

Section 3-2

- 3-1. Continuous
- 3-2. Discrete
- 3-3. Continuous
- 3-4. Discrete
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- 3-6. Continuous
- 3-7. Discrete
- 3-8. Continuous
- 3-9. Continuous

Section 3-3

- 3-10. a) Engineers with at least 36 months of full-time employment.
b) Samples of cement blocks with compressive strength of at least 7000 kg per square centimeter.
c) Measurements of the diameter of forged pistons that conform to engineering specifications.
d) Cholesterol levels at most 180 or at least 220.
- 3-11. The intersection of A and B is empty, therefore $P(X \in A \cap B) = 0$.
a) Yes, since $P(X \in A \cap B) = 0$.
b) $P(X \in A') = 1 - P(X \in A) = 1 - 0.4 = 0.6$
c) $P(X \in B') = 1 - P(X \in B) = 1 - 0.6 = 0.4$
d) $P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B) = 0.4 + 0.6 - 0 = 1$
- 3-12. a) $P(X \in A') = 1 - P(X \in A) = 1 - 0.3 = 0.7$
b) $P(X \in B') = 1 - P(X \in B) = 1 - 0.25 = 0.75$
c) $P(X \in C') = 1 - P(X \in C) = 1 - 0.6 = 0.4$
d) A and B are mutually exclusive if $P(X \in A \cap B) = 0$. To determine if A and B are mutually exclusive, solve the following for $P(X \in A \cap B)$:
 $P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B)$
 $0.55 = 0.3 + 0.25 - P(X \in A \cap B)$
 $0.55 = 0.55 - P(X \in A \cap B)$ and $P(X \in A \cap B) = 0$.
Therefore, A and B are mutually exclusive.
e) B and C are mutually exclusive if $P(X \in B \cap C) = 0$. To determine if B and C are mutually exclusive, solve the following for $P(X \in B \cap C)$:
 $P(X \in B \cup C) = P(X \in B) + P(X \in C) - P(X \in B \cap C)$
 $0.70 = 0.25 + 0.60 - P(X \in B \cap C)$
 $0.70 = 0.85 - P(X \in B \cap C)$ and $P(X \in B \cap C) = 0.15$.
Therefore, B and C are not mutually exclusive.
- 3-13. a) $P(X > 15) = 1 - P(X \leq 15) = 1 - 0.3 = 0.7$
b) $P(X \leq 24) = P(X \leq 15) + P(15 < X \leq 24) = 0.3 + 0.6 = 0.9$
c) $P(15 < X \leq 20) = P(X \leq 20) - P(X \leq 15) = 0.5 - 0.3 = 0.2$
d) $P(X \leq 18) = P(15 < X \leq 18) + P(X \leq 15)$
where $P(15 < X \leq 18) = P(15 < X \leq 24) - P(18 < X \leq 24) = 0.6 - 0.4 = 0.2$
Therefore, $P(X \leq 18) = P(15 < X \leq 18) + P(X \leq 15) = 0.2 + 0.3 = 0.5$
Alternatively, $P(X \leq 18) = P(X \leq 24) - P(18 < X \leq 24) = 0.9 - 0.4 = 0.5$.
- 3-14. A - Overfilled, B - Medium filled, C - Underfilled
a) $P(X \in C') = 1 - P(X \in C) = 1 - 0.20 = 0.8$
b) $P(X \in A \cup C) = P(X \in A) + P(X \in C) - P(X \in A \cap C) = 0.45 + 0.20 - 0 = 0.65$ ($P(X \in A \cap C) = 0$ since A and C are mutually exclusive)
- 3-15. a) $P(X \leq 8000) = 1 - P(X > 8000) = 1 - 0.45 = 0.55$
b) $P(X > 6000) = 1 - P(X \leq 6000) = 1 - 0.05 = 0.95$
c) $P(6000 < X \leq 8000) = P(X \leq 8000) - P(X \leq 6000) = 0.55 - 0.05 = 0.50$
- 3-16. a) Probability that a component does not fail: $P(E_1') = 1 - P(E_1) = 1 - 0.15 = 0.85$
b) $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = 0.15 + 0.30 = 0.45$

- c) $P(E_1 \text{ or } E_2)' = 1 - 0.45 = 0.55$
- 3-17. a) $P(X = 2 \text{ or } X = 3) = 0.25 + 0.25 = 0.50$
 b) $P(X < 2) = P(X = 0) + P(X = 1) = 0.10 + 0.15 = 0.25$
 c) $P(X > 3) = P(X = 4) + P(X = 5) = 0.15 + 0.10 = 0.25$
 d) $P(X > 0) = 1 - P(X = 0) = 1 - 0.1 = 0.9$
- 3-18. a) $P(X < 1) = P(X = 0) = 0.7$
 b) $P(X > 3) = 1 - P(X \leq 3) = 1 - 1 = 0$
 c) $P(X > 0) = 1 - P(X = 0) = 1 - 0.7 = 0.3$
 d) $P(X = 0) = 0.7$
- 3-19. a) $P(X > 50) = 1 - P(X \leq 50)$. Because $P(0 \leq X \leq 49) = 1$, then $P(X \leq 50) = 1$ and $P(X > 50) = 1 - 1 = 0$
 b) $P(10 \leq X \leq 19) = P(X \leq 19) - P(X < 10) = P(X \leq 19) - P(X \leq 9) = 0.7 - 0.4 = 0.3$
 c) $P(20 \leq X \leq 29) = P(X \leq 29) - P(X < 20) = P(X \leq 29) - P(X \leq 19) = 0.8 - 0.7 = 0.1$
 d) $P(\text{More than 39 unique visitors}) = P(X > 39) = 1 - P(X \leq 39) = 1 - 0.9 = 0.1$
- 3-20. a) $A \cap B =$ visit to hospital 4 that result in LWBS = 242. Then $P(A \cap B) = 242/22,252 = 0.011$
 b) $A' =$ visit to hospital 1, 2, or 3 = 5,292 + 6,991 + 5,640 = 17,926. Then $P(A') = 17,926/22,252 = 0.805$
 c) $A \cup B =$ visit to hospital 4 or a visit that results in LWBS, or both = 195 + 270 + 246 + 4329 = 5,040. Then $P(A \cup B) = 5,040/22,252 = 0.227$
 d) $A \cup B' =$ visit to hospital 4 or a visit that does not result in LWBS, or both = (5,292 - 195) + (6,991 - 270) + (5,640 - 246) + 4,329 = 21,541. Then $P(A \cup B') = 21,541/22,252 = 0.968$.
 e) $A' \cap B' =$ visit to hospital 1, 2, or 3 and does not result in LWBS = (5,292 - 195) + (6,991 - 270) + (5,640 - 246) = 17,212. Then $P(A' \cap B') = 17,212/22,252 = 0.774$. Another approach is to write $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.227 = 0.773$.

