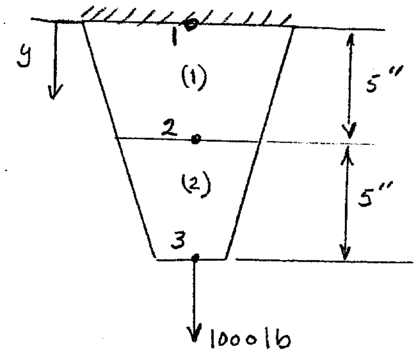
$\boxed{1.1}$

(a) Using two elements



$A(y) = 0.25 - 0.0125y$

$A_1 = 0.25 \text{ in}^2 \qquad A_2 = 0.1875 \text{ in}^2$

$A_3 = 0.125 \text{ in}^2$

$$K_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

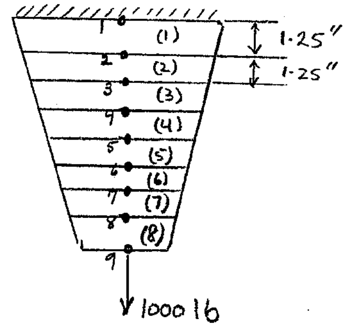
$$K_1 = \frac{(0.25 + 0.1875)(10.4 \times 10^6)}{2\,(5)} = 455{,}000 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = \frac{(0.1875 + 0.125)(10.4 \times 10^6)}{2(5)} = 325{,}000 \ \frac{\text{lb}}{\text{in}}$$

$$10^3 \begin{array}{c} \begin{array}{ccc} 1 & 0 & \end{array} \\ \begin{bmatrix} \cancel{455} & \cancel{-455} & \\ \cancel{-455}\ 0 & 455+325 & -325 \\ & -325 & 325 \end{bmatrix} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1000 \end{Bmatrix}$$

$u_1 = 0, \quad \underline{\underline{u_2 = 0.002197 \text{ in}}} \quad \underline{\underline{u_3 = 0.005274 \text{ in}}}$ ⇦

(b)



$A_1 = 0.25 \text{ in}^2$

$A_2 = 0.234375 \text{ in}^2$

$A_3 = 0.21875 \text{ in}^2$

$A_4 = 0.203125 \text{ in}^2$

$A_5 = 0.1875 \text{ in}^2$

$A_6 = 0.171875 \text{ in}^2$

$A_7 = 0.15625 \text{ in}^2$

$A_8 = 0.140625 \text{ in}^2$

$A_9 = 0.125 \text{ in}^2$

1.1 Cont.

$K_1 = 2015000\ \frac{lb}{in}$ $\quad K_2 = 1885000\ \frac{lb}{in}$ $\quad K_3 = 1755000\ \frac{lb}{in}$

$K_4 = 1625000\ \frac{lb}{in}$ $\quad K_5 = 1495000\ \frac{lb}{in}$ $\quad K_6 = 1365000\ \frac{lb}{in}$

$K_7 = 1235000\ \frac{lb}{in}$ $\quad K_8 = 1105000\ \frac{lb}{in}$

$$\begin{bmatrix} \overset{1}{\cancel{2015000}} & \overset{0}{\cancel{-2015000}} & & & & & & & \\ \underset{0}{\cancel{-2015000}} & \begin{matrix}2015000\\+1885000\end{matrix} & -1885000 & & & & & & \\ & -1885000 & \begin{matrix}1885000\\+1755000\end{matrix} & -1755000 & & & & & \\ & & -1755000 & \begin{matrix}1755000\\+1625000\end{matrix} & -1625000 & & & & \\ & & & -1625000 & \begin{matrix}1625000\\+1495000\end{matrix} & -1495000 & & & \\ & & & & -1495000 & \begin{matrix}1495000+\\1365000\end{matrix} & -1365000 & & \\ & & & & & -1365000 & \begin{matrix}1365000+\\1235000\end{matrix} & -1235000 & \\ & & & & & & -1235000 & \begin{matrix}1235000+\\1105000\end{matrix} & -1105000 \\ & & & & & & & -1105000 & 1105000 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1000 \end{Bmatrix}$$

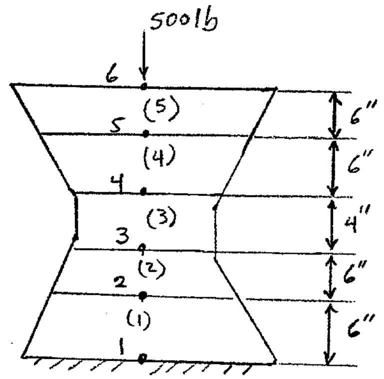
$u_1 = 0$, $\quad u_2 = 0.00049628$, $\quad u_3 = 0.0010268$, $\quad u_4 = 0.0015965$

$u_5 = 0.0022119$, $\quad u_6 = 0.0028808$, $\quad u_7 = 0.0036134$, $\quad u_8 = 0.0044232$

$u_9 = 0.0053281$ in.

