

# Chapter 1

## Introduction: Basic Principles

**1.1.(a)** Air flows adiabatically through a long straight circular section duct, 0.25 m diameter at a measured mass flow rate of 40 kg/s. At a particular section along the duct the values of static temperature  $T = 150^{\circ}\text{C}$  and static pressure  $p = 550\text{ kPa}$ . Determine the average velocity of the airflow and its stagnation temperature.

(b) At another station further along the duct, measurements reveal that the static temperature has dropped to  $147^{\circ}\text{C}$  as a consequence of wall friction. Determine the average velocity and the static pressure of the airflow at this station.

Also determine the change in entropy per unit of mass flow between the two stations.

For air assume that  $R = 287\text{J}/(\text{kgK})$  and  $\gamma = 1.4$

### Solution

(a) From the gas law

$$\rho_1 = \frac{p_1}{RT_1} = \frac{550 \times 10^3}{287 \times 423} = 4.53 \text{ kg/m}^3$$

From the conservation of mass law

$$c_1 = \frac{\dot{m}}{\rho A} = \frac{40}{4.53 \times 0.04908} = 180 \text{ m/s Ans}$$

$$\text{where } A = \left(\frac{\pi}{4}\right) d^2 = 0.04908 \text{ m}^2$$

From the conservation of energy equation the stagnation temperature is

$$T_{01} = T_1 + \frac{c_1^2}{2C_p} = 423 + \frac{179.9^2}{2 \times 1004.5} = 439.1 \text{ K Ans}$$

(b)  $T_2 = 147 + 273 = 420 \text{ K}$

As the flow is adiabatic, the energy of the air is conserved along the pipe, and  $h_{02} = h_{01}$

$$\therefore C_p T_2 + \frac{c_2^2}{2} = C_p T_1 + \frac{c_1^2}{2} \quad \therefore c_2^2 = c_1^2 + 2C_p (-3) = 26373$$

$$\therefore c_2 = 162.4 \text{ m/s} \quad \mathbf{ANS}$$

From the continuity equation

$\dot{m} = \rho_1 c_1 A = \rho_2 c_2 A$  and the gas law  $\rho = p / (RT)$  we arrive at the following useful expression:

$$\frac{p_2}{p_1} = \frac{c_1 T_2}{c_2 T_1} = \frac{180 \times 420}{185.5 \times 423} = 0.9635$$

$$\therefore p_2 = 530 \text{ kPa}$$

(c) The change in entropy between the two stations is:

$$\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = R \left[ \frac{\gamma}{\gamma-1} \ln \frac{T_2}{T_1} - \ln \frac{p_2}{p_1} \right]$$

$$= 3.81 \text{ J/(kg K)} \quad \mathbf{Ans}$$

**N.B.** In this type of flow (called a FANNO FLOW) the entropy will always increase.

**1.2.** Nitrogen gas at a stagnation temperature of 300 K and a static pressure of 2 bar flows through a pipe duct of 0.3 m diameter. At a particular station along the duct length the Mach number is 0.6. Assuming the flow is frictionless, determine

- (1) the static temperature and stagnation pressure of the flow,
- (2) the mass flow of gas if the duct diameter is 0.3 m.

For nitrogen gas take

$$R = 297 \text{ J/(kgK)} \text{ and } \gamma = 1.4$$

### Solution

(1) From eqn. (1.35)

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = 1.2755 \therefore p_0 = 2.551 \text{ bar Ans.}$$

and 
$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 = 1.072 \therefore T = \frac{T_0}{1.072} = 279.9 \text{ K Ans.}$$

(2) From eqn. (1.38)

$$\frac{\dot{m} \sqrt{C_p T_0}}{A_n p_0} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2} \left(\frac{\gamma+1}{\gamma-1}\right)}$$

and we have  $A_n = \frac{\pi}{4} d^2 = 0.07069 \text{ m}^2$  and  $C_p = \frac{\gamma R}{\gamma-1} = 1040 \text{ J/(kg K)}$

Substituting values (and part evaluating) we have, for the LHS of the above equation

$$\dot{m} \sqrt{1040 \times 300} \div (0.07069 \times 2.551 \times 10^5) = 0.03097 \dot{m}$$

and for the RHS of the equation

$$\frac{\gamma M}{\sqrt{\gamma-1}} \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{2} \left(\frac{\gamma+1}{\gamma-1}\right)} = \frac{1.4 \times 0.6}{\sqrt{0.4}} (1.072)^{-3} = 1.0782$$

$$\therefore \dot{m} = 27.16 \text{ kg/s ANS.}$$

1.3. Air flows adiabatically through a horizontal duct and at a section numbered (1) the static pressure  $p_1 = 150$  kPa, the static temperature  $T_1 = 200$  °C and the velocity  $c_1 = 100$  m/s. At a station further downstream the static pressure  $p_2 = 50$  kPa and the static temperature  $T_2 = 150$  °C. Determine the velocity  $c_2$  and the change in entropy per unit mass of air.