Section 3-4

- 3-21. a) $\int_0^4 kx^2 dx = k \frac{x^3}{3} \Big|_0^4 = k \frac{64}{3}$. Therefore, $k = 3/64$.
- $$E(X) = \int_0^4 \frac{3}{64} x^3 dx = \frac{3}{64} \frac{x^4}{4} \Big|_0^4 = 3$$
- $$V(X) = \int_0^4 \frac{3}{64} x^2 (x-3)^2 dx = \frac{3}{64} \int_0^4 (x^4 - 6x^3 + 9x^2) dx$$
- $$= \frac{3}{64} \left(\frac{x^5}{5} - \frac{6x^4}{4} + \frac{9x^3}{3} \right) \Big|_0^4 = 0.6$$
- b) $\int_0^2 k(1+2x) dx = k(x+x^2) \Big|_0^2 = k6$. Therefore, $k = 1/6$.
- $$E(X) = \int_0^2 \frac{1}{6} x(1+2x) dx = \frac{1}{6} \int_0^2 (x+2x^2) dx = \frac{1}{6} \left(\frac{x^2}{2} + \frac{2x^3}{3} \right) \Big|_0^2 = 11/9$$
- $$V(X) = \int_0^2 \frac{1}{6} (1+2x)(x - \frac{11}{9})^2 dx$$
- $$= \frac{1}{6} \int_0^2 \left(x^2 - \frac{22}{9}x + \frac{121}{81} + 2x^3 - \frac{44}{9}x^2 + \frac{242}{81}x \right) dx$$
- $$= \frac{1}{6} \left(\frac{x^3}{3} - \frac{22}{9} \frac{x^2}{2} + \frac{121}{81}x + 2 \frac{x^4}{4} - \frac{44}{9} \frac{x^3}{3} + \frac{242}{81} \frac{x^2}{2} \right) \Big|_0^2 = \frac{1.704}{6} = 0.284$$

$$c) \int_0^{\infty} ke^{-x} dx = k(-e^{-x}) \Big|_0^{\infty} = k. \text{ Therefore, } k = 1.$$

$$E(X) = \int_0^{\infty} xe^{-x} dx, \text{ using integration by parts } E(X) = -xe^{-x} \Big|_0^{\infty} + \int_0^{\infty} e^{-x} dx = -e^{-x} \Big|_0^{\infty} = 1$$

$$V(X) = \int_0^{\infty} (x-1)^2 e^{-x} dx = \int_0^{\infty} (x^2 e^{-x} - 2xe^{-x} + e^{-x}) dx. \text{ Now, using integration by parts}$$

$$\int_0^{\infty} x^2 e^{-x} dx = 2 \int_0^{\infty} xe^{-x} dx. \text{ Therefore, } V(X) = \int_0^{\infty} e^{-x} dx = 1, \text{ because } e^{-x} \text{ is a probability density function.}$$

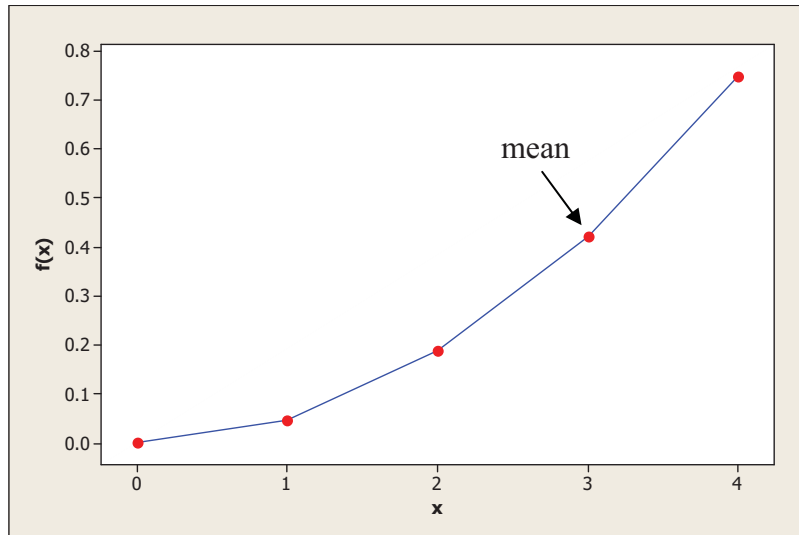
$$d) \int_{100}^{100+k} k dx = kx \Big|_{100}^{100+k} = k^2, k^2 = 1, \text{ so } k = 1 \text{ (where } k > 0)$$

$$E(X) = \int_{100}^{101} 1 dx = 100.5$$

$$V(X) = \int_{100}^{101} (x-100.5)^2 dx = 0.08333$$

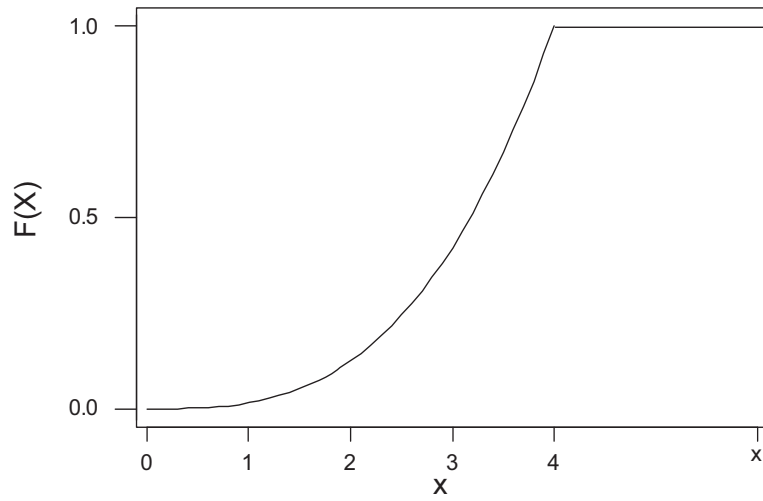
3-22. For 3-21a:

$$a) f(x) = \frac{3x^2}{64}, \quad 0 < x < 4$$



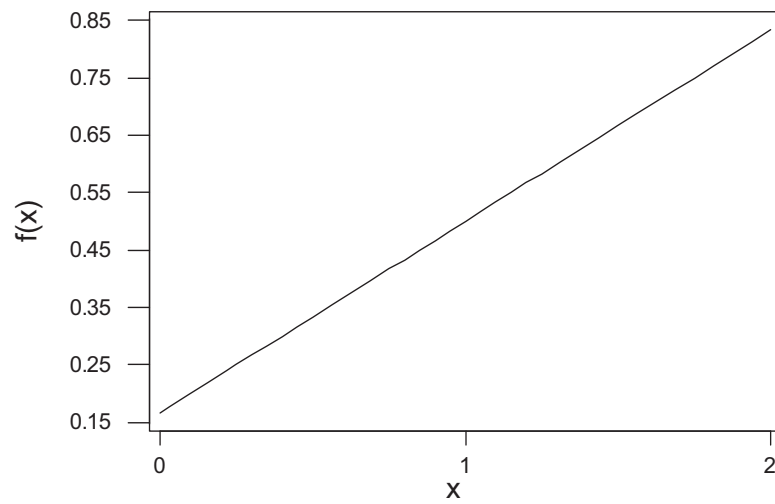
$$b) F(X) = \int_0^x f(t) dt = \int_0^x \frac{3t^2}{64} dt = \frac{x^3}{64}, \quad 0 < x < 4$$

c)



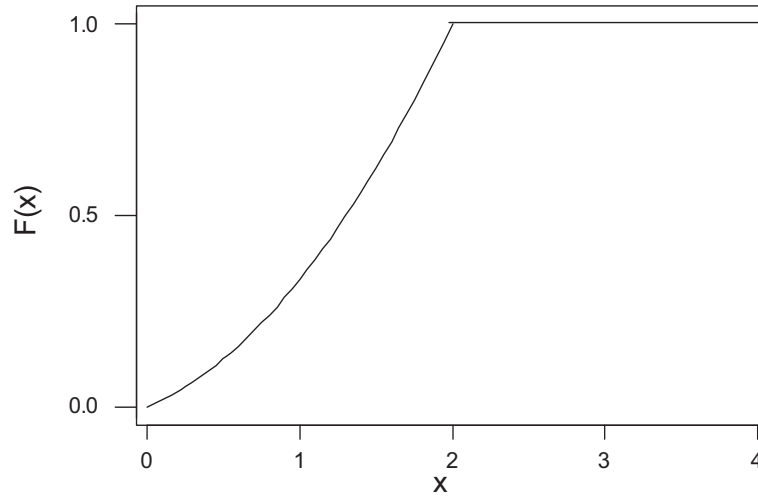
3-21b:

$$a) f(x) = \frac{(1+2x)}{6}, \quad 0 < x < 2$$



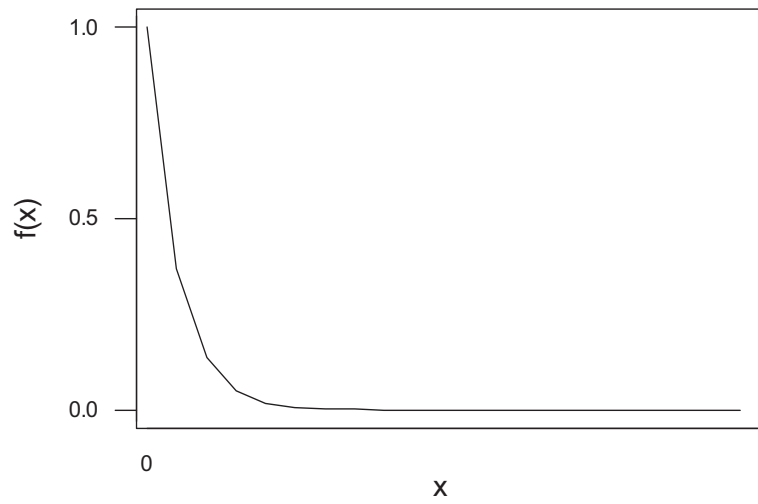
$$b) F(x) = \int_0^x \frac{(1+2t)}{6} dt = \frac{1}{6} (x+x^2), \quad 0 < x < 2$$

c)



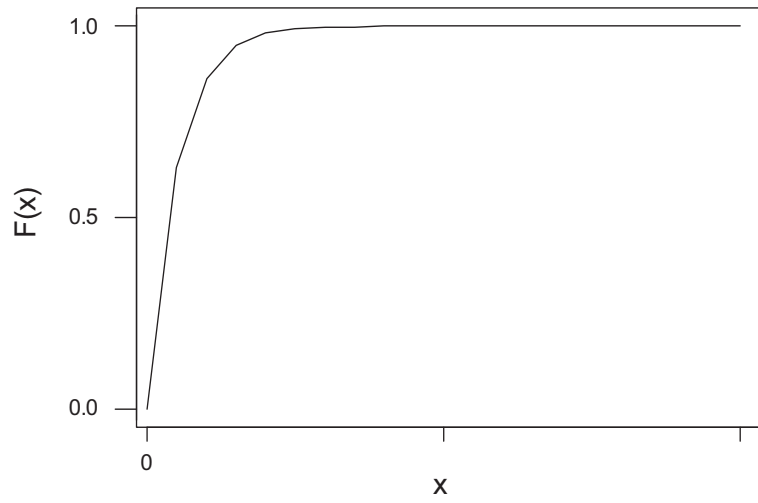
3-21c: $f(x) = e^{-x}$, $x > 0$

a)



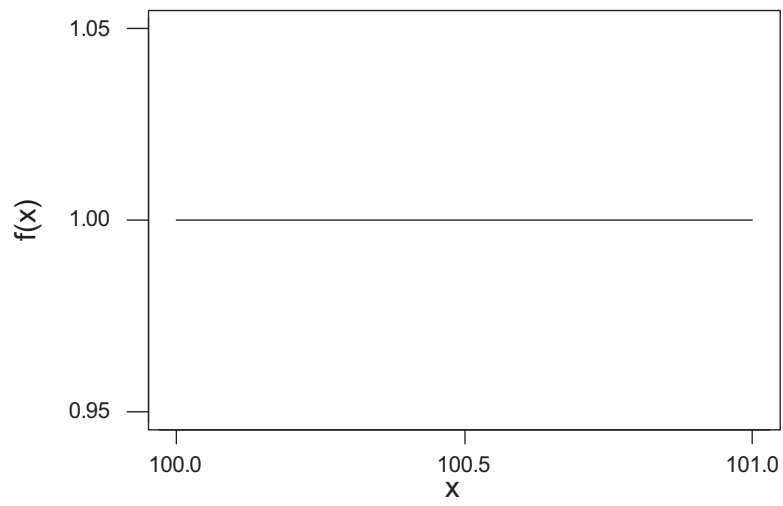
b) $F(x) = 1 - e^{-x}$, $x > 0$

c)



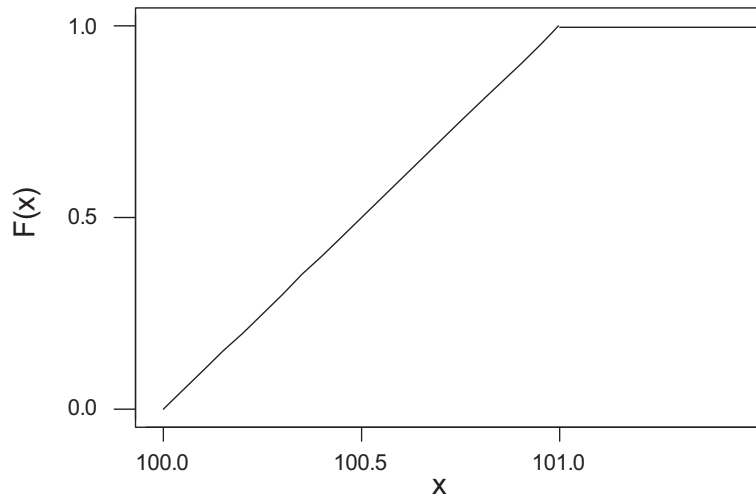
3-21 d: $f(x) = 1, 100 < x < 101$

a)



b) $F(X) = x - 100, 100 < x < 101$

c)



$$3-23. \quad a) P(X > 6) = \int_6^{\infty} e^{-(x-6)} dx = 1$$

$$b) P(6 \leq X < 8) = \int_6^8 e^{-(x-6)} dx = e^0 - 0.1353 = 0.8647$$

$$c) P(X < 8) = \int_6^8 e^{-(x-6)} dx = 0.8647$$

$$d) P(X > 8) = 1 - P(X \leq 8) = 1 - P(X < 8) = 1 - 0.8647 = 0.1353$$

$$e) P(X < x) = \int_6^x e^{-(x-6)} dx = 0.95$$

$$\int_6^x e^{-(x-6)} dx = 1 - e^{-(x-6)}$$

$$1 - e^{-(x-6)} = 0.95$$

$$x = -\ln(0.05) + 6$$

$$x = 9$$

3-24. a) $P(0 < X) = 0.5$, by symmetry.

$$b) P(0.5 < X) = \int_{0.5}^1 1.5x^2 dx = 0.5x^3 \Big|_{0.5}^1 = 0.5 - 0.0625 = 0.4375$$

$$c) P(-0.5 \leq X \leq 0.5) = \int_{-0.5}^{0.5} 1.5x^2 dx = 0.5x^3 \Big|_{-0.5}^{0.5} = 0.125$$

$$d) P(X < -2) = 0$$

$$e) P(X < 0 \text{ or } X > -0.5) = 1$$

$$f) P(x < X) = \int_x^1 1.5x^2 dx = 0.5x^3 \Big|_x^1 = 0.5 - 0.5x^3 = 0.05$$

$$\text{Then, } x = 0.9655$$

$$3-25. \quad a) P(X > 1000) = \int_{1000}^{\infty} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.7788$$

$$b) P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.1723$$

$$c) P(X < 4000) = \int_0^{4000} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.6321$$

$$d) P(X < x) = \int_0^x \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.10$$

$$\int_0^x \frac{e^{-\frac{x}{4000}}}{4000} dx = -e^{-\frac{x}{4000}} \Big|_0^x = 1 - e^{-\frac{x}{4000}} = 0.1$$