y (in)	exact defl.	Two-element	eight element
0	0	0	0
1.25	0.00049645		0.00049628
2.5	0.0010272		0.0010268
3.75	0.0015972		0.0015965
5.0	0.0022129	0.002197	0.0022119
6.25	0.0028822		0.0028808
7.5	0.0036154		0.0036134
8.75	0.0044259		0.0044232
10.0	0.0053319	0.005274	0.0053281

$$\boxed{1.2}$$



$$K = \frac{A_{avg.}E}{l}$$

$$K_1 = K_5 = \frac{\left(\frac{10.5+12}{2}\right)(3)(3.27\times10^{6})}{6}$$

$$K_1 = K_5 = 18,393,750\ \frac{lb}{in}$$

$$K_2 = K_4 = \frac{\left(\frac{10.5+9}{2}\right)(3)(3.27\times10^{6})}{6} = 15,941,250\ \frac{lb}{in}$$

$$K_3 = \frac{(9)(3)(3.27\times10^{6})}{4} = 22,072,500\ \frac{lb}{in}$$

$$\begin{bmatrix} \cancel{18393750}\ 1 & \cancel{-18393750}\ 0 & & & & \\ \cancel{-18393750}\ 0 & 18393750+15941250 & -15941250 & & & \\ & -15941250 & 15941250+22072500 & -22072500 & & \\ & & -22072500 & 22072500+15941250 & -15941250 & \\ & & & -15941250 & 15941250+18393750 & -18393750 \\ & & & & -18393750 & 18393750 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -500 \end{Bmatrix}$$

$$\{u\} = \begin{Bmatrix} 0 \\ -2.71831\times10^{-5} \\ -5.85483\times10^{-5} \\ -8.12009\times10^{-5} \\ -1.12566\times10^{-4} \\ -1.39749\times10^{-4} \end{Bmatrix} \text{in} \Leftarrow$$

$$\sigma = \left(\frac{u_{i+1}-u_i}{l}\right)E$$

$$\sigma^{(1)} = \frac{(-2.71831\times10^{-5}-0)(3.27\times10^{6})}{6} = \underline{\underline{14.8\text{ Psi C}}}$$

$$\sigma^{(5)} = \frac{(-1.39749+1.12566)\times10^{-4}\times(3.27\times10^{6})}{6} = \underline{\underline{14.8\text{ Psi C}}}$$

as a check:

$$\sigma^{(1)} = \sigma^{(5)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+12}{2}\right)(3)} = 14.8\text{ psi C}$$

$$\sigma^{(2)} = \frac{(-5.85483+2.71831)\times10^{-5}\times(3.27\times10^{6})}{6} = \underline{\underline{17.1\text{ Psi C}}}$$

$$\sigma^{(4)} = \frac{(-1.12566\times10^{-4}+8.12009\times10^{-5})(3.27\times10^{6})}{6} = \underline{\underline{17.1\text{ PsiC}}}$$

as a check:

$$\sigma^{(2)} = \sigma^{(4)} = \frac{F}{A} = \frac{500}{\left(\frac{10.5+9}{2}\right)(3)} = 17.1\text{ Psi}$$

$$\sigma^{(3)} = \frac{(-8.12009\times10^{-5}+5.85483)\times10^{-5}\times(3.27\times10^{6})}{4}$$

$$\sigma^{(3)} = \underline{\underline{18.5\text{ Psi C}}}$$

as a check:

$$\sigma^{(3)} = \frac{F}{A} = \frac{500}{(9)(3)} = 18.5\text{ Psi}$$

$\boxed{1.3}$

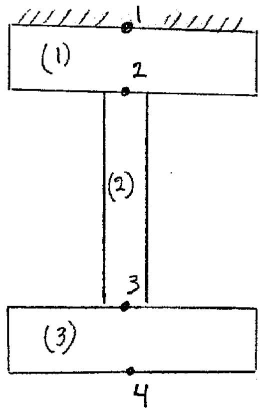
$$K = \frac{AE}{\ell}$$

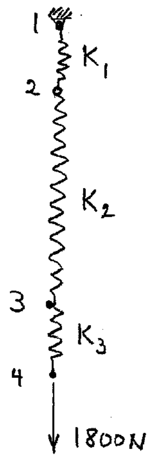
$$K_1 = K_3 = \frac{(0.08)(0.006)(68.9 \times 10^9)}{0.025}$$

$$K_1 = K_3 = 1.32288 \times 10^9 \ \frac{N}{m}$$

$$K_2 = \frac{(0.02)(0.006)(68.9 \times 10^9)}{0.1}$$

$$K_2 = 0.08268 \times 10^9 \ \frac{N}{m}$$





$$\begin{array}{ccccc} & 1 & 0 & & \\ 10^9 & \cancel{1.32288} & \cancel{-1.32288} & & \\ & \cancel{-1.32288} & 1.32288 + 0.08268 & -0.08268 & \\ & & -0.08268 & 0.08268 + 1.32288 & -1.32288 \\ & & & -1.32288 & 1.32288 \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1800 \end{Bmatrix}$$

$$\begin{bmatrix} 1.40556 & -0.08268 & 0 \\ -0.08268 & 1.40556 & -1.32288 \\ 0 & -1.32288 & 1.32288 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1800 \times 10^{-9} \end{Bmatrix}$$

$u_2 = 1.360668 \times 10^{-6}$ m $u_3 = 2.313135 \times 10^{-5}$ m $u_4 = 2.449202 \times 10^{-5}$ m

$$\sigma^{(1)} = E \frac{u_2 - u_1}{\ell} = (68.9 \times 10^9)\left(\frac{1.360668 \times 10^{-6} - 0}{0.025}\right) = 3750000 = 3.75 \text{ MPa}$$

as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{1800 \text{ N}}{(0.08)(0.006)} = 3.75 \text{ MPa}$

$$\sigma^{(2)} = E \frac{u_3 - u_2}{\ell} = (68.9 \times 10^9)\left(\frac{2.313135 \times 10^{-5} - 1.360668 \times 10^{-6}}{0.1}\right) = 15 \text{ MPa}$$

as a check $\sigma^{(2)} = \dfrac{1800 \text{ N}}{(0.02)(0.006)} = 15 \text{ MPa}$

$$\sigma^{(3)} = E \frac{u_4 - u_3}{\ell} = (68.9 \times 10^9) \frac{(2.449202 - 2.313135) \times 10^{-5}}{0.025} = 3.75 \text{ MPa}$$

