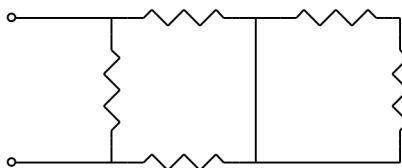


# Chapter 1

## The Basics

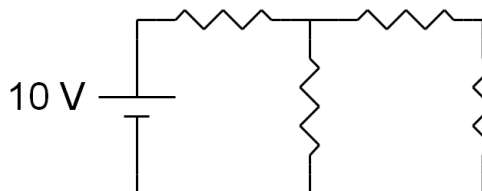
### 1.1 Exercises

**Exercise 1.1** Calculate the equivalent resistance of the network in Figure 1.9 (in the textbook, shown below). All resistors have a resistance  $R$ .



Solution: The middle branch is a straight wire, so it nullifies the two resistors in series that it shorts. We are then left with two branches in parallel: one with resistance  $R$  and the other with resistance  $2R$ . The equivalent resistance is then  $R_{eq} = (1/R + 1/2R)^{-1} = 2R/3$ .

**Exercise 1.2** Use the concepts of resistors in series and parallel with a voltage divider to calculate the voltage across each resistor in the circuit of Figure 1.14 (in the textbook, shown next). Each resistor has a value of  $1 \text{ k}\Omega$ .



Solution: There are two junctions in the circuit. The equivalent resistance between them is  $R_{junc} = (1/R + 1/2R)^{-1} = 2R/3$ . Using the voltage divider Eq., the voltage between the two junctions is  $V_{junc} = [2R/3] \cdot V_0/[R + 2R/3] = 2V_0/5 = 4 \text{ V}$ . One of

the branches between the two junctions has two resistors in series. Since they have the same resistance value, the voltage across each is 2 V. The resistors between the junctions are, as a whole, in series with a resistance  $R$ . The voltage across the latter is 6 V, because the sum of voltages in series has to be 10 V.

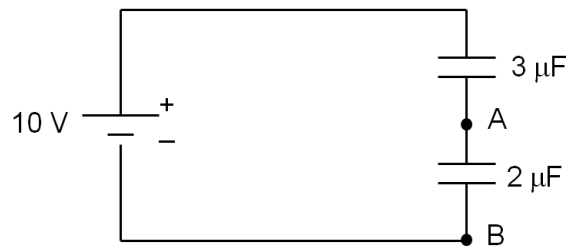
**Exercise 1.3** *A capacitor is charged to 200 V.*

1. *What capacitance does it have if it stores a charge of 10 mC?*
2. *If we connect a 1 k $\Omega$  resistor across the capacitor, what is the maximum current that will flow as the capacitor discharges?*

Solution: (a) The capacitance is  $C = q/V_C = 50\mu\text{F}$ . (b) The current decays exponentially as the capacitor discharges, so maximum current occurs when the capacitor has its maximum voltage, or  $V_C/R = 0.2$  A.

## 1.2 Problems

1. Two capacitors with capacitances  $3\mu\text{F}$  and  $2\mu\text{F}$  are initially discharged. They are connected in series, and then the two ends of the combination are connected to a 10-V battery, as shown in Figure 1.24 (in the textbook, shown next). The negative end of the battery is at zero potential.



- (a) How much charge does the battery deliver? You can get started by figuring out the relationship between the charges in each capacitor and the voltage across each of them.

Solution: Because the two capacitors are in series, they have the same charge  $Q$ . The source delivers  $Q$ . The voltages across each capacitor add up to the source voltage  $V_0 = 10$  V. Therefore  $Q/C_{top} + Q/C_{bot} = V_0$ , or  $Q = 12\mu\text{C}$ .

- (b) If we connect a single capacitor to the same battery, what would be the capacitance of the capacitor so that it draws the same charge from the battery?

Solution:  $C = Q/V_0 = 12\mu\text{C}/10\text{V} = 1.2 \mu\text{F}$ , or using equivalent capacitance:  $C_{eq} = (1/C_{top} + 1/C_{bot})^{-1} = 1.2 \mu\text{F}$ .

- (c) Find the charges on each capacitor.

Solution: The charge on each capacitor is  $Q = 12 \mu\text{C}$ .

- (d) Find the voltage across each capacitor.