$$e^{-\frac{x}{4000}} = 0.9$$

$$x = 421.6$$

$$e) E(X) = \int_0^{\infty} x \frac{e^{-\frac{x}{4000}}}{4000} dx = 4000$$

$$V(X) = (4000)^2 = 16000000$$

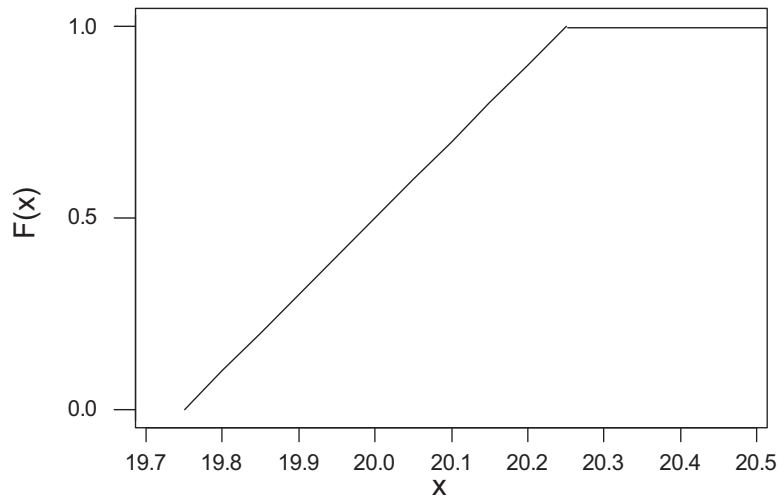
$$3-26. \quad a) P(19.75 < X < 20) = \int_{19.75}^{20} 2.0 dx = 0.5$$

$$b) P(19.8 < X < 20.0) = \int_{19.8}^{20.0} 2.0 dx = 0.4$$

$$c) E(X) = \int_{19.75}^{20.25} 2.0x dx = 20$$

$$V(X) = \int_{19.75}^{20.25} 2.0x^2 dx - [E(X)]^2 = 400.02 - (20)^2 = 0.02$$

$$d) F(x) = \int_{19.75}^x 2.0 dx = 2x - 39.5, \quad 19.75 < x < 20.25$$



- 3-27. a) $P(X < 905) = F(905) = 0.1(905) - 90 = 0.5$
 b) $P(900 < X < 905) = F(905) - F(900) = 0.5 - 0 = 0.5$
 c) $P(X > 908) = 1 - P(X \leq 908) = 1 - [0.1(908) - 90] = 0.2$
 d) In specification = $P(802 < X < 808) = F(808) - F(802) = 0.8 - 0.2 = 0.6$.
 Therefore, out of specification is $1 - 0.6 = 0.4$.

- 3-28. a) $P(X \leq 2.0080) = F(2.0080) = 200(2.0080) - 401 = 0.6$
 b) $P(X > 2.0055) = 1 - P(X \leq 2.0055) = 1 - F(2.0055) = 1 - 0.1 = 0.9$
 c) $P(2.0080 < X < 2.0090) = F(2.0090) - F(2.0080) = 0.8 - 0.6 = 0.2$

- 3-29. a) Show that $\int_1^{\infty} 2x^{-3} dx = 1$:

$$\int_1^{\infty} 2x^{-3} dx = -x^{-2} \Big|_1^{\infty} = 0 - (-1^{-2}) = 1$$

$$b) F(X) = \int_1^x 2x^{-3} dx = -x^{-2} \Big|_1^x = 1 - x^{-2}$$

$$c) E(X) = \int_1^{\infty} 2x^{-2} dx = 2.0$$

$$d) P(X < 5) = \int_1^5 2x^{-3} dx = 0.96$$

$$e) P(X > 7) = \int_7^{\infty} 2x^{-3} dx = 0.0204$$

- 3-30. a) $E(X) = \int_5^{\infty} x 10e^{-10(x-5)} dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)} dx$, we obtain

$$E(X) = -xe^{-10(x-5)} \Big|_5^\infty + \int_5^\infty e^{-10(x-5)} dx = 5 - \frac{e^{-10(x-5)}}{10} \Big|_5^\infty = 5.1$$

Now, $V(X) = \int_5^\infty (x-5.1)^2 10e^{-10(x-5)} dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$$dv = 10e^{-10(x-5)}, \text{ we obtain } V(X) = -(x-5.1)^2 e^{-10(x-5)} \Big|_5^\infty + 2 \int_5^\infty (x-5.1)e^{-10(x-5)} dx.$$

From the definition of $E(X)$, the integral above is recognized to equal 0. Therefore, $V(X) = (5-5.1)^2 = 0.01$.

$$\text{b) } P(X > 5.1) = \int_{5.1}^\infty 10e^{-10(x-5)} dx = -e^{-10(x-5)} \Big|_{5.1}^\infty = e^{-10(5.1-5)} = 0.3679$$

$$3-31. \quad \text{a) } P(X < 1408) = F(1408) = 0.1(1408) - 140 = 0.8$$

$$\text{b) } P(1395 < X < 1405) = F(1405) - F(1395) = [0.1(1405) - 140] - 0 = 0.5$$

$$3-32. \quad \text{a) } E(X) = \int_{100}^{120} x \frac{600}{x^2} dx = 600 \ln x \Big|_{100}^{120} = 600(\ln 120 - \ln 100) = 109.39 \mu\text{m}$$

$$V(X) = \int_{100}^{120} x^2 \frac{600}{x^2} dx - E(x)^2 = 600x \Big|_{100}^{120} - 109.39^2 = 33.19 \mu\text{m}^2$$

$$\text{b) Average Cost} = \text{Coating Cost} \times \text{Mean of the Coating Thickness} \\ = 0.50 \times 109.39 = \$54.70$$

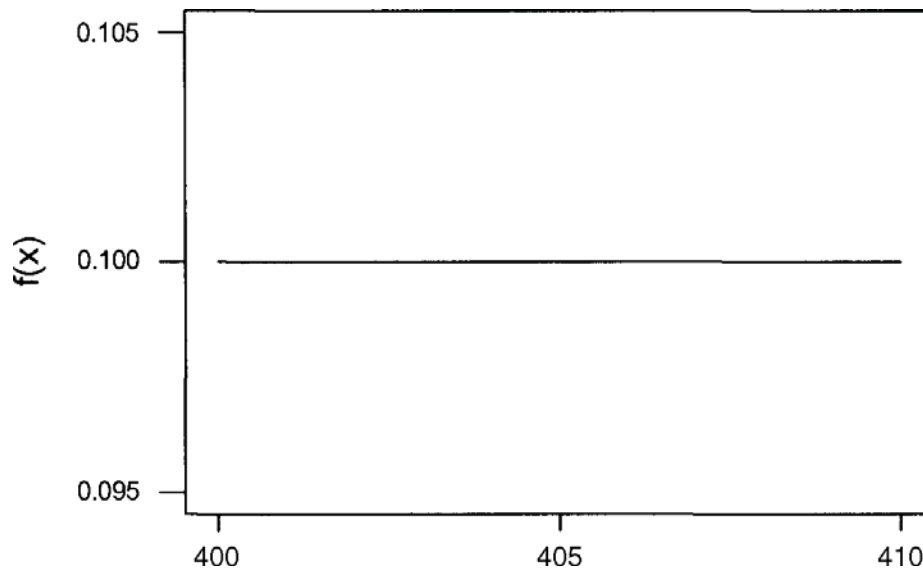
$$3-33. \quad \text{a) } P(X < 409) = 0.1(409) - 40 = 0.9$$

