{3} + \frac{t^2}{4} + C$$

Initial conditions t=0, s_o=0:

$$s = \frac{t^3}{3} + \frac{t^2}{4}$$

When t=3s:

$$s = \frac{3^3}{3} + \frac{3^2}{4} = 11.25\,\text{m}$$

Acceleration:

$$a = \frac{dv}{dt} = (2t^2 + 0.5) = 2 \times 3^2 + 0.5 = 6.5\ \text{m/s}^2$$

Q1.12 Acceleration:

$$a = \frac{dv}{dt} = -0.5v^3 \Rightarrow \frac{dv}{-0.5v^3} = dt$$

Integrating:

$$\frac{1}{v^2} = t + C$$

Applying initial condition at t=0, v_o=100 m/s:

$$\frac{1}{100^2} = 0 + C \Rightarrow C = 10^{-4}$$

At t=3s:

$$\frac{1}{v^2} = t + 10^{-4} \Rightarrow \frac{1}{v^2} = 3 + 10^{-4} \Rightarrow v = 0.577\,\text{m/s}$$

Q1.13 $x = 2t^3 - 10t^2 + 2t + 2$, $y = 3t^3 - 4t^2 + 5t + 3$

a) Differentiating with respect to time to derive velocity and acceleration:

$\dot{x} = 6t^2 - 20t + 2$ $\dot{y} = 9t^2 - 8t + 5$

$\ddot{x} = 12t - 20$ $\ddot{y} = 18t - 8$

When t=4 s

$\dot{x} = 6\times 4^2 - 20\times 4 + 2 = 18\,\text{m/s}$ $\dot{y} = 9\times 4^2 - 8\times 4 + 5 = 117\,\text{m/s}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(18)^2 + (117)^2} = 118.38\,\text{m/s}$$

b) $\ddot{x} = 12\times 4 - 20 = 28\ \text{m/s}^2$ $\ddot{y} = 18\times 4 - 8 = 64\ \text{m/s}^2$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(28)^2 + (64)^2} = 69.86\,\text{m/s}^2$$

Q1.14 $x = 8t$, $y = 0.03x^2 = 0.03\times(8t)^2 = 1.92t^2$

a) Differentiate to derive velocity and acceleration:

$\dot{x} = 8$ $\dot{y} = 3.84t$

$\ddot{x} = 0$ $\ddot{y} = 3.84$

When t=3 s

$\dot{x} = 8\,\text{m/s}$ $\dot{y} = 3.84\times 3 = 11.52\,\text{m/s}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(8)^2 + (11.52)^2} = 14.03\,\text{m/s}$$

b) $\theta = \tan^{-1}\dfrac{\dot{y}}{\dot{x}} = \tan^{-1}\dfrac{11.52}{\dot{8}} \Rightarrow \theta = 55.22°$

c) $\ddot{x} = 0\ \text{m/s}^2$ $\ddot{y} = 3.84\ \text{m/s}^2$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(0)^2 + (3.84)^2} = 3.84\,\text{m/s}^2$$

Q1.15 a) $x = 3t^2$, $y = 0.06t^3$

When t=10 s:

$x = 3t^2 = 3\times 10^2 = 300\,\text{m}$

$y = 0.06t^3 = 0.06\times 10^3 = 60\ \text{m}$

$$L = \sqrt{x^2 + y^2} = \sqrt{(300)^2 + (60)^2} = 305.94\,\text{m}$$

b) Differentiate to derive velocity and acceleration:

$\dot{x} = 6t$ $\dot{y} = 0.18t^2$

$\ddot{x} = 6$ $\ddot{y} = 0.36t$

When t=10 s

$\dot{x} = 6 \times 10 = 60\,\text{m/s}$ $\dot{y} = 0.18 \times 10^2 = 18\,\text{m/s}$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(60)^2 + (18)^2} = 62.64\,\text{m/s}$$

c) $\ddot{x} = 6$ m/s^2 $\ddot{y} = 0.36 \times 10 = 3.6$ m/s^2

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(6)^2 + (3.6)^2} = 7\text{ m/s}^2$$