$\boxed{1.4}$

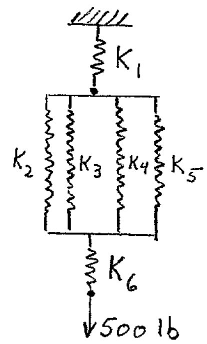
$$K_1 = K_6 = \frac{(4)\,(0.125)(28\times10^6)}{2}$$

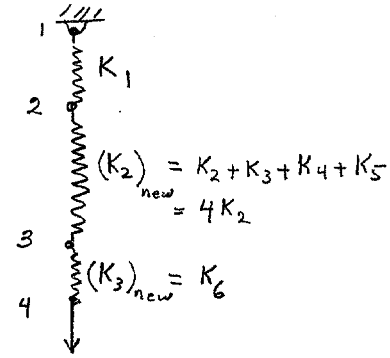
$$K_1 = K_6 = 7\times10^6 \ \frac{\text{lb}}{\text{in}}$$

$$K_2 = K_3 = K_4 = K_5 = \frac{(0.625)(0.125)(28\times10^6)}{8}$$

$$K_2 = 273438 \ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_{new} = (4)(273438) = 1093750 \ \frac{\text{lb}}{\text{in}}$$





$$\begin{bmatrix} \overset{1}{\cancel{7\times10^6}} & 0 & \overset{0}{\cancel{-7\times10^6}} & \\ \cancel{-7\times10^6}\ 0 & 7\times10^6+1093750 & -1093750 & \\ & -1093750 & 1093750+7\times10^6 & -7\times10^6 \\ & & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 500 \end{Bmatrix}$$

$$\begin{bmatrix} 8093750 & -1093750 & 0 \\ -1093750 & 8093750 & -7\times10^6 \\ 0 & -7\times10^6 & 7\times10^6 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500 \end{Bmatrix}$$

$u_2 = \underline{\underline{7.142857\times10^{-5} \text{ in}}}$ $\quad u_3 = \underline{\underline{5.285714\times10^{-4} \text{ in}}}$ $\quad u_4 = \underline{\underline{6\times10^{-4} \text{ in}}}$

$$\sigma^{(1)} = E\left(\frac{u_2 - u_1}{l}\right) = 28\times10^6\left(\frac{7.142857\times10^{-5} \text{ in}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

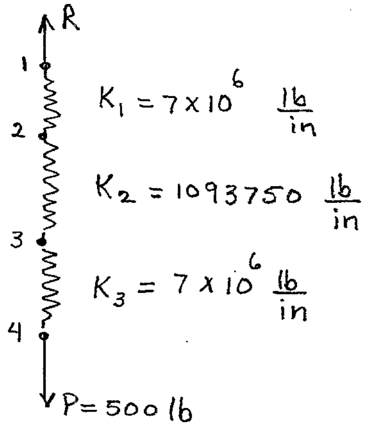
as a check: $\sigma^{(1)} = \dfrac{F}{A} = \dfrac{500}{(4)(0.125)} = 1000 \ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(2)} = E\left(\frac{u_3 - u_2}{l}\right) = 28\times10^6\left(\frac{5.285714\times10^{-4} - 7.142857\times10^{-5}}{8}\right) = \underline{\underline{1600 \ \frac{\text{lb}}{\text{in}^2}}}$$

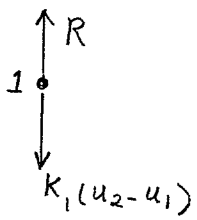
as a check: $\sigma^{(2)} = \dfrac{500}{(2.5)(0.125)} = 1600 \ \dfrac{\text{lb}}{\text{in}^2}$

$$\sigma^{(3)} = E\left(\frac{u_4 - u_3}{l}\right) = 28\times10^6\left(\frac{6\times10^{-4} - 5.285714\times10^{-4}}{2}\right) = \underline{\underline{1000 \ \frac{\text{lb}}{\text{in}^2}}}$$

$\boxed{1.5}$

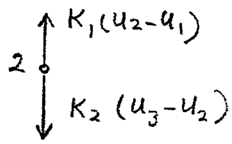


node 1:



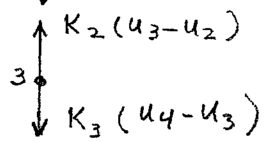
$$R - K_1(u_2 - u_1) = 0$$

node 2:



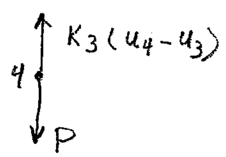
$$K_1(u_2 - u_1) - K_2(u_3 - u_2) = 0$$

node 3:



$$K_2(u_3 - u_2) - K_3(u_4 - u_3) = 0$$

node 4:

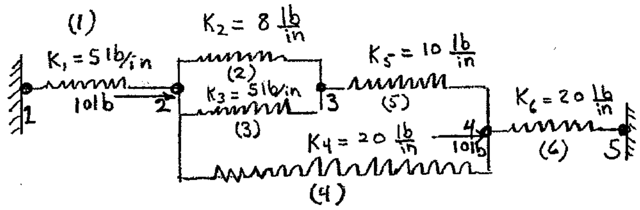


$$K_3(u_4 - u_3) - P = 0$$

In matrix form:

$$\begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1 + K_2 & -K_2 & \\ & -K_2 & K_2 + K_3 & -K_3 \\ & & K_3 & K_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -R \\ 0 \\ 0 \\ P \end{Bmatrix}$$