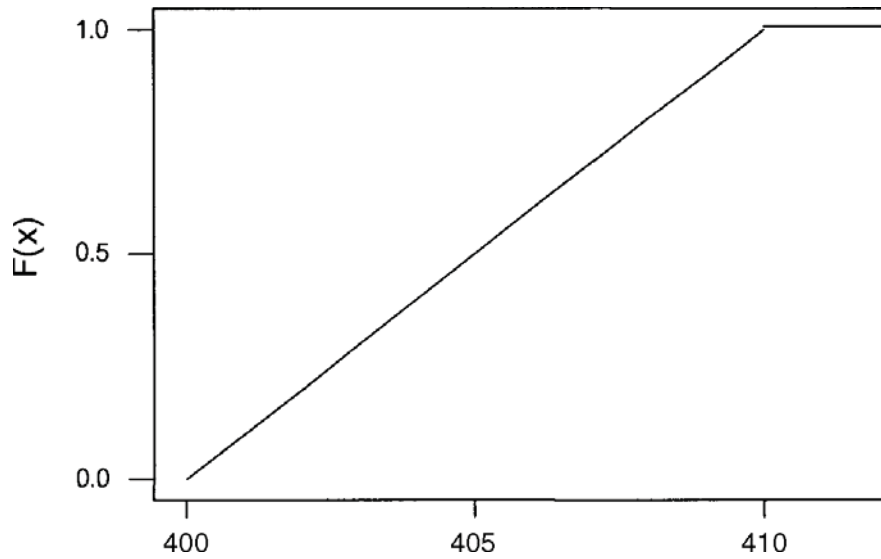
$$\text{b) } P(400 < X < 408) = F(408) - F(400) = 0.8 - 0 = 0.8$$

$$\text{c) } P(X > 409) = 1 - F(409) = 1 - 0.9 = 0.1$$

$$\text{d) } f(x) = 0.1 \text{ for } 400 \leq x \leq 410$$

e)





f) $E(X) = 205, V(X) = 8.3333$

3-34. Now, $f(x) = \frac{e^{-x/10}}{10}$ for $0 < x$ and

$$F_x(x) = \frac{1}{10} \int_0^x e^{-u/10} du = -e^{-u/10} \Big|_0^x = 1 - e^{-x/10} \text{ for } 0 < x.$$

$$\text{Then, } F_x(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-x/10}, & x > 0 \end{cases}$$

a) $P(X < 60) = F(60) = 1 - e^{-6} = 1 - 0.002479 = 0.9975$

b) $\frac{1}{10} \int_{15}^{30} e^{-x/10} dx = e^{-1.5} - e^{-3} = 0.173343$

c) $\frac{1}{10} \int_0^x e^{-u/10} du = 0.5 \rightarrow x \approx 6.93147$

d) $P(15 < X < 30) = F(30) - F(15) = e^{-1.5} - e^{-3} = 0.173343$

e) $\mu = E(X) = \int_0^{\infty} xf(x) dx = \frac{1}{10} \int_0^{\infty} xe^{-x/10} dx = e^{-x/10}(-x-10) \Big|_0^{\infty} = 0 + 10 = 10$

$$\begin{aligned} \sigma^2 = V(X) &= \int_0^{\infty} x^2 f(x) dx - \mu^2 = \frac{1}{10} \int_0^{\infty} x^2 e^{-x/10} dx = e^{-x/10}((-x-20)x-200) \Big|_0^{\infty} - (10)^2 \\ &= 0 + 200 - 100 = 100 \end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{100} = 10$$

3-35. a) $P(1 < X < 5) = \int_1^5 f(x) dx = -\frac{32}{31x} \Big|_1^5 = 0.826$

b) $E(X) = \int_1^{32} xf(x) dx = \int_1^{32} \frac{32}{31x} dx = \frac{32}{31} \ln x \Big|_1^{32} = 3.578$

$$E(X^2) = \int_1^{32} x^2 f(x) dx = \int_1^{32} \frac{32}{31} dx = 32$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 32 - 12.8021 = 19.1979$$

c) $3 * 3.578 = 10.73$

- 3-36. a) $P(X < 1) = F(1) = 1 - \exp(-1/2) = 1 - 0.607 = 0.393$
 b) $P(X > 2) = 1 - F(2) = 1 - (1 - \exp(-2/2)) = 1 - (1 - 0.368) = 0.368$
 c) $P(1 \leq X < 2) = F(2) - F(1) = 0.632 - 0.393 = 0.239$
 d) $P(X < x) = 0.95 = 1 - \exp(-x/2)$. Then, $\exp(-x/2) = 0.05$ and $x = 6$
 e) $f(x) = 0.5e^{-x/2}$ for $0 \leq x$

3-37. $F_x(x) = \int_0^x f(u)du = \left(\frac{1}{18}u^2\right)\Big|_0^x = \frac{1}{18}x^2$ for $0 < x < 3$
 $F_x(x) = \int_0^3 f(u)du + \int_3^x f(u)du = 0.5 + \left(\frac{2}{3}u - \frac{1}{18}u^2\right)\Big|_3^x$
 $= \frac{2}{3}x - \frac{1}{18}x^2 - 1$, for $3 < x < 6$.

Then, $F_x(x) = \begin{cases} \frac{1}{18}x^2, & 0 < x < 3 \\ \frac{2}{3}x - \frac{1}{18}x^2 - 1, & 3 < x < 6 \end{cases}$

a) $P(X < 4) = F(4) = \frac{2}{3}(4) - \frac{1}{18}(4)^2 - 1 = \frac{7}{9}$

b) $P(X > 5) = 1 - F(5) = 1 - [\frac{2}{3}(5) - \frac{1}{18}(5)^2 - 1] = 1 - \frac{17}{18} = 1 - 0.944 = 0.056$

c) $P(X \leq 0.5) = F(0.5) = \frac{1}{18}(0.5^2) = 0.014$

d) $P(X > x) = 1 - F(x) = 0.1$. Therefore $F(x) = 0.9$.

From part (b), $F(5) = 0.944$, the range of $3 < x < 6$ are selected.

$F(x) = 0.9 = \frac{2}{3}(x) - \frac{1}{18}(x)^2 - 1$. Then, $\frac{2}{3}(x) - \frac{1}{18}(x)^2 = 1.9$.

Solving the equation above to obtain $x = 4.658$

e) $E(X) = \int_0^3 xf(x)dx + \int_3^6 xf(x)dx = \int_0^3 x \frac{1}{9}x dx + \int_3^6 x(\frac{2}{3} - \frac{1}{9}x)dx = (\frac{1}{27}x^3 \Big|_0^3) + (\frac{1}{3}x^2 - \frac{1}{27}x^3 \Big|_3^6) = 1 + (4 - 2) = 3$

Section 3-5

- 3-38. a) $P(-1 < Z < 1) = P(Z < 1) - P(Z < -1) = 0.841345 - 0.158655 = 0.68269$
 b) $P(-2 < Z < 2) = P(Z < 2) - P(Z < -2) = 0.97725 - 0.02275 = 0.9545$
 c) $P(-3 < Z < 3) = P(Z < 3) - P(Z < -3) = 0.998650 - 0.00135 = 0.997300$
 d) $P(Z < -3) = 0.00135$
 e) $P(0 < Z \leq 3) = P(Z < 3) - P(Z < 0) = 0.998650 - 0.5 = 0.498650$

3-39. a) $P(Z < z) = 0.500000$
 $z = 0$

b) $P(Z < z) = 0.001001$
 $z = -3.09$

c) $P(Z > z) = 0.881000$
 Can be rewritten as: $1 - P(Z < z) = 0.881000$
 $P(Z < z) = 0.119000$
 $z = -1.18$

d) $P(Z > z) = 0.866500$
 Can be rewritten as: $1 - P(Z < z) = 0.866500$
 $P(Z < z) = 0.133500$
 $z = -1.11$

e) $P(-1.3 < Z < z) = 0.863140$

$$\begin{aligned} P(Z < z) - P(Z < -1.3) &= 0.863140 \\ P(Z < z) - 0.096801 &= 0.863140 \\ P(Z < z) &= 0.959941, Z = 1.75 \end{aligned}$$