Q1.16 a) $(y-50)^2 = 169\,x$,

When y=100 m:

$$x = \frac{(100-50)^2}{169} = 14.7829 \text{ m}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{(14.7829)^2 + (100)^2} = 101.088\,\text{m}$$

b) Differentiate to derive velocity and acceleration:

$$2\dot{y}\,y - 100\,\dot{y} = 169\,\dot{x}$$

$$2\ddot{y}\,y + 2\dot{y}^2 - 100\,\ddot{y} = 169\,\ddot{x}$$

When y=100 m

$$\dot{x} = \frac{2 \times 200 \times 100 - 100 \times 200}{169} = 118.343\,\text{m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(118.343)^2 + (200)^2} = 232.39\,\text{m/s}$$

c) $\ddot{x} = \dfrac{0 + 2 \times 200^2 - 0}{169} = 473.37$ m/s^2

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(0)^2 + (473.37)^2} = 473.37\,\text{m/s}^2$$

Q1.17 $\dot{y}_o = v_o \sin 30 = 9 \times \sin 30 = 4.5$ m/s

a) $y = y_o + \dot{y}_o t - \frac{1}{2}gt^2 \Rightarrow -1 = 0 + 4.5\times t - \frac{1}{2}\times 9.81\times t^2 \Rightarrow t = 1.1023$ s

b) $x = x_o + \dot{x}_o t \Rightarrow L = 0 + 9\times\cos 30\times 1.1023 = 8.59$ m

c) $\dot{y}^2 = \dot{y}_o^2 - 2g(y - y_o) \Rightarrow 0 = (9\times\sin 30)^2 - 2\times 9.81\times(h-1)$

From which *h*=2.03 m

Q1.18 a) $x = x_o + \dot{x}_o t \Rightarrow x = x_o + v_o\cos\theta\times t \Rightarrow 9 = 0 + 15\times\cos 20\times t$

From which *t*=0.64 s

b) $\dot{y} = \dot{y}_o - gt \Rightarrow \dot{y} = 15\times\sin 20 - 9.81\times 0.64 = -1.148$ m/s

$\dot{x} = \dot{x}_o = 15\times\cos 20 = 14.095$ m/s

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(14.095)^2 + (-1.148)^2} = 14.14$ m/s

$\theta = \tan^{-1}\frac{-1.148}{14.095} = -4.66°$

c) $y = y_o + \dot{y}_o t - \frac{1}{2}gt^2 = 1 + 15\times\sin 20\times 0.64 - \frac{1}{2}\times 9.81\times 0.64^2$

From which:

y=2.27 m

Q1.19 a) $\dot{x}_o = v_o\cos\theta = 4\times\cos 60 = 2$ m/s

$\dot{y}_o = v_o\sin\theta = 4\times\sin 60 = 3.4641$ m/s

$x = x_o + \dot{x}_o t \Rightarrow x = 0 + 2\times t \Rightarrow x = 2t$ (1)

$y = y_o + \dot{y}_o t - \frac{1}{2}gt^2 \Rightarrow 0.012x^2 = 0 + 3.4641t - \frac{1}{2}\times 9.81\times t^2$ (2)

Substitute Equation (1) into Equation (2) and solve for *t*:

t=0.6994 s = 0.7 s

b) Substitute by t in (1):

$x = 2t = 2\times 0.6994 = 1.398$ m

And

$$y = 0.012\,x^2 = 0.012 \times 1.398^2 = 0.02345 \text{ m}$$

Q1.20 $r = 3t^2 + 3$ $\qquad \theta = \frac{t}{2}\sin\frac{\pi t}{4}$

Differentiate to derive velocity and acceleration:

$\dot{r} = 6t$ $\qquad \dot{\theta} = \frac{1}{2}\sin\frac{\pi t}{4} + \frac{t}{2}\times\frac{\pi}{4}\cos\frac{\pi t}{4} = \frac{1}{2}\sin\frac{\pi t}{4} + \frac{\pi t}{8}\cos\frac{\pi t}{4}$