$\boxed{1.6}$



Size of the global matrix: 5x5

$$[K]^{(1)} = \begin{array}{cc} ① & ② \\ \end{array}\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{array}{c} ① \\ ② \end{array} \qquad [K]^{(2)} = \begin{array}{cc} ② & ③ \\ \end{array}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{array}{c} ② \\ ③ \end{array}$$

$$[K]^{(3)} = \begin{array}{cc} ② & ③ \\ \end{array}\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}\begin{array}{c} ② \\ ③ \end{array} \qquad [K]^{(4)} = \begin{array}{cc} ② & ④ \\ \end{array}\begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}\begin{array}{c} ② \\ ④ \end{array}$$

$$[K]^{(5)} = \begin{array}{cc} ③ & ④ \\ \end{array}\begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}\begin{array}{c} ③ \\ ④ \end{array} \qquad [K]^{(6)} = \begin{array}{cc} ④ & ⑤ \\ \end{array}\begin{bmatrix} 20 & -20 \\ -20 & 20 \end{bmatrix}\begin{array}{c} ④ \\ ⑤ \end{array}$$

$$[K]^{(G)} = \begin{bmatrix} 5 & -5 & & & \\ -5 & 5+8+5+20 & -8-5 & -20 & \\ & -8-5 & 8+5+10 & -10 & \\ & -20 & -10 & 20+10+20 & -20 \\ & & & -20 & 20 \end{bmatrix}$$

applying B.Cs and loads; $u_1 = u_5 = 0$ $\quad F_2 = 10 lb$, $\quad F_4 = 10$ lb

$$\begin{bmatrix} 38 & -13 & -20 \\ -13 & 23 & -10 \\ -20 & -10 & 50 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 10 \\ 0 \\ 10 \end{Bmatrix}$$

$\underline{\underline{u_2 = 0.962 \text{ in}}}$ $\qquad u_3 = \underline{\underline{0.874 \text{ in}}}$ $\qquad u_4 = \underline{\underline{0.760 \text{ in}}}$

node 1:

$R_1 \leftarrow \underset{1}{\bullet} \rightarrow K_1(u_2 - u_1)$ $\qquad R_1 = 5\,(0.962 - 0) = \underline{\underline{4.8 \text{ lb}}}$

node 5

$K(u_4 - u_5) \rightarrow \underset{5}{\bullet} \leftarrow R_5$ $\qquad R_5 = 20\,(0.760 - 0) = \underline{\underline{15.2 \text{ lb}}}$

note also as a check, $\quad R_1 + R_5 = \sum F_{external}$ $\qquad 4.8 + 15.2 = 10 + 10$

$\boxed{1.7}$

$$
150\begin{bmatrix}
\overset{\frac{1}{150}}{\cancel{5.88}} & \overset{0}{\cancel{5.88}} & & & & & & \\
-5.88 & 5.88+2.27 & -2.27 & & & & & \\
 & -2.27 & 2.27+10 & -10 & & & & \\
 & & -10 & 10+0.581 & -0.581 & & & \\
 & & & -0.581 & 0.581+0.781 & -0.781 & & \\
 & & & & -0.781 & 0.781+2.22 & -2.22 & \\
 & & & & & -2.22 & 2.22+1.47 & -1.47 \\
 & & & & & & \underset{0}{\cancel{-1.47}} & \underset{\frac{1}{150}}{\cancel{1.47}}
\end{bmatrix}
\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{Bmatrix}
=
\begin{Bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}
$$

$$
\{T\} = \begin{Bmatrix} 10 \\ 12.04 \\ 17.31 \\ 18.51 \\ 39.12 \\ 54.46 \\ 59.85 \\ 68 \end{Bmatrix} {}^{\circ}\mathrm{F}
$$

$$q = UA\Delta T$$

$$q = (1.47)(150)(68-59.85) = \underline{\underline{1800\ \frac{\text{Btu}}{\text{hr}}}}$$

or as another example:

$$q = (0.781)(150)(54.46-39.12) = 1800\ \frac{\text{Btu}}{\text{hr}}$$

or

$$q = \frac{1}{\sum R_{\text{esistance}}} A\left(T_{in} - T_{out}\right) = \frac{(150)(68-10)}{0.17+0.44+0.1+1.72+1.28+0.45+0.68} = 1800\ \frac{\text{Btu}}{\text{hr}}$$

$\boxed{1.8}$

$$150\begin{bmatrix} \cancel{5.88}\ \tfrac{1}{150} & \cancel{-5.88}\ 0 & & & & & \\ -5.88 & 5.88+1.23 & -1.23 & & & & \\ & -1.23 & 1.23+0.76 & -0.76 & & & \\ & & -0.76 & 0.76+0.053 & -0.053 & & \\ & & & -0.053 & 0.053+2.22 & -2.22 & \\ & & & & -2.22 & 2.22+1.47 & -1.47 \\ & & & & & \cancel{-1.47}\ 0 & \cancel{1.47}\ \tfrac{1}{150} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 68 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 20 \\ 20.37 \\ 22.12 \\ 24.95 \\ 65.57 \\ 66.54 \\ 68 \end{Bmatrix} {}^{\circ}F \Leftarrow$$

$$q = UA\,\Delta T$$

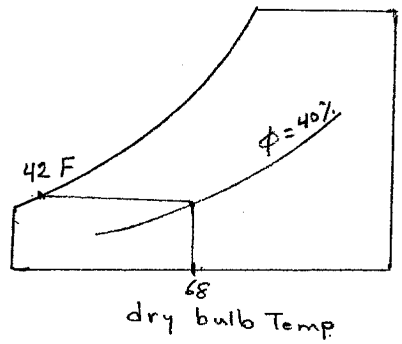
as an example:

$$q = (1.47)(150)(68-66.54) = \underline{\underline{320\ \frac{Btu}{hr}}}$$

or

$$q = \frac{1}{\sum R_{esistance}} A\,(T_{in} - T_{out}) = \frac{(150)(68-20)}{0.17+0.81+1.32+19+0.45+0.68} = 320\ \frac{Btu}{hr}$$

$\boxed{1.9}$



with the help of a Psychometric chart, using a dry bulb Temp. of 68°F and $\phi = 40\%$, we identify Condensation temperature to be 42°F. Thus Condensation will occur between surfaces 4 and 5.