- 3-40. a) $P(-z < Z < z) = P(Z < z) - P(Z < -z) = 1 - 2P(Z < -z) = 0.90$
 So $P(Z < -z) = 0.5(1 - 0.90) = 0.05$
 $z = 1.65$
- b) $P(-z < Z < z) = P(Z < z) - P(Z < -z) = 1 - 2P(Z < -z) = 0.97$
 So $P(Z < -z) = 0.5(1 - 0.97) = 0.015$
 $z = 2.17$
- c) $P(-z < Z < z) = P(Z < z) - P(Z < -z) = 1 - 2P(Z < -z) = 0.61$
 So $P(Z < -z) = 0.5(1 - 0.61) = 0.195$
 $z = 0.86$
- d) $P(-z < Z < z) = P(Z < z) - P(Z < -z) = 1 - 2P(Z < -z) = 0.99$
 So $P(Z < -z) = 0.5(1 - 0.99) = 0.005$
 $z = 2.58$
- 3-41. a) $P(X < 34) = P(Z < 2) = 0.97725$
 b) $P(X > 28) = P(Z > -1) = 0.841345$
 c) $P(28 < X < 32) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1) = 0.841345 - 0.158655 = 0.68269$
 d) $P(24 < X < 36) = P(-3 < Z < 3) = 0.99865 - 0.00135 = 0.9973$
 e) $P(26 < X < 30) = P(-2 < Z < 0) = P(Z < 0) - P(Z < -2) = 0.5 - 0.02275 = 0.47725$
 f) $P(30 < X < 36) = P(0 < Z < 3) = P(Z < 3) - P(Z < 0) = 0.99865 - 0.50 = 0.49865$

- 3-42. a) $P(X > x) = P(Z > z) = 0.5$ where $z = \frac{x-20}{2} = 0$. Thus $x = 20$
- b) $P(X > x) = P(Z > z) = 0.95$ where $z = \frac{x-20}{2} = -1.645$. Thus $x = 16.71$
- c) $P(x < X < 20) = P(Z < 0) - P(Z < z) = 0.5 - P(Z < z) = 0.2$
 So $P(Z < z) = 0.3$ where $z = \frac{x-20}{2} = -0.53$. Thus $x = 18.94$

- 3-43. a) $P(X < 21) = P(Z < -3) = 0.001350$
 b) $P(X > 20) = P(Z > -3.5) = 0.999767$
 c) $P(23 < X < 27) = P(-2 < Z < 0) = P(Z < 0) - P(Z < -2) = 0.5 - 0.02275 = 0.47725$
 d) $P(22 < X < 29) = P(-2.5 < Z < 1) = 0.841345 - 0.00621 = 0.835135$
 e) $P(20 < X < 28) = P(-3.5 < Z < 0.5) = 0.691462 - 0.000233 = 0.691229$

- 3-44. a) $P(X > x) = 1 - P(X < x)$
 $= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.50.$

$$\text{And, } P\left(Z < \frac{x-6}{3}\right) = 0.50$$

$$\text{Therefore, } \frac{x-6}{3} = 0 \text{ and } x = 6.$$

- b) $P(X > x) = 1 - P(X < x)$
 $= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.95.$

$$\text{And, } P\left(Z < \frac{x-6}{3}\right) = 0.05$$

$$\text{Therefore, } \frac{x-6}{3} = -1.645 \text{ and } x = 1.065.$$

- c) $P(x < X < 9) = 0.20$

$$P\left(\frac{x-6}{3} < Z < \frac{9-6}{3}\right) = P\left(\frac{x-6}{3} < Z < 1\right)$$

$$P(Z < 1) - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$0.841345 - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.841345 - 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.641345$$

Therefore, $\frac{x-6}{3} = 0.36$, and $x = 7.08$.

d) $P(3 < X < x) = 0.80$

$$P\left(\frac{3-6}{3} < Z < \frac{x-6}{3}\right) = P\left(-1 < Z < \frac{x-6}{3}\right)$$

$$P\left(Z < \frac{x-6}{3}\right) - P(Z < -1) = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) - 0.158655 = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.958655$$

Therefore, $\frac{x-6}{3} \cong 1.74$, and $x = 11.22$.

3-45. a) $P(X < 5250) = P\left(Z < \frac{5250 - 5000}{100}\right)$
 $= P(Z < 2.5)$
 $= 0.9938$

b) $P(4800 < X < 4900) = P(-2 < Z < -1)$
 $= P(Z < -1) - P(Z < -2)$
 $= 0.1359$

c) $P(X > x) = P\left(Z > \frac{x - 5000}{100}\right) = 0.95.$

Therefore, $\frac{x-5000}{100} = -1.64$ and $x = 4835.51$.

3-46. a) $P(X < 269) = P(Z < 2) = 0.97725$
b) $P(X < 200) = P(Z < -3) = 0.00170$

3-47. a) $P(X > 0.62) = P\left(Z > \frac{0.62 - 0.5}{0.05}\right)$
 $= P(Z > 2.4)$
 $= 1 - P(Z < 2.4)$
 $= 0.0082$
b) $P(0.47 < X < 0.63) = P(-0.6 < Z < 2.6)$
 $= P(Z < 2.6) - P(Z < -0.6)$
 $= 0.99534 - 0.27425$

$$= 0.7211$$

$$\text{c) } P(X < x) = P\left(Z < \frac{x-0.5}{0.05}\right) = 0.90.$$

$$\text{Therefore, } \frac{x-0.5}{0.05} = 1.28 \text{ and } x = 0.5641.$$

$$3-48. \quad \text{a) } P(X < 355) = P\left(Z < \frac{355-367}{3}\right) = P(Z < -4) \cong 0$$

$$\text{b) } P(X < 358) = P\left(Z < \frac{358-367}{3}\right)$$

$$= P(Z < -3)$$

$$= 0.00135$$

and

$$P(X > 373) = P\left(Z > \frac{373-367}{3}\right)$$

$$= P(Z > 2)$$

$$= 0.02275$$

Therefore, the proportion of cans scrapped is $0.00135 + 0.02275 = 0.0241$.

$$\text{c) } P(367 - x < X < 367 + x) = 0.99.$$

$$\text{Therefore, } P\left(-\frac{x}{3} < Z < \frac{x}{3}\right) = 0.99$$

$$\text{Consequently, } P\left(Z < \frac{x}{3}\right) = 0.995 \text{ and } x = 3(2.58) = 7.74.$$

The specifications are (359.26; 374.74).

$$3-49. \quad \text{d) If } P(X > 355) = 0.999, \text{ then } P\left(Z > \frac{355-\mu}{3}\right) = 0.999.$$

$$\text{Therefore, } \frac{355-\mu}{3} = -3.09 \text{ and } \mu = 364.27.$$

$$\text{e) If } P(X > 355) = 0.999, \text{ then } P\left(Z > \frac{355-\mu}{1.5}\right) = 0.999.$$

$$\text{Therefore, } \frac{355-\mu}{1.5} = -3.09 \text{ and } \mu = 359.635.$$

$$3-50. \quad \text{a) } P(X > 0.5) = P\left(Z > \frac{0.5-0.4}{0.05}\right)$$

$$= P(Z > 2)$$

$$= 1 - 0.97725$$

$$= 0.02275$$

$$\text{b) } P(0.4 < X < 0.5) = P(0 < Z < 2)$$

$$= P(Z < 2) - P(Z < 0)$$

$$= 0.47725$$

$$\text{c) } P(X > x) = 0.90. \text{ Therefore, } \frac{x-0.4}{0.05} = -1.28 \text{ and } x = 0.336.$$

$$3-51. \quad \text{a) } P(90.3 < X) = P\left(\frac{90.3-90.2}{0.1} < Z\right)$$

$$= P(1 < Z)$$

$$= P(Z > 1)$$

$$\begin{aligned}
&= 1 - P(Z < 1) \\
&= 1 - 0.841345 \\
&= 0.158655
\end{aligned}$$

$$\begin{aligned}
P(X < 89.7) &= P\left(Z < \frac{89.7 - 90.2}{0.1}\right) \\
&= P(Z < -5) \\
&\cong 0.
\end{aligned}$$

Therefore, the answer is 0.1587.

b) The process mean should be set at the center of the specifications; that is, at $\mu = 90.0$.