$\ddot{r} = 6 \text{ m/s}^2$ $\qquad \ddot{\theta} = \frac{\pi}{8}\cos\frac{\pi t}{4} + \frac{\pi}{8}\cos\frac{\pi t}{4} - \frac{\pi t}{8}\times\frac{\pi}{4}\sin\frac{\pi t}{4} = \frac{\pi}{4}\cos\frac{\pi t}{4} - \frac{\pi^2 t}{32}\sin\frac{\pi t}{4}$

When t=2 s

a) $r = 15 \text{ m}$

$\dot{r} = 12$ m/s $\qquad \dot{\theta} = \frac{1}{2}\sin\frac{\pi\times 2}{4} + \frac{\pi\times 2}{8}\cos\frac{\pi\times 2}{4} = 0.5$ rad/s

$\ddot{r} = 6$ $\qquad \ddot{\theta} = \frac{\pi}{4}\cos\frac{\pi\times 2}{4} - \frac{\pi^2\times 2}{32}\sin\frac{\pi\times 2}{4} = -\frac{\pi^2}{16}$ = -0.617 rad/s^2

b) Radial and tangential velocities:

$v_r = \dot{r} = 12$ m/s $\qquad v_t = r\dot{\theta} = 15\times 0.5 = 7.5$ m/s

Total magintude of velocity:

$$v = \sqrt{v_r^{\,2} + v_t^{\,2}} == \sqrt{(12)^2 + (7.5)^2} = 14.15 \text{ m/s}$$

c) Radial and tangential velocities and accelerations:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 6 - 15\times(0.5)^2 = 2.25 \text{ m/s}^2$$

$$a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 15\times -\frac{\pi^2}{16} + 2\times 12\times(0.5) = 2.747 \text{ m/s}^2$$

Total magintude of acceleration:

$$a = \sqrt{a_r^{\,2} + a_t^{\,2}} = \sqrt{(2.25)^2 + (2.747)^2} = 3.55 \text{ m/s}^2$$

Q1.21 a) $v_t = r\dot{\theta} \Rightarrow 15 = 50\times\dot{\theta} \Rightarrow \dot{\theta} = 0.3 \text{ rad/s}$

b) $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 50\times(0.3)^2 = -4.5 \text{ m/s}^2$

Q1.22 a) $x = r\cos\theta$

$\mathrm{y} = r\sin\theta$

$$v_r = \dot{r} = 1 \text{ m/s}$$

$$v_t = r\dot{\theta} = 2 \times \left(20 \times \frac{2\pi}{360} \right) = 0.698 \text{ m/s}$$

Referring to Figure 1.22 in Chapter 1, the velocity components in the Cartesian co-ordinate system are:

$$\dot{x} = v_r \cos\theta - v_t \sin\theta = 1 \times \cos 30 - 0.698 \times \sin 30 = 0.517 \text{ m/s}$$

$$\dot{y} = v_r \sin\theta + v_t \cos\theta = 1 \times \sin 30 + 0.698 \times \cos 30 = 1.105 \text{ m/s}$$

b) $a_r = \ddot{r} - r\dot{\theta}^2 = 3 - 2 \times \left(20 \times \frac{2\pi}{360} \right)^2 = 2.756 \text{ m/s}^2$

$$a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2 \times \left(50 \times \frac{2\pi}{360} \right) + 2 \times 1 \times \left(20 \times \frac{2\pi}{360} \right) = 2.44 \text{ m/s}^2$$

Referring to Figure 1.23 in Chapter 1, the acceleration components in the Cartesian co-ordinate system are:

$$\ddot{x} = a_r \cos\theta - a_t \sin\theta = 2.756 \times \cos 30 - 2.44 \times \sin 30 = 1.167 \text{ m/s}^2$$

$$\dot{y} = v_r \sin\theta + v_t \cos\theta = 2.756 \times \sin 30 + 2.44 \times \cos 30 = 3.49 \text{ m/s}^2$$