Solution: For the top capacitor:  $V_{top} = Q/C_{top} = 4 \text{ V}$ ; and for the bottom capacitor,  $V_{bot} = Q/C_{bot} = 6 \text{ V}$ .

- (e) If point B is at zero potential, what is the potential at point A between the two capacitors?

Solution:  $V_A = V_B + 6 \text{ V} = 6 \text{ V}$ .

2. A battery with potential  $V_0$  is connected to a capacitor  $C$ . We label the charge in the top plate as  $q_1$  and the charge in the bottom plate as  $q_2$ . Which statement is correct?

- (a)  $q_1 = q_2 = CV_0$   
 (b)  $q_1 = -q_2 = CV_0$   
 (c)  $q_1 + q_2 = CV_0$   
 (d)  $q_1 = -q_2 = 2CV_0$   
 (e)  $q_1 = q_2 = CV_0/2$

Solution: Answer is (b), because the capacitor's top plate has a charge  $Q$ , and the bottom plate has a charge  $-Q$ . Their absolute value is equal to  $CV_0$ .

3. Suppose that we have two capacitors with capacitances  $0.1 \mu\text{F}$  and  $0.2 \mu\text{F}$ . We connect them in parallel and apply a voltage of  $10 \text{ V}$  to their ends.

- (a) What is the charge on each capacitor?

Solution: Because the capacitors are in parallel, the voltage across them is the same. If  $C_1 = 0.1 \mu\text{F}$  and  $C_2 = 0.2 \mu\text{F}$ , then  $q_1 = C_1V_0 = 1 \mu\text{C}$  and  $q_2 = C_2V_0 = 2 \mu\text{C}$ .

- (b) What is the total amount of charge that the source has to supply to charge the two capacitors?

Solution: The source has to supply charge to both capacitors:  $q = q_1 + q_2 = 3 \mu\text{C}$ .

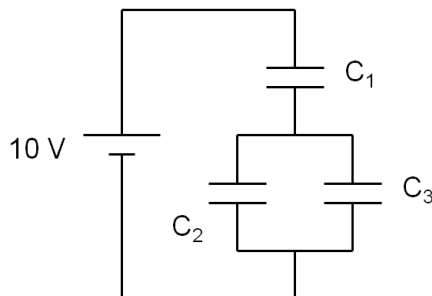
- (c) If we replaced both capacitors with one so that it draws the same amount of charge from the supply, what capacitance would it have?

Solution:  $C_{eq} = (q_1 + q_2)/V_0 = 0.3\mu\text{F}$ .

- (d) Can you generalize the previous situation for finding the equivalent capacitance of any two capacitors connected in parallel?

Solution:  $C_{eq} = C_1 + C_2$ .

4. In the circuit of Figure 1.25 (in the textbook, shown below),  $C_1 = 0.2 \mu\text{F}$  and  $C_2 = 0.15 \mu\text{F}$ . When a voltage of 10 V is applied to the system of capacitors, we find that the charge of capacitor 1 is  $1 \mu\text{C}$ .



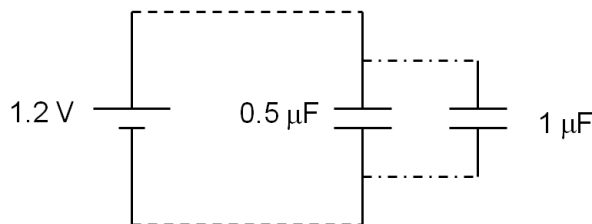
- (a) Find the value of  $C_3$ , and the charge on it.

Solution: The voltage across capacitor 1 is  $V_1 = q_1/C_1 = 5 \text{ V}$ , so the voltage across capacitors 2 and 3 must also be 5 V (to add to the supply voltage). The charge on capacitor 1 must be split onto capacitors 2 and 3 because they are in parallel (i.e., the wire that connects the bottom plate of 1 to the top plates of 2 and 3 is initially neutral, so  $q_2 + q_3 - q_1 = 0$ ). The charge on capacitor 2 is  $q_2 = (5\text{V})(0.15\mu\text{F}) = 0.75 \mu\text{C}$ . Thus  $q_3 = 0.25 \mu\text{C}$ . The capacitance of 3 is then  $C_3 = (0.25\mu\text{C})/(5\text{V}) = 0.05 \mu\text{F}$ .

- (b) What is the equivalent capacitance of the arrangement?