$$\begin{aligned}
\text{c) } P(89.7 < X < 90.3) &= P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right) \\
&= P(-3 < Z < 3) = 0.9973.
\end{aligned}$$

$$\text{3-52. a) } P(X > 1140) = P\left(Z > \frac{1140 - 1000}{60}\right) = P(Z > 2.33) = P(Z < -2.33) = 0.009903$$

$$\text{b) } P(X < 900) = P\left(Z > \frac{900 - 1000}{60}\right) = P(Z > -0.17) = 0.432505$$

$$\text{3-53. a) } P(X > 9) = P(Z > 1.34) = 0.09012$$

$$\text{b) } P(5.5 < X < 8.5) = P(-0.35 < Z < 1.10) = 0.864334 - 0.363169 = 0.501165$$

$$\text{c) Threshold} = \mu + 3.75\sigma = 6.23 + 3.75(2.064) = 13.97$$

$$\begin{aligned}
\text{3-54. a) } P(X < 5000) &= P\left(Z < \frac{5000 - 7000}{600}\right) \\
&= P(Z < -3.33) \\
&= 0.0004.
\end{aligned}$$

$$\text{b) } P(X > x) = 0.95. \text{ Therefore, } P\left(Z > \frac{x - 7000}{600}\right) = 0.95 \text{ and } \frac{x - 7000}{600} = -1.64. \text{ Consequently, } x = 6016.$$

$$\begin{aligned}
\text{3-55. a) } P(X > 0.066) &= P\left(Z > \frac{0.066 - 0.05}{0.01}\right) \\
&= P(Z > 1.6) \\
&= 0.0548.
\end{aligned}$$

$$\begin{aligned}
\text{b) } P(0.036 < X < 0.066) &= P(-1.4 < Z < 1.6) \\
&= 0.8644.
\end{aligned}$$

$$\begin{aligned}
\text{c) } P(0.036 < X < 0.066) &= P\left(\frac{0.036 - 0.05}{\sigma} < Z < \frac{0.066 - 0.05}{\sigma}\right) \\
&= P\left(\frac{-0.014}{\sigma} < Z < \frac{0.016}{\sigma}\right) = 0.995
\end{aligned}$$

$$\text{Therefore, } P\left(Z < \frac{0.016}{\sigma}\right) = 0.995. \text{ Consequently, } \frac{0.016}{\sigma} = 2.81 \text{ and } \sigma = 0.00569.$$

$$\text{3-56. a) } P(X > 62) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.977250 = 0.02275$$

$$\begin{aligned}
\text{b) } P(X < 62) &= 0.999 \\
P\left(Z < \frac{62 - 59}{\hat{\sigma}}\right) &= 0.999 \\
\frac{62 - 59}{\hat{\sigma}} &= 3.09 \\
\hat{\sigma} &= 0.97087
\end{aligned}$$

$$c) P(X < 62) = P(Z < z) = 0.999 \text{ where } z = \frac{62 - \hat{\mu}}{1.5} = 3.09. \text{ Thus } \hat{\mu} = 57.365$$

3-57. X is a lognormal distribution with $\theta=5$ and $\omega^2=9$

$$a) P(X < 13300) = P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13300) - 5}{3}\right) \\ = \Phi(1.50) = 0.9332$$

b) Find the value for which $P(X \leq x) = 0.95$

$$P(X \leq x) = P(e^W \leq x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 5}{3}\right) = 0.95$$

$$\frac{\ln(x) - 5}{3} = 1.65 \quad x = e^{1.65(3)+5} = 20952.2$$

$$c) \mu = E(X) = e^{\theta + \omega^2/2} = e^{5+9/2} = e^{9.5} = 13359.7$$

$$V(X) = e^{2\theta + \omega^2} (e^{\omega^2} - 1) = e^{10+9} (e^9 - 1) = e^{19} (e^9 - 1) = 1.45 \times 10^{12}$$

3-58. a) X is a lognormal distribution with $\theta=2$ and $\omega^2=4$

$$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500) - 2}{2}\right) \\ = \Phi(2.11) = 0.9826$$

b)

$$P(500 < X < 1000) = P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000)) = \Phi\left(\frac{\ln(1000) - 2}{2}\right) - \Phi\left(\frac{\ln(500) - 2}{2}\right) \\ = \Phi(2.45) - \Phi(2.11) = 0.99286 - 0.9826 \\ = 0.01026$$

c)

$$P(1500 < X < 2000) = P(1500 < e^W < 2000) = P(\ln(1500) < W < \ln(2000)) = \Phi\left(\frac{\ln(2000) - 2}{2}\right) - \Phi\left(\frac{\ln(1500) - 2}{2}\right) \\ = \Phi(2.80) - \Phi(2.66) = 0.99744 - 0.99609 \\ = 0.00135$$

d) The product has significantly degraded over the first 500 hours. The degradation is less significant after 500 hours.

3-59. X is a lognormal distribution with $\theta=0.5$ and $\omega^2=1$

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1 - \Phi\left(\frac{\ln(10) - 0.5}{1}\right) \\ = 1 - \Phi(1.80) = 1 - 0.96407 = 0.03593$$

$$b) P(X \leq x) = P(e^W \leq x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x) - 0.5}{1}\right) = 0.50$$

$$\frac{\ln(x) - 0.5}{1} = 0 \quad x = e^{0(1)+0.5} = 1.65 \text{ seconds}$$

$$c) \mu = E(X) = e^{\theta + \omega^2/2} = e^{0.5+1/2} = e^1 = 2.7183$$

$$V(X) = e^{2\theta + \omega^2} (e^{\omega^2} - 1) = e^{1+1} (e^1 - 1) = e^2 (e^1 - 1) = 12.6965$$

3-60. a) Find the values of θ and ω^2 given that $E(X) = 10000$ and $\sigma = 20,000$

$$10000 = e^{\theta + \omega^2/2} \quad 20000^2 = e^{2\theta + \omega^2} (e^{\omega^2} - 1)$$

Let $x = e^\theta$ and $y = e^{\omega^2}$ then (1) $10000 = x\sqrt{y}$ and (2) $20000^2 = x^2y(y-1) = x^2y^2 - x^2y$

Square (1): $10000^2 = x^2y$ and substitute into (2)

$$20000^2 = 10000^2(y-1)$$

$$y = 5$$

Substitute y into (1) and solve for x : $x = \frac{10000}{\sqrt{5}} = 4472.1360$

$$\theta = \ln(4472.1360) = 8.4056 \quad \text{and} \quad \omega^2 = \ln(5) = 1.6094$$

$$\text{b) } P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000) - 8.4056}{1.2686}\right)$$

$$= 1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$$

$$\text{c) } P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\frac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$$

$$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \quad x = e^{-1.28(1.2686) + 8.4056} = 881.65 \text{ hours}$$

3-61. $\beta = 0.2$ and $\delta = 100$ hours

$$E(X) = 100\Gamma\left(1 + \frac{1}{0.2}\right) = 100 \times 5! = 12,000$$

$$V(X) = 100^2\Gamma\left(1 + \frac{2}{0.2}\right) - 100^2[\Gamma\left(1 + \frac{1}{0.2}\right)]^2 = 3.61 \times 10^{10}$$

3-62. a) $P(X < 10000) = F_X(10000) = 1 - e^{-100^{0.2}} = 1 - e^{-2.512} = 0.9189$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-50^{0.2}} = 0.1123$

3-63. Let X denote lifetime of a bearing. $\beta=2$ and $\delta=10000$ hours

$$\text{a) } P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^2} = e^{-0.8^2} = 0.5273$$

b)

$$\begin{aligned} E(X) &= 10000\Gamma\left(1 + \frac{1}{2}\right) = 10000\Gamma(1.5) \\ &= 10000(0.5)\Gamma(0.5) = 5000\sqrt{\pi} = 8862.3 \\ &= 8862.3 \text{ hours} \end{aligned}$$