$\boxed{1.10}$

$$1000\begin{bmatrix} \cancel{1.47}\ \tfrac{1}{1000} & \cancel{-1.47}\ 0 & & & \\ -1.47 & 1.47+0.053 & -0.053 & & \\ & -0.053 & 0.053+2.22 & -2.22 & \\ & & -2.22 & 2.22+1.47 & -1.47 \\ & & & \cancel{-1.47}\ 0 & \cancel{1.47}\ \tfrac{1}{1000} \end{bmatrix}\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 0 \\ 0 \\ 0 \\ 70 \end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix} 15 \\ 16.81 \\ 66.99 \\ 68.19 \\ 70 \end{Bmatrix} °F$$

$q = UA\,\Delta T$

as an example:

$$q = (1.47)(1000)(70-68.19) = \underline{\underline{2660\ \frac{Btu}{hr}}}$$

or

$$q = (1.47)(1000)(16.81-15) = 2660\ \frac{Btu}{hr}$$

$\boxed{1.11}$

$$22.5\begin{bmatrix} \cancel{5.88}\ \frac{1}{22.5} & \cancel{-5.88}\ 0 & & \\ -5.88 & 5.88+2.56 & -2.56 & \\ & -2.56 & 2.56+1.47 & -1.47 \\ & & \cancel{-1.47}\ 0 & \cancel{1.47}\ \frac{1}{22.5} \end{bmatrix}\begin{Bmatrix}T_1\\T_2\\T_3\\T_4\end{Bmatrix}=\begin{Bmatrix}20\\0\\0\\70\end{Bmatrix}$$

$$\{T\} = \begin{Bmatrix}20\\26.85\\42.59\\70\end{Bmatrix} \ {}^\circ F$$

$$q = U_1 A\, \Delta T = (5.88)(22.5)(26.85-20) = 910 \ \frac{Btu}{hr}$$

or as another example:

$$q = U_2 A\, \Delta T = (2.56)(22.5)(42.59-26.85) = 910 \ \frac{Btu}{hr}$$

or

$$q = \frac{1}{\sum \text{Resistance}} A \ (T_{in} - T_{out}) = \frac{1}{(0.17+0.39+0.68)}(22.5)(70-20)$$

$$\underline{\underline{q = 910 \ \frac{Btu}{hr}}}$$

$\boxed{1.12}$

$$(A_1)_c = (A_5)_c = \left(\frac{12+10.5}{2}\right)(3) - (3)\frac{\pi}{4}(0.5)^2 = 33.161 \text{ in}^2$$

$$(A_2)_c = (A_4)_c = \frac{(10.5+9)}{2}(3) - 3\,\frac{\pi}{4}(0.5)^2 = 28.661 \text{ in}^2$$

$$(A_3)_c = (9)(3) - 3\,\frac{\pi}{4}(0.5)^2 = 26.411 \text{ in}^2$$

$$(K_1)_c = (K_5)_c = \frac{(33.161)(3.27\times10^6)}{6} = 18{,}072{,}745 \ \frac{\text{lb}}{\text{in}}$$

$$(K_2)_c = (K_4)_c = \frac{(28.661)(3.27\times10^6)}{6} = 15{,}620{,}245 \ \frac{\text{lb}}{\text{in}}$$

$$(K_3)_c = \frac{(26.411)(3.27\times10^6)}{4} = 21{,}590{,}992 \ \frac{\text{lb}}{\text{in}}$$

$$(K_1)_s = (K_2)_s = (K_4)_s = (K_5)_s = \frac{(3)(\frac{\pi}{4})(0.5)^2(29\times10^6)}{6} = 2{,}847{,}068 \ \frac{\text{lb}}{\text{in}}$$

$$(K_3)_s = \frac{(3)(\frac{\pi}{4})(0.5)^2(29\times10^6)}{4} = 4{,}270{,}602 \ \frac{\text{lb}}{\text{in}}$$

The Combined stiffnesses:

$$K_1 = K_5 = 18{,}072{,}745 + 2{,}847{,}068 = 20{,}919{,}813$$

$$K_2 = K_4 = 15{,}620{,}245 + 2{,}847{,}068 = 18{,}467{,}313$$

$$K_3 = 21{,}590{,}992 + 4{,}270{,}602 = 25{,}861{,}594$$

$$\begin{bmatrix} \cancel{20{,}919{,}813}\ 1 & \cancel{-20{,}919{,}813}\ 0 & & & & \\ \cancel{-20{,}919{,}813}\ 0 & 20{,}919{,}813 + 18{,}467{,}313 & -18{,}467{,}313 & & & \\ & -18{,}467{,}313 & 18{,}467{,}313 + 25{,}861{,}594 & -25{,}861{,}594 & & \\ & & -25{,}861{,}594 & 25{,}861{,}594 + 18{,}467{,}313 & -18{,}467{,}313 & \\ & & & -18{,}467{,}313 & 18{,}467{,}313 + 20{,}919{,}813 & -20{,}919{,}813 \\ & & & & -20{,}919{,}813 & 20{,}919{,}813 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1000 \end{Bmatrix}$$

$u_1 = 0$ $\quad u_2 = -4.78016\times10^{-5}$ in $\quad u_3 = -1.01951\times10^{-4}$ in