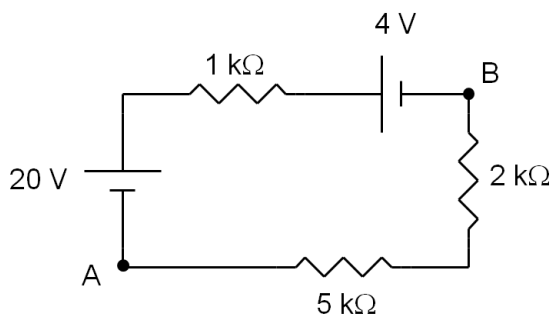
Solution: The capacitance of 2 and 3 in parallel is the sum of the two:  $C_{23} = C_2 + C_3 = 0.2 \mu\text{F}$ . They are in series with capacitor 1, so  $C_{eq} = (1/C_1 + 1/C_{23})^{-1} = 0.1 \mu\text{F}$ . Check: The source must deliver the charge  $Q = C_{eq}V_0 = 1 \mu\text{C}$ , which is also equal to the charge on capacitor 1.

5. We apply 1.2 V to the  $0.5 \mu\text{F}$  capacitor of Figure 1.26 (in the textbook, shown next). We then disconnect the capacitor from the power supply. Subsequently, we connect an uncharged  $1\text{-}\mu\text{F}$  capacitor in parallel with the other capacitor. Find the charge on the  $1\text{-}\mu\text{F}$  capacitor. (Note that the voltage across it is not 1.2 V.)



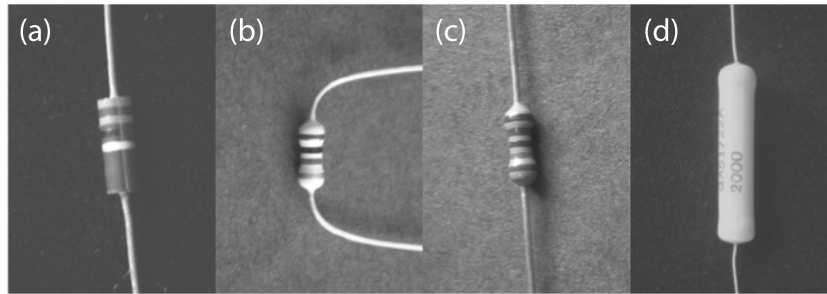
Solution: When we connect the supply to the first capacitor, it gets a charge  $q_1 = (0.5\mu\text{F})(1.2\text{V}) = 0.6 \mu\text{C}$ . We then disconnect it from the supply and connect it to the second capacitor. The charges rearrange so that the two capacitors have the same voltage across them. We can also think that the two capacitors now act as a single capacitor of capacitance  $C' = C_1 + C_2 = 1.5 \mu\text{F}$ . The charge on it is the initial one, so the new voltage across the capacitors is  $V' = q_1/C' = 0.4 \text{ V}$ . The charges on each capacitor are then  $q'_1 = C_1V' = 0.2 \mu\text{C}$  and  $q_2 = C_2V' = 0.4 \mu\text{C}$ . Check: The two charges indeed add to the original charge.

6. The potential of point A in Figure 1.27 (in the textbook, shown below) is 0 V. Find the potential of point B.



Solution: We have a single loop. The sum of voltages must add to zero. The total supply voltage is  $20\text{V} - 4\text{V} = 16 \text{ V}$ . The voltage drops across the resistors, their value times the current, must be equal to this voltage. Since the resistors are in series in the loop, then their combined resistance is  $8 \text{ k}\Omega$ . The current is then  $I = (16\text{V})/(8\text{k}\Omega) = 2 \text{ mA}$ . The potential of B is the potential on A plus the drops on the  $5\text{-k}\Omega$  and  $2\text{-k}\Omega$  resistors,  $(5\text{k}\Omega)(2\text{mA}) = 10 \text{ V}$  and  $(2\text{k}\Omega)(2\text{mA}) = 4 \text{ V}$ , respectively. Therefore  $V_B = 14 \text{ V}$ .

7. The colors of the bands of the resistors of Figure 1.4 (in the textbook, shown next) are: (a) Orange, orange, red, white; (b) Brown, black, red, gold; (c) Brown, red, blue, orange, green.



