SOLUTIONS MANUAL

Engineering Materials I
An Introduction to Properties, Applications and Design, Fourth Edition

Solutions to Examples

2. 1. (a) For commodity A $P(t) = C_A \exp\frac{r_A}{100}t,$

and for commodity B $Q(t) = C_B \exp\frac{r_B}{100}t,$

where C_A and C_B are the current rates of consumption ($t = t_0$) and $P(t)$ and $Q(t)$ are the values at $t = t$. Equating and solving for t gives

$$t = \frac{100}{r_B - r_A}\ln\left(\frac{C_A}{C_B}\right).$$

(b) The doubling time, t_D, is calculated by setting $C(t = t) = 2C_0$, giving

$$t_D = \frac{100}{r}\ln 2 \approx \frac{70}{r}.$$

Substitution of the values given for r in the table into this equation gives the doubling times as 35, 23 and 18 years respectively.

(c) Using the equation of Answer (a) we find that aluminium overtakes steel in 201 years; polymers overtake steel in 55 years.

2.2. Principal conservation measures (see Section 2.7):

Substitution

Examples: aluminium for copper as a conductor; reinforced concrete for wood, stone or cast-iron in construction; plastics for glass or metals as containers. For many applications, substitutes are easily found at small penalty of cost. But in certain specific uses, most elements are not easily replaced. Examples: tungsten in cutting tools and lighting (a fluorescent tube contains more tungsten, as a starter filament, than an incandescent bulb!); lead in lead-acid batteries; platinum as a catalyst in chemical processing; etc. A long development time (up to 25 years) may be needed to find a replacement.

Recycling

The fraction of material recycled is obviously important. Products may be re-designed to make recycling easier, and new recycling processes developed, but development time is again important.

More Economic Design

Design to use proportionally smaller amounts of scarce materials, for example, by building large plant (economy of scale); using high-strength materials; use of surface coatings to prevent metal loss by corrosion (e.g. in motor cars).

2.3. (a) If the current rate of consumption in tonnes per year is C then exponential growth means that

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{r}{100}C,$$

where r is the fractional rate of growth in % per year. Integrating gives

$$C = C_0 \exp\left\{\frac{r(t-t_0)}{100}\right\},$$

where C_0 was the consumption rate at time $t = t_0$.

(b) Set

$$\frac{Q}{2} = \int_0^{t_{1/2}} C\mathrm{d}t,$$

where

$$C = C_0 \exp\left\{\frac{rt}{100}\right\}.$$

Then

$$\frac{Q}{2} = \left[C_0 \frac{100}{r} \exp\left\{\frac{rt}{100}\right\}\right]_0^{t_{1/2}},$$

which gives the desired result.

2.4. See Chapter 2 for discussion with examples.

3.1. Refer to the results at the end of Chapter 3 for the elastic buckling of struts (pp 52 and 53), and second moments of area (pp 49 and 50). Appropriate situation is probably Case 2 (left hand side drawing).

$$F_{\mathrm{cr}} = 9.87\left(\frac{EI}{l^2}\right),\ I = \frac{\pi r^4}{4},$$

$$F_{\mathrm{cr}} = \frac{9.87\pi E}{4}\left(\frac{r}{l}\right)^2 r^2 = \frac{9.87\pi \times 2\times 10^4\ \mathrm{N\ mm^{-2}}}{4}\left(\frac{8.5\ \mathrm{mm}}{750\ \mathrm{mm}}\right)^2 8.5^2\ \mathrm{mm}^2$$

$$= 1439\ \mathrm{N} = 148\ \mathrm{kgf}.$$

This gives a factor of safety of about 148/90 = 1.65, so he should be OK.

3.2. Refer to the results at the end of Chapter 3 for the mode 1 natural vibration frequencies of beams (pp 50 and 51), and second moments of area (pp 49 and 50).

The appropriate situation is Case 2.

$$f = 0.560\sqrt{\frac{EI}{Ml^3}},\ I = \frac{bd^3}{12},\ M = \rho lbd.$$
$$f = 0.560\left(\frac{Ed^2}{12\rho l^4}\right)^{1/2} = 0.1617\left(\frac{d}{l^2}\right)\left(\frac{E}{\rho}\right)^{1/2},$$
$$E = \left(\frac{f}{0.1617}\right)^2\left(\frac{l^2}{d}\right)^2\rho.$$

Because the frequency of natural vibration involves a force acting on a mass to give it an acceleration, it is crucial when real numbers are put into the governing equation that the basic SI units are used as follows

$$E = \left(\frac{440}{0.1617}\right)^2\left(\frac{0.085^2}{0.00386}\right)^2 \times 7.85 \times 10^3 = 204 \times 10^9\ \text{N m}^{-2} = 204\ \text{GN m}^{-2}.$$

3.3. This is a consequence of the equations of static equilibrium.

3.4. Principal planes have no components of shear stress acting on them. Principal directions are normal to principal planes. Principal stresses are normal stresses acting on principal planes. The shear stress components all vanish.

3.5. (a) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, (b) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, (c) $\begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix}$, (d) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$.

3.6. $\begin{pmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. No.

3.7. $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0.5\sigma_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. There are no shear stress components normal to these axes.

3.8. Because the two shear strain terms on any given axis plane are defined so there is no rotation.

3.9. Principal strains are axial strains. The shear strain components all vanish.

3.10. (a) $\begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & -\upsilon\varepsilon_1 & 0 \\ 0 & 0 & -\upsilon\varepsilon_1 \end{pmatrix}$. (b) $\begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$.