$u_4 = -1.40618\times10^{-4}$ $\quad u_5 = -1.94768\times10^{-4}$ in $\quad u_6 = -2.42570\times10^{-4}$ in

$\boxed{\begin{matrix}1.12\\ \text{Cont.}\end{matrix}}$

$$\sigma^{(1)}_{\text{Concrete}} = E_c\left(\frac{u_2-u_1}{\ell}\right) = \frac{(3.27\times10^{6})(-4.78016\times10^{-5})}{6} = \underline{\underline{26\ \frac{\text{lb}}{\text{in}^2}\ \text{C}}}$$

$$\sigma^{(1)}_{\text{Steel}} = E_s\left(\frac{u_2-u_1}{\ell}\right) = \underline{\underline{231\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

as a check:

$$\sigma^{(1)}_{\text{Concrete}}(A_1)_c + \sigma^{(1)}_s(A_1)_s = (26)(33.161)+(231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = \underline{\underline{1000\ \text{lb}}}$$

$$\sigma^{(2)}_c = E_c\left(\frac{u_3-u_2}{\ell}\right) = \frac{(3.27\times10^{6})(-1.01951\times10^{-4}+4.78016\times10^{-5})}{6} = \underline{\underline{30\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma^{(2)}_s = 231\ \frac{\text{lb}}{\text{in}^2}\ \text{C}$$

check: $\sigma^{(2)}_c(A_2)_c + \sigma^{(2)}_s(A_2)_s = (30)(28.661)+(231)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000\ \text{lb}$

$$\sigma^{(3)}_c = E_c\left(\frac{u_4-u_3}{\ell}\right) = \frac{(3.27\times10^{6})(-1.40618\times10^{-4}+1.01951\times10^{-4})}{4} = \underline{\underline{32\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma^{(3)}_s = E_s\left(\frac{u_4-u_3}{\ell}\right) = \frac{(29\times10^{6})(-1.40618\times10^{-4}+1.01951\times10^{-4})}{4} = \underline{\underline{280\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

check: $\sigma^{(3)}_c(A_3)_c + \sigma^{(3)}_s(A_3)_s = (32)(26.411)+(280)(3)\left(\frac{\pi}{4}\right)(0.5)^2 = 1000\ \text{lb}$

$$\sigma^{(4)}_c = E_c\left(\frac{u_5-u_4}{\ell}\right) = \frac{(3.27\times10^{6})(-1.94768\times10^{-4}+1.40618\times10^{-4})}{6} = \underline{\underline{30\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma^{(4)}_s = 231\ \frac{\text{lb}}{\text{in}^2}\ \text{C}$$

$$\sigma^{(5)}_c = E_c\left(\frac{u_6-u_5}{\ell}\right) = \frac{(3.27\times10^{6})(-2.42570+1.94768)\times10^{-4}}{6} = \underline{\underline{26\ \frac{\text{lb}}{\text{in}^2}}}\ \text{C}$$

$$\sigma^{(5)}_s = 231\ \frac{\text{lb}}{\text{in}^2}\ \text{C}$$

note $\sigma^{(1)}_c = \sigma^{(5)}_c$ and $\sigma^{(2)}_c = \sigma^{(4)}_c$ as expected.

$\boxed{1.13}$

$$\Lambda^{(e)} = \frac{A_{avg}\,E}{2\ell}(u_{i+1}-u_i)^2$$

$$\Lambda_{total} = \Lambda^{(1)} + \Lambda^{(2)} + \Lambda^{(3)} + \Lambda^{(4)} + \Lambda^{(5)}$$

$$\Lambda^{(1)} = \Lambda^{(5)} = \frac{(A_1)_c E_c}{2\ell}(u_{i+1}-u_i)^2 + \frac{(A_1)_s E_s}{2\ell}(u_{i+1}-u_i)^2$$

$$= \frac{(A_1)_c E_c + (A_1)_s E_s}{2\ell}(u_2-u_1)^2$$

$$\Lambda^{(1)} = \left[\frac{(33.161)(3.27\times10^6) + 3(\frac{\pi}{4})(0.5)^2(29\times10^6)}{(2)(6)}\right](-4.78016\times10^{-5})^2 = 0.024 \text{ lb.in}$$

or $\Lambda^{(5)} = [\quad // \quad]\left[(-2.4257+1.94768)\times10^{-4}\right]^2 = 0.024$ ✓

$$\Lambda^{(2)} = \frac{(A_2)_c E_c + (A_2)_s E_s}{2\ell}(u_3-u_2)$$

$$= \left[\frac{(28.661)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(6)}\right]\left[(-1.01951\times10^{-4} + 4.78016\times10^{-5})\right]^2 = 0.027$$

or $\Lambda^{(4)} = [\quad // \quad]\left[(-1.94768+1.40618)\times10^{-4}\right]^2 = 0.027$ ✓

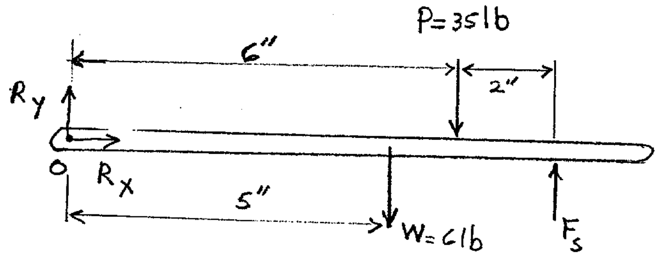
$$\Lambda^{(3)} = \frac{(A_3)_c E_c + (A_3)_s E_s}{2\ell}(u_4-u_3)$$

$$\Lambda^{(3)} = \left[\frac{(26.411)(3.27\times10^6) + 3\frac{\pi}{4}(0.5)^2(29\times10^6)}{2(4)}\right]\left[(-1.40618+1.01951)\times10^{-4}\right]^2$$

$$= 0.019 \text{ lb.in}$$

$$\Lambda_{total} = (2)(0.024) + 0.019 + 2(0.027) = \underline{\underline{0.121 \text{ lb.in}}} \Leftarrow$$