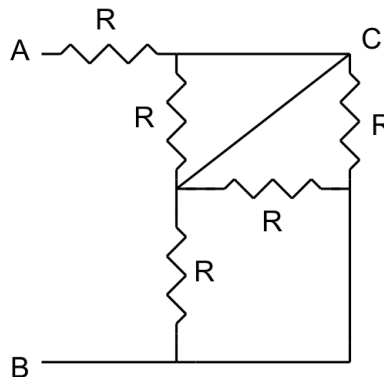
(a) Determine their value and tolerance.

Solution: (a)  $33 \times 10^2 \Omega = 3.3\text{k}\Omega$ , 20%; (b)  $10 \times 10^2 \Omega = 1\text{k}\Omega$ , 5%; and (c)  $126 \times 10^3 \Omega = 126\text{k}\Omega$ , 0.5%.

(b) A measurement of the actual resistance of (b) gave  $992 \Omega$ . Should we return it as defective? Explain.

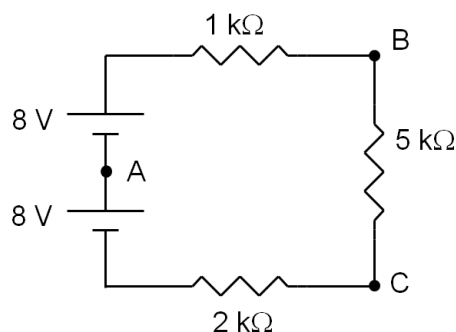
Solution: No, because the difference between the expected and measured is 0.8%, which is within the 5% tolerance of the resistor.

8. Calculate the equivalent resistance of the network in Figure 1.28 (in the textbook, shown below).



Solution: In the upper-right side of the circuit is a wire oriented diagonally. Notice that it meets with another wire to short a resistor. The potential of that wire is the same. Let us call this potential,  $V_C$ . Between the wire at potential  $A$  and the one at  $C$  there is only one resistor. Between  $C$  and  $B$  there are three resistors in parallel. Therefore we can reduce the arrangement as two resistors in series:  $R$  between  $A$  and  $C$ , and  $R/3$  between  $C$  and  $B$ . The equivalent resistance between  $A$  and  $B$  is  $4R/3$ .

9. If point  $A$  is at zero potential in Figure 1.29 Figure 1.28 (in the textbook, shown next), what is the potential at points  $B$  and  $C$ ?



Solution: We can calculate the current flowing through the loop by adding the voltage sources, 16 V, and dividing that by the equivalent resistance of three resistors in series, or 8 k $\Omega$ . We get  $I = 2$  mA. The potential of the positive side of the battery above  $A$  is 8 V. The potential of point  $B$  is one resistor drop below:  $V_B = 8\text{V} - (1\text{k}\Omega)(2\text{mA}) = 6$  V. Point  $C$  is another resistor drop below  $B$ :  $V_C = V_B - (5\text{k}\Omega)(2\text{mA}) = -4$  V. Check: The potential of the negative side of the battery below  $A$  is  $-8$  V. The potential of point  $C$  is one resistor drop above this:  $V_C = -8\text{V} + (2\text{k}\Omega)(2\text{mA}) = -4$  V.

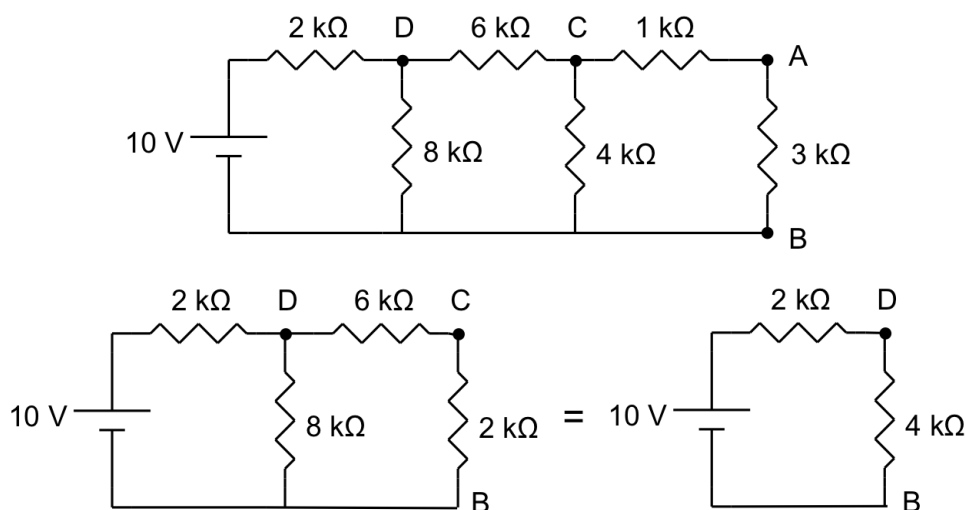
10. Many of the circuits that you will use in the lab will be powered by +12 V power supplies. What is the smallest value 1/8-W resistor that we can apply the full voltage of the power supply without burning it?