Q 1.23 a) $v_t = r\dot{\theta} = 180 \times (0.03) = 5.4 \text{ m/s}$

$$v_r = 0 \text{ m/s}$$

Total magintude of velocity:

$$v = \sqrt{v_r^2 + v_t^2} == \sqrt{(0)^2 + (5.4)^2} = 5.4 \text{ m/s}$$

c) Radial and tangential velocities and accelerations:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 180 \times (0.03)^2 = -0.162 \text{ m/s}^2$$

$$a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 180 \times -0.001 + 0 = 0.18 \text{ m/s}^2$$

Total magintude of acceleration:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.162)^2 + (0.18)^2} = 0.242 \text{ m/s}^2$$

Q1.24 a) $v_r = 0 \text{ m/s}$

$$v_t = r\dot{\theta} = 3 \times (0.2) = 0.6 \text{ m/s}$$

b) $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 3\times(0.2)^2 = -0.12$ m/s^2

$a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3\times 0 + 0 = 0$ m/s^2

Q1.25 $v_t = r\dot{\theta} \Rightarrow 20\times\cos30 = \frac{866}{\cos30}\times\dot{\theta} \Rightarrow \dot{\theta} = 0.0173$ rad/s $\times\frac{180}{\pi} = 1^\circ/\text{s}$

Q1.26 a) $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(0.08x)^2\right]^{3/2}}{|0.08|}$

$\rho = \frac{\left[1+(0.08\times 15)^2\right]^{3/2}}{|0.08|} = 47.64$ m

b) $a_n = \frac{v^2}{\rho} = \frac{5.5^2}{47.64} = 0.635$ m/s^2

c) $a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.2^2 + 0.635^2} = 2.29$ m/s^2

Q1.27 a) $a_n = \frac{v^2}{\rho} = \frac{9^2}{100} = 0.81$ m/s^2

b) $v = u + at \Rightarrow 9 = 0 + 2t \Rightarrow t = 4.5$ s

Q1.28 $\rho = \frac{\left[1+(0.6\times 6)^2\right]^{3/2}}{|0.6|} = 86.93$ km

$a_n = \frac{v^2}{\rho} = \frac{220^2}{86.93\times 10^3} = 0.5567$ m/s^2

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.5567^2} = 0.748$ m/s^2

Q1.29 a) $v = u + at \Rightarrow 30 = 14 + a_t\times 4 \Rightarrow a_t = 4$ m/s^2

b) $a_n = \frac{v^2}{\rho} = \frac{25^2}{250} = 2.5$ m/s^2

Q1.30 a) $a_t = 15 \times \cos 60 = 7.5$ m/s^2

b) $a_n = 15 \times \sin 60 = 12.99$ m/s^2

$a_n = \frac{v^2}{\rho} \Rightarrow 12.99 = \frac{22^2}{\rho} \Rightarrow \rho = 37.26$ m

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Q1.31 a) $\dot{v} = 1.1t$

Integrating:

$v = 0.55t^2$

When t=3 s, v=4.95 m/s

b) $a_n = \frac{v^2}{\rho} = \frac{4.95^2}{11} = 2.2275$ m/s^2

$a_t = \dot{v} = 1.1 \times 3 = 3.3$ m/s^2

c) $\dot{\theta} = \frac{v}{r} = \frac{0.55t^2}{11} = 0.05t^2$

$\dot{\theta} = 0.05 \times 3^2 = 0.45$ rad/s

d) Integrating $\dot{\theta}$:

$\theta = \frac{0.05t^3}{3}$

$\theta = \frac{0.05 \times 3^3}{3} = 0.45$ rad$= 0.45 \times \frac{180}{\pi} = 25.78°$

Q1.32 $a_n = \frac{v^2}{\rho} \Rightarrow 2 = \frac{5000^2}{(h + 12713/2) \times 10^3} \Rightarrow h = 6143.5$ m

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Q1.33 $a_n = \frac{v^2}{\rho} = \frac{5000^2}{(7000 + 12713/2) \times 10^3} = 1.87$ m/s^2