$\boxed{1.14}$



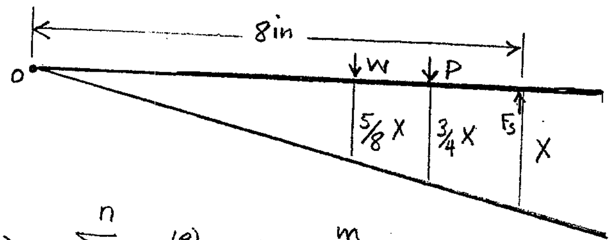
(a) $\circlearrowleft^{+} \sum M_o = 0$

$$(-6\ \text{lb})(5\ \text{in}) + (35\ \text{lb})(6\ \text{in}) + F_s(8\ \text{in}) = 0$$

$$F_s = 30\ \text{lb}$$

$F_s = KX \qquad 30\ \text{lb} = 60X \qquad \underline{\underline{X = 0.5\ \text{in}}}$

(b)



$$\frac{\partial \pi}{\partial u_i} = \frac{\partial}{\partial u_i}\sum_{i=1}^{n} \Lambda^{(e)} - \frac{\partial}{\partial u_i}\sum_{i=1}^{m} F_i u_i = 0$$

$$\frac{\partial}{\partial X}\left(\frac{1}{2}KX^2\right) - \frac{\partial}{\partial X}\left(35\left(\frac{3}{4}X\right)\right) - (6)\left(\frac{5}{8}X\right) = 0$$

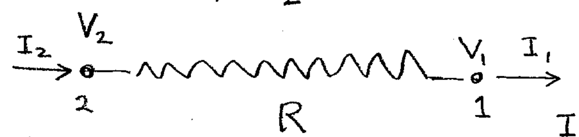
$$KX - (35)\left(\frac{3}{4}\right) - (6)\left(\frac{5}{8}\right) = 0$$

$$X = \frac{(35)\left(\frac{3}{4}\right) + (6)\left(\frac{5}{8}\right)}{60}$$

$$\underline{\underline{X = 0.5\ \text{in}}}$$

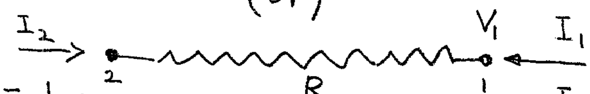
1.15

$$I_1 = I_2$$



$$I_2 = \frac{1}{R}(V_2 - V_1) \qquad I_1 = \frac{1}{R}(V_2 - V_1)$$

(or)



$$I_2 = \frac{1}{R}(V_2 - V_1) \qquad I_1 = \frac{1}{R}(V_1 - V_2)$$

Because of the fact that charge is conserved in a circuit (Kirchhoff's current law), at any time, the algebraic sum of the currents entering any node must be zero. Thus we can write

$$I_1 = \frac{1}{R}(V_1 - V_2)$$

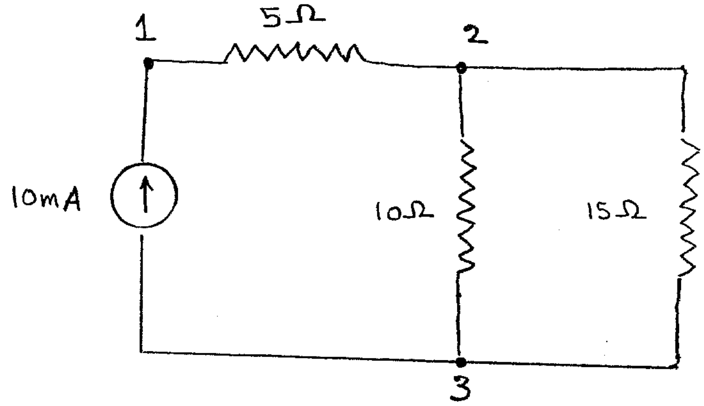
$$I_2 = \frac{1}{R}(V_2 - V_1)$$

Note $\quad I_1 + I_2 = 0$

In matrix form:

$$\frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

$\boxed{1.16}$



$$[K]^{(1)} = \frac{1}{5}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(2)} = \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[K]^{(3)} = \frac{1}{15}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$; Apply $V_3 = 0$ as a boundary condition

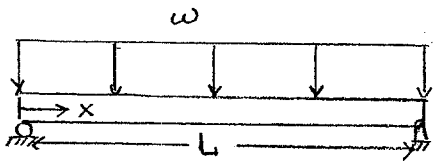
$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5}+\frac{1}{10}+\frac{1}{15} & -\frac{1}{10}-\frac{1}{15} \\ 0 & \cancel{-\frac{1}{10}-\frac{1}{15}}\ 0 & \cancel{\frac{1}{10}+\frac{1}{15}}\ 1 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 0.01 \\ 0 \\ 0 \end{Bmatrix}$$

$V_1 = 0.11$ $\qquad$ $V_2 = 0.06$

$\underline{V_1 - V_2 = 0.05 \text{ Volts}}$ ⇐

$\underline{\underline{V_2 - V_3 = 0.06 \text{ Volts}}}$ ⇐

1-17



$$\frac{d^2Y}{dx^2} = \frac{M(x)}{EI} = \frac{\omega x(L-x)}{2EI}$$

$$\frac{dY}{dx} = \frac{1}{2EI}\left(\frac{\omega x^2 L}{2} - \frac{\omega x^3}{3}\right) + C_1$$

$$Y = \frac{1}{2EI}\left(\frac{\omega x^3 L}{6} - \frac{\omega x^4}{12}\right) + C_1 x + C_2$$