Solution: Given are:  $V_0 = 12$  V, and  $P_{max} = 0.125$  W. We know that  $P = V^2/R$ , so  $R_{min} = V^2/P_{max} = 1152\Omega$ .

11. Be aware of the dangers of electricity. Human skin can exhibit large variations in electrical resistance. Although dry skin may have a resistance of 100 k $\Omega$ , wet and tender skin may have resistances as low as 1 k $\Omega$ . If electrocution is caused by currents above 50 mA, what applied voltages would cause electrocution for (a) dry and (b) wet skin?

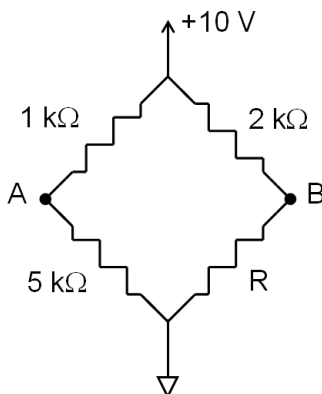
Solution: Lethal current is  $I_{leth} = 50$  mA. (a) For dry skin  $V_{leth} = (50\text{mA})(100\text{k}\Omega) = 5000$  V. (b) For wet skin  $V_{leth} = (50\text{mA})(1\text{k}\Omega) = 50$  V.

12. Using *only* the concepts of equivalent resistance and voltage divider, calculate the voltage between points A and B of Figure 1.30 (in the textbook, shown below). Hint: First find the voltage drop across the 8 k resistor, but do not ignore the resistor ladder to the right of it.



Solution: The two resistors of branch  $CAB$  are in parallel with the  $4\text{ k}\Omega$  resistor, so the resistance between  $C$  and  $B$  is  $R_{CB} = 2\text{ k}\Omega$  (see the figure above). The resistance between  $D$  and  $B$  is the resistance in the branch  $DCB$  in parallel with the  $8\text{ k}\Omega$  resistor. The equivalent resistance between  $D$  and  $B$  is then  $R_{DB} = 4\text{ k}\Omega$  (see the figure above). Applying the voltage divider argument in the reduced one-loop circuit, we get  $V_{DB} = (4/6)V_0 = 6.67\text{ V}$ . Applying the voltage divider argument between  $D$  and  $B$ , we get  $V_{CB} = (2/8)V_{DB} = (1/6)V_0 = 1.67\text{ V}$ . Finally, Applying the voltage divider argument between  $C$  and  $B$ , we get  $V_{AB} = (3/4)V_{CB} = (1/8)V_0 = 1.25\text{ V}$ .

13. Find the value of  $R$  in the circuit of Figure 1.31 (in the textbook, shown below) if points  $A$  and  $B$  are at the same potential.



Solution: The ratio of the resistance on the two branches must be the same:

$$\frac{1\text{ k}\Omega}{5\text{ k}\Omega} = \frac{2\text{ k}\Omega}{R}$$

or  $R = 10 \text{ k}\Omega$ .

14. Two resistors are connected in series to a 20-V battery. The ratio of their resistances is 1:4.

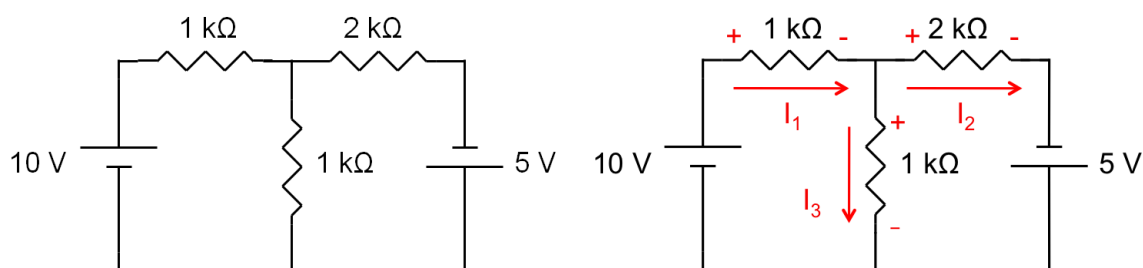
(a) Find the voltage across the larger resistor.