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a binomial random variable with $n = 10$ and $p = 0.5273$.

$$P(Y = 10) = \binom{10}{10} 0.5273^{10} (1 - 0.5273)^0 = 0.00166.$$

3-64. a) $E(X) = \delta\Gamma\left(1 + \frac{1}{\beta}\right) = 900\Gamma\left(1 + \frac{1}{3}\right) = 900\Gamma\left(\frac{4}{3}\right) = 900(0.89298) = 803.68 \text{ hours}$

b)

$$\begin{aligned} V(X) &= \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2 = 900^2\Gamma\left(1 + \frac{2}{3}\right) - 900^2\left[\Gamma\left(1 + \frac{1}{3}\right)\right]^2 \\ &= 900^2(0.90274) - 900^2(0.89298)^2 = 85314.64 \text{ hours}^2 \end{aligned}$$

$$\text{c) } P(X < 500) = F_X(500) = 1 - e^{-\left(\frac{500}{900}\right)^3} = 0.1576$$

3-65. $E(X) = \delta\Gamma\left(1 + \frac{1}{2}\right) = 2.5$

$$\text{So } \delta = \frac{2.5}{\Gamma\left(1 + \frac{1}{2}\right)} = \frac{5}{\sqrt{\pi}}$$

$$\text{Var}(X) = \delta^2 \Gamma(2) - (E(X))^2 = \frac{25}{\pi} - 2.5^2 = 1.7077$$

$$\text{Stdev}(X) = 1.3068$$

3-66. $\Gamma(r) = \int_0^{\infty} x^{r-1} e^{-x} dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$$\Gamma(r) = -x^{r-1} e^{-x} \Big|_0^{\infty} + (r-1) \int_0^{\infty} x^{r-2} e^{-x} dx = (r-1) \Gamma(r-1).$$

3-67. a) $\Gamma(6) = 5! = 120$

$$\text{b) } \Gamma\left(\frac{5}{2}\right) = \frac{3}{2} \Gamma\left(\frac{3}{2}\right) = \frac{3}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3}{4} \pi^{1/2} = 1.32934$$

$$\text{c) } \Gamma\left(\frac{9}{2}\right) = \frac{7}{2} \Gamma\left(\frac{7}{2}\right) = \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{105}{16} \pi^{1/2} = 11.6317$$

3-68. $E(X) = 6/3 = 2$; $V(X) = 6/3^2 = 0.6667$

3-69. $E(X) = 3.2/2.5 = 1.28$; $V(X) = 3.2/(2.5)^2 = 0.512$

3-70. $r/\lambda = 3$, so $r = 3\lambda$; Also, $r/\lambda^2 = 1.5$; Using substitution, $3\lambda/\lambda^2 = 1.5$, and $\lambda = 2$;
Giving $r = 6$.

3-71. $r/\lambda = 11.4$, so $r = 11.4\lambda$; Also, $r/\lambda^2 = 40.32$; Using substitution, $\frac{11.4\lambda}{\lambda^2} = 40.32$, and $\lambda = 0.28$;
Giving $r = 3.19$.

3-72. $r/\lambda = 4$, so $r = 4\lambda$; Also, $r/\lambda^2 = 2$; Using substitution, $4\lambda/\lambda^2 = 2$, and $\lambda = 2$;
Giving $r = 8$.

Section 3-6

$$\begin{aligned} 3-73. \text{ a) } P(X < 0.25) &= \int_0^{0.25} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} dx = \int_0^{0.25} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} dx \\ &= \int_0^{0.25} \frac{\Gamma(3.5)}{\Gamma(2.5)\Gamma(1)} x^{1.5} dx = \frac{(2.5)(1.5)(0.5)\sqrt{\pi}}{(1.5)(0.5)\sqrt{\pi}} \frac{x^{2.5}}{2.5} \Big|_0^{0.25} = 0.25^{2.5} = 0.0313 \end{aligned}$$

$$\begin{aligned} \text{b) } P(0.25 < X < 0.75) &= \int_{0.25}^{0.75} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} dx = \int_{0.25}^{0.75} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} dx \\ &= \int_{0.25}^{0.75} \frac{\Gamma(3.5)}{\Gamma(2.5)\Gamma(1)} x^{1.5} dx = \frac{(2.5)(1.5)(0.5)\sqrt{\pi}}{(1.5)(0.5)\sqrt{\pi}} \frac{x^{2.5}}{2.5} \Big|_{0.25}^{0.75} = 0.75^{2.5} - 0.25^{2.5} = 0.4559 \end{aligned}$$

$$\text{c) } \mu = E(X) = \frac{\alpha}{\alpha + \beta} = \frac{2.5}{2.5 + 1} = 0.7143$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)} = \frac{2.5}{(3.5)^2 (4.5)} = 0.0454$$

$$3-74. \quad \text{a) } P(X < 0.25) = \int_0^{0.25} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} = \int_0^{0.25} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

$$= \int_0^{0.25} \frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)} (1-x)^{3.2} = \frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2) - (1-x)^{4.2}}{(3.2)(2.2)(1.2)\Gamma(1.2) \cdot 4.2} \Big|_0^{0.25} = -(0.75)^{4.2} + 1 = 0.7013$$

$$\text{b) } P(0.5 < X) = \int_{0.5}^1 \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} = \int_{0.5}^1 \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

$$= \int_{0.5}^1 \frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)} (1-x)^{3.2} = \frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2) - (1-x)^{4.2}}{(3.2)(2.2)(1.2)\Gamma(1.2) \cdot 4.2} \Big|_{0.5}^1 = 0 + (0.5)^{4.2} = 0.0544$$

$$\text{c) } \mu = E(X) = \frac{\alpha}{\alpha + \beta} = \frac{1}{1 + 4.2} = 0.1923$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{4.2}{(5.2)^2(6.2)} = 0.0251$$

$$3-75. \quad \text{a) Mode} = \frac{\alpha - 1}{\alpha + \beta - 2} = \frac{2}{3 + 1.4 - 2} = 0.8333$$

$$\mu = E(X) = \frac{\alpha}{\alpha + \beta} = \frac{3}{3 + 1.4} = 0.6818$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{4.2}{(4.4)^2(5.4)} = 0.0402$$

$$\text{b) Mode} = \frac{\alpha - 1}{\alpha + \beta - 2} = \frac{9}{10 + 6.25 - 2} = 0.6316$$

$$\mu = E(X) = \frac{\alpha}{\alpha + \beta} = \frac{10}{10 + 6.25} = 0.6154$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{62.5}{(16.25)^2(17.25)} = 0.0137$$

c) Both mean and variance from part a) are larger than part b).

$$3-76. \quad \text{a) } P(X > 0.9) = \int_{0.9}^1 \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} = \int_{0.9}^1 \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

$$= \int_{0.9}^1 \frac{\Gamma(11)}{\Gamma(10)\Gamma(1)} x^9 = \frac{(10)(9)\Gamma(9)}{(9)\Gamma(9)} \frac{x^{10}}{10} \Big|_{0.9}^1 = 1 - (0.9^{10}) = 0.6513$$

$$\text{b) } P(X < 0.5) = \int_0^{0.5} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} = \int_0^{0.5} \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

$$= \int_0^{0.5} \frac{\Gamma(11)}{\Gamma(10)\Gamma(1)} x^9 = \frac{(10)(9)\Gamma(9)}{(9)\Gamma(9)} \frac{x^{10}}{10} \Big|_0^{0.5} = 0.5^{10} = 0.0010$$

$$\text{c) } \mu = E(X) = \frac{\alpha}{\alpha + \beta} = \frac{10}{10 + 1} = 0.9091$$

$$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} = \frac{10}{(11)^2(12)} = 0.0069$$

3-77. Let X denotes the complete time which is a proportion time of the maximum time
 $X = 2/2.5 = 0.8$