Applying Boundary Conditions:

$Y = 0$ @ $x = 0$ $\qquad C_2 = 0$

$Y = 0$ @ $x = L$ $\qquad C_1 = -\dfrac{\omega L^3}{24EI}$

$$Y_{exact} = -\frac{\omega x}{24EI}\left(x^3 - 2Lx^2 + L^3\right)$$

(a) Note that the assumed Solution satisfies the boundary Conditions. $Y = C_1\left[\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right]$

$$\frac{dY}{dx} = C_1\left[\frac{2x}{L^2} - \frac{1}{L}\right]$$

$$\frac{d^2Y}{dx^2} = C_1\left(\frac{2}{L^2}\right)$$

$$\frac{2C_1}{L^2} - \frac{\omega x(L-x)}{2EI} = R$$

We may force the error function to equal zero at $x = L/2$

$$\frac{2C_1}{L^2} - \frac{\omega \frac{L}{2}\left(L - \frac{L}{2}\right)}{2EI} = 0 \rightarrow C_1 = \frac{\omega L^4}{16EI}$$

$$Y = \frac{\omega L^4}{16EI}\left[\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right]$$

1.17 Cont.

(b) $$\int_0^L R\,dx = 0$$

$$\int_0^L \left[\frac{2C_1}{L^2} - \frac{\omega x(L-x)}{2EI}\right] dx = 0$$

$$\frac{2C_1}{L^2}L - \frac{\omega}{2EI}\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = 0 \longrightarrow C_1 = \frac{\omega L^4}{24EI}$$

$$Y = \frac{\omega L^4}{24EI}\left(\left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)\right)$$

For W24x104 Beam $I = 3100 \text{ in}^4$; $E = 29\times10^6 \frac{\text{lb}}{\text{in}^2}$

Exact

$$Y_{max} = \frac{-5\omega L^4}{384EI} = \frac{-5\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ ft}}{12\text{ in}}\right)\left(20\text{ ft}\times\frac{12\text{ in}}{\text{ft}}\right)^4}{(384)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\text{ in}^4)} = -0.20 \text{ in}$$

Collocation

$$Y_{max} = \frac{-\omega L^4}{64EI} = \frac{-\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ ft}}{12\text{ in}}\right)\left(20\text{ ft}\times\frac{12\text{ in}}{\text{ft}}\right)^4}{(64)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\text{ in}^4)} = -0.24 \text{ in}$$

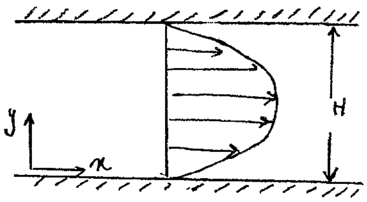
Subdomain

$$Y_{max} = \frac{-\omega L^4}{96EI} = \frac{-\left(5000\frac{\text{lb}}{\text{ft}}\times\frac{1\text{ ft}}{12\text{ in}}\right)\left(20\text{ ft}\times\frac{12\text{ in}}{\text{ft}}\right)^4}{(96)\left(29\times10^6\frac{\text{lb}}{\text{in}^2}\right)(3100\text{ in}^4)} = -0.16 \text{ in}$$

1.18

See Section 1.7

1.19



$$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx}\,y + c_1$$

$$u(y) = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + c_1 y + C_2$$

applying B.Cs $u(0)=0$ and $u(H)=0$

$$0 = 0 + 0 + C_2 \qquad \text{and} \qquad 0 = \frac{1}{\mu}\frac{dP}{dx}\frac{H^2}{2} + c_1 H$$

$$C_2 = 0 \qquad \text{and} \qquad c_1 = -\frac{H}{2\mu}\frac{dP}{dx}$$

$$\underline{\underline{u(y) = -\frac{1}{2\mu}\frac{dP}{dx}(Hy - y^2)}}$$

note because Pressure drops in the direction of flow, $\frac{dP}{dx} < 0$ in velocity equation.

(a) $$\mu \frac{d^2u}{dy^2} = \frac{dP}{dx} \qquad u_{assumed} = c_1 \sin\left(\frac{\pi y}{H}\right)$$

note the assumed solution Satisfies the boundary Conditions.

$$\frac{du}{dy} = \frac{d}{dy}\left(c_1 \sin\left(\frac{\pi y}{H}\right)\right) = c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)$$

$$\frac{d^2u}{dy^2} = \frac{d}{dy}\left(c_1 \frac{\pi}{H}\cos\left(\frac{\pi y}{H}\right)\right) = -c_1\left(\frac{\pi}{H}\right)^2 \sin\left(\frac{\pi y}{H}\right)$$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2\sin\left(\frac{\pi y}{H}\right)\right] - \frac{dP}{dx} = \mathcal{R}$$

We may force the error function to equal zero at $y = \frac{H}{2}$

$$\mu\left[-c_1\left(\frac{\pi}{H}\right)^2\sin\left(\frac{\pi}{H}\frac{H}{2}\right)\right] - \frac{dP}{dx} = 0$$

$$c_1 = -\frac{H^2}{\mu\pi^2}\frac{dP}{dx} \qquad \text{then,} \quad \underline{\underline{u(y) = -\frac{H^2}{\mu\pi^2}\frac{dP}{dx}\left(\sin\left(\frac{\pi y}{H}\right)\right)}} \Leftarrow$$