Solution: If one resistor has a resistance  $R$  and the other one  $4R$ , then the voltage across the larger one is  $V = (4R/5R)(20\text{V}) = 16 \text{ V}$ .

(b) If the total current flowing through the resistors is 10 mA, what is the value of the smaller resistor?

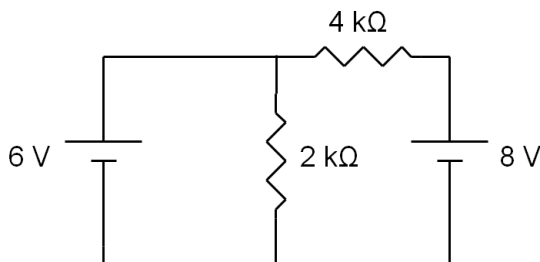
Solution: The voltage across the smaller resistor (of resistance  $R$ ) is 4 V. Then  $R = (4\text{V})/(10\text{mA}) = 400 \Omega$ .

15. Find the current flowing through each resistor, and the voltage drop across the resistor in the middle branch of the circuit of Figure 1.32 (in the textbook, shown on the left below).



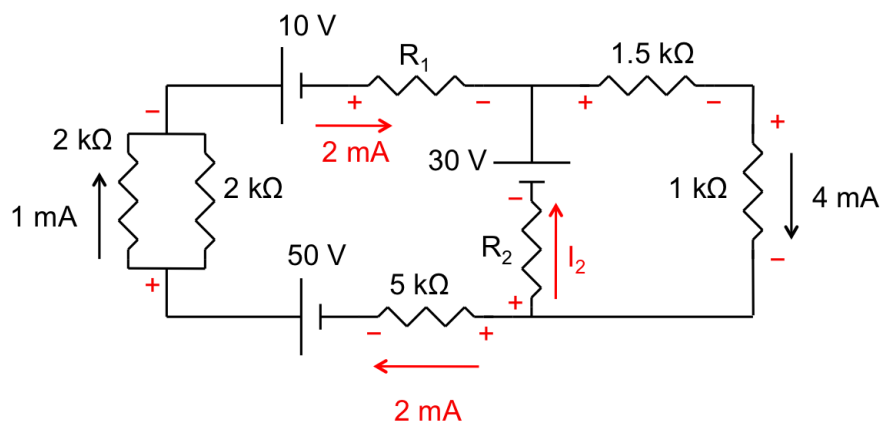
Solution: Let us assume that the currents are given in mA, resistances in  $\text{k}\Omega$  and voltages in V. The equations for the left-hand and right-hand loops are:  $10 - I_1 - I_3 = 0$  and  $5 - I_3 - 2I_2 = 0$ . The equation for the currents is  $I_1 = I_2 + I_3$ . Solving the system of equations, we get  $I_1 = 7 \text{ mA}$ ,  $I_2 = 4 \text{ mA}$  and  $I_3 = 3 \text{ mA}$ . The voltage drops across the middle resistor is  $(1\text{k}\Omega)(3\text{mA}) = 3 \text{ V}$ .

16. Find the current flowing through the 4-k $\Omega$  resistor in Figure 1.33 (in the textbook, shown below).



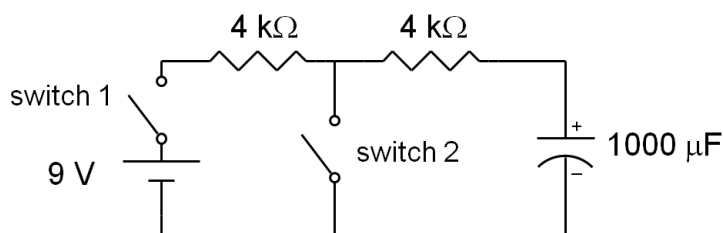
Solution: The 6-V supply is straight across the 2-k $\Omega$  resistor, so the voltage across the 4-k $\Omega$  resistor is 2 V. The current is then  $(2\text{V})/(4\text{k}\Omega) = 0.5$  mA.

17. Find the value of the resistors  $R_1$  and  $R_2$  in Figure 1.34 (in the textbook, shown below with solution).



Solution: We can calculate the voltage across the right branch (of total resistance 2.5 k $\Omega$ ):  $(4\text{mA})(2.5\text{k}\Omega) = 10$  V. Next we focus on the two resistors in parallel in the left-most vertical branch. The resistors have the same resistance, so if the current on the left resistor is 1 mA, then the right resistor carries the same current. The voltage drop across them is 2 V. Up to this point we know all the drops on the left loop, so we can find the voltage across  $R_2$ : 18 V. Its value is then  $R_1 = 9$  k $\Omega$ . From the sum of currents at the joints we get that  $I_2 = 2$  mA. The voltage across  $R_2$  is 20 V, so  $R_2 = 10$  k $\Omega$ .

18. The capacitor in the circuit of Figure 1.36 (in the textbook, shown below) is discharged.



- (a) Switch 1 is closed at  $t = 0$ . At what time will the voltage across the capacitor reach 6 V?

Solution: The equation for charging the capacitor is:  $V_C = (9\text{V})(1 - e^{-t/RC})$ , where  $RC = (8\text{k}\Omega)(1000\mu\text{F}) = 8$  s. We solve for  $t$  when  $V_C = 6$  V:  $t = -(8\text{s}) \ln(1 - 6/9) = 8.8$  s.

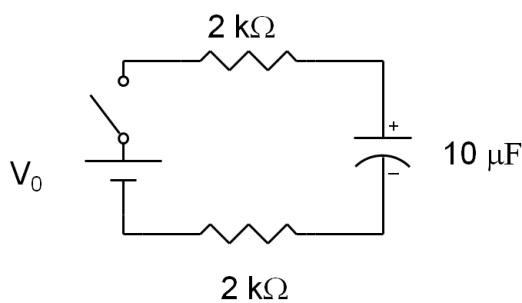
- (b) Switch 1 is opened when the voltage across the capacitor reaches 6 V. What was the current flowing through the circuit before the switch was opened?

Solution: The drop across each resistor is 1.5 V, so the current flowing through them is  $I = (1.5\text{V})/(4\text{k}\Omega) = 0.375\text{ mA}$ .

- (c) We close switch 2. At what time after closing the switch is the voltage across the capacitor equal to 3 V?

Solution: The time constant for the discharge of the capacitor is 4 s. The time is:  $t = -(4\text{s}) \ln(3/6) = 2.8\text{ s}$ .

19. At  $t = 0$ , the switch of Figure 1.36 (in the textbook, shown below) is closed. At  $t = 80\text{ ms}$ , the voltage across the capacitor is 5 V. Calculate  $V_0$ .

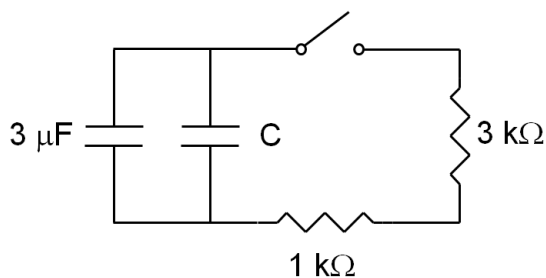


Solution: The time constant for discharging the capacitor is

$$\tau = RC = (2\text{k}\Omega + 2\text{k}\Omega)(10\mu\text{F}) = 40\text{ms}$$

The discharge equation is  $V_t = V_0 e^{-t/\tau}$ . The condition of the problem is  $V_t = 5\text{ V}$  at  $t = 80\text{ ms}$ , so  $5\text{V} = V_0 e^{-2}$ . Solving for  $V_0$  we get  $V_0 = 36.9\text{ V}$ .

20. In Figure 1.37 (in the textbook, shown below), the capacitors are initially charged. As soon as the switch is closed the current is 0.5 mA. After 14 ms the current has dropped to 0.25 mA. Find  $C$ .



Solution: As soon as the switch is closed, the voltage across the capacitors,  $V_0$ , is applied to the resistors, so the current flowing through them at that initial time is  $I = V_0/R_{total}$ . The problem gives  $I_0 = 0.5$  mA at  $t = 0$ , and  $R_{total} = (3\text{k}\Omega + 1\text{k}\Omega) = 4\text{k}\Omega$ . Therefore,  $V_0 = I_0 R_{total} = 2$  V. At  $t = 14$  ms the current is  $I_t = 0.25$  mA. Therefore, the voltage across the capacitors at that time is  $V_t = I_t R_{total} = 1$  V. So from the discharge of the capacitor we have  $V_t = V_0 e^{-t/\tau}$ . Solving for  $\tau$  we get  $\tau = -t/\ln(1/2) = 20$  ms. Since  $\tau = R_{total} C_{total}$ , then  $C_{total} = 5\mu\text{F}$ . The answer will be  $C = C_{total} - 3\mu\text{F} = 2\mu\text{F}$ .

# Chapter 2

## Introduction to Digital Electronics

### 2.1 Exercises

**Exercise 2.1** *What are the decimal representations of  $33_8$  and  $33_{16}$ ?*

Solution:  $33_8 = 3 \times 8 + 3 = 27$ , and  $33_{16} = 3 \times 16 + 3 = 51$ .

**Exercise 2.2** *What is the value of  $1F_{16}$  in decimal?*

Solution:  $1F_{16} = 1 \times 16 + 15 = 31$ .

**Exercise 2.3** *Convert  $B3F_{16}$  to binary.*

Solution: We know:  $B = 1011_2$ ,  $3 = 0011_2$ ,  $F = 1111_2$ . So  $B3F_{16} = 101100111111_2$ .

**Exercise 2.4** *Convert  $101101110_2$  to hexadecimal.*

Solution:  $101101110_2 = 1\ 0110\ 1110_2 = 16E_{16}$ .

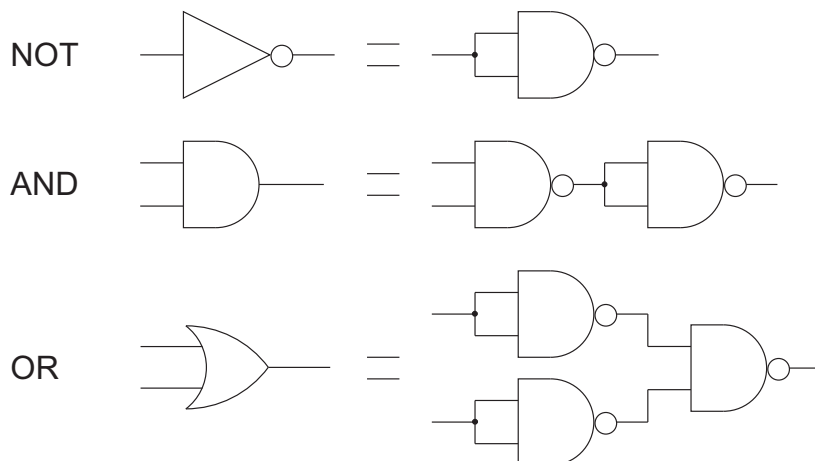
**Exercise 2.5** *Calculate the sum and difference of  $101101_2$  and  $001110_2$ .*

Solution:  $101101_2 + 001110_2 = 111011_2$ ,  $101101_2 - 001110_2 = 011111_2$ .

**Exercise 2.6** *What character does the ASCII code  $0101\ 1101$  represent?*

Solution:  $01011101_2 = 5D = ]$

**Exercise 2.7** *Verify the relationship between the OR gate and NAND gates of Figure ?? (in the textbook, shown next) using the truth table.*



Solution:

A	B	$A \uparrow B$	$\overline{A \uparrow B}$	AB	A'	B'	$A' \uparrow B'$	A+B
0	0	1	0	0	1	1	0	0
0	1	1	0	0	1	0	1	1
1	0	1	0	0	0	1	1	1
1	1	0	1	1	0	0	1	1

**Exercise 2.8** Find the matrix for the AND gate.

Solution:

$$\text{AND} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**Exercise 2.9** Suppose we have a gate  $G$  where one input,  $|A\rangle$ , goes through a NOT gate and then into an AND gate, and the other input,  $|B\rangle$ , goes straight into the AND gate.

1. Find the matrix for  $G$ .
2. Fill in the truth table for gate  $G$  by calculating its output using matrix operations.

Solution: (1)

$$(\text{NOT}) \otimes (\text{I}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\text{AND} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$