For air take  $\gamma = 1.4$  and  $R = 287$  J/(kgK)

### Solution

From eqn. (1.12)

$$h_{01} = h_{02} \quad \therefore h_1 + \frac{1}{2}c_1^2 = h_2 + \frac{1}{2}c_2^2$$

$$\therefore c_2^2 = 2C_p(T_1 - T_2) + c_1^2 \quad \text{where } C_p = \frac{\gamma R}{\gamma - 1} = 3.5 \times 287 = 1004.5 \text{ J/kgK}$$

$$\therefore c_2^2 = 2 \times 1004.5(200 - 150) + 100^2 = 100450$$

$$c_2 = 316.9 \text{ m/s} \quad \text{Ans}$$

From eqn.(1.28a)

$$\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = 1.0045 \ln \frac{150}{200} - 0.287 \ln \left( \frac{50}{150} \right)$$

$$\therefore \Delta s = -0.289 + 0.315 = 0.0263 \text{ kJ/kg} \quad \text{ANS}$$

1.4. For the adiabatic expansion of a perfect gas through a turbine, show that the overall efficiency  $\eta_t$  and small stage efficiency  $\eta_p$  are related by

$$\eta_t = (1 - \varepsilon^{\gamma_p}) / (1 - \varepsilon)$$

where  $\varepsilon = r^{(1-\gamma)/\gamma}$ , and  $r$  is the expansion pressure ratio,  $\gamma$  is the ratio of specific heats.

An axial flow turbine has a small stage efficiency of 86%, an overall pressure ratio of 4.5 to 1 and a mean value of  $\gamma$  equal to 1.333. Calculate the overall turbine efficiency.

Solution. The overall efficiency of a turbine is assumed to mean the total to total efficiency defined, eqn. (2.21), by

$$\eta_t = (h_{01} - h_{02}) / (h_{01} - h_{02s})$$

For a perfect gas,  $h = C_p T$ , so that

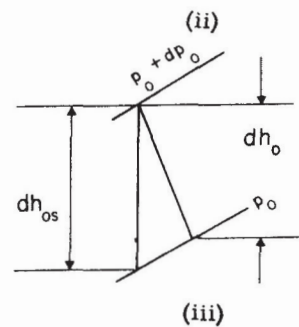
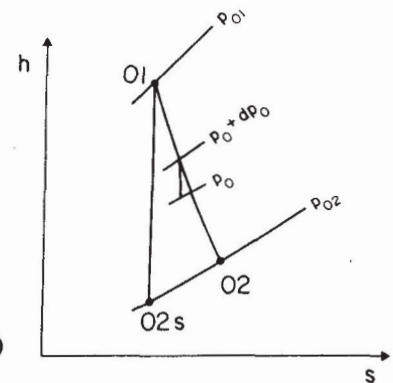
$$\begin{aligned} \eta_t &= (T_{01} - T_{02}) / (T_{01} - T_{02s}) \\ &= (1 - T_{02}/T_{01}) / (1 - T_{02s}/T_{01}) \end{aligned} \quad \dots (i)$$

The overall total pressure ratio is

$$\begin{aligned} r &= p_{01}/p_{02} = (T_{01}/T_{02s})^{\gamma/(\gamma-1)} \\ \therefore \varepsilon &= T_{02s}/T_{01} = r^{(1-\gamma)/\gamma} \end{aligned}$$

Consider a small part of the expansion process as shown in the sketch. This expansion is best imagined as a small stage with an enthalpy drop  $dh_o$  and corresponding pressure drop  $dp_o$ . The small stage efficiency is defined as

$$\eta_p = dh_o / dh_{os}$$



Now an elementary change in specific entropy can be related, using the laws of thermodynamics, to the elementary changes in other properties. From eqn. (1.27)

$$Tds = dh - dp/\rho$$

which is applicable to both reversible and irreversible processes on a pure substance.

For a constant entropy process it follows that

$$dh_{os} = dp_o/\rho_o$$

Substituting this result and the perfect gas relations,  $p_o/\rho_o = RT_o$  and  $dh_o = C_p dT_o$  into eqn. (iii)

$$\eta_p = \rho_o C_p dT_o/dp_o = (p_o/RT_o) \gamma R/(\gamma-1) dT_o/dp_o$$

where  $C_p = \gamma R/(\gamma-1)$ . After rearranging the above equation

$$dT_o/T_o = [\eta_p(\gamma-1)/\gamma] dp_o/p_o$$

Integrating and putting in the limits for the overall process,

$$\begin{aligned} T_{o2}/T_{o1} &= (p_{o2}/p_{o1})^{\eta_p(\gamma-1)/\gamma} \\ &= r^{\eta_p(1-\gamma)/\gamma} = \epsilon^{\eta_p} \end{aligned} \quad (iv)$$

Substituting eqns. (ii) and (iv) into eqn. (i), the required relation is obtained,

$$\eta_t = (1 - \epsilon^{\eta_p})/(1 - \epsilon)$$

With  $r = 4.5$ ,  $\gamma = 1.333$  and  $\eta_p = 0.86$ ,

$$\begin{aligned} \epsilon &= r^{(1-\gamma)/\gamma} = 1/4.5^{0.2498} = 1/1.456 = 0.6868. \\ \epsilon^{\eta_p} &= 0.6868^{0.86} = 0.7239 \\ \therefore \eta_t &= (1 - 0.7239)/(1 - 0.6868) \\ &= \underline{88.16 \text{ per cent}} \end{aligned}$$

1.5 Air is expanded in a multi-stage axial flow turbine, the pressure drop across each stage being very small. Assuming that air behaves as a perfect gas with ratio of specific heats  $\gamma$ , derive pressure-temperature relationships for the following processes:

- (i) reversible adiabatic expansion;
- (ii) irreversible adiabatic expansion, with small stage efficiency  $\eta_p$ ;

(iii) reversible expansion in which the heat loss in each stage is a constant fraction  $k$  of the enthalpy drop in that stage;

(iv) reversible expansion in which the heat loss is proportional to the absolute temperature  $T$ .

Sketch the first three processes on a  $T, s$  diagram.

If the entry temperature is 1100 K, and the pressure ratio across the turbine is 6 to 1, calculate the exhaust temperatures in each of these three cases. Assume that  $\gamma$  is 1.333, that  $\eta_p = 0.85$ , and that  $k = 0.1$ .

Solution. (i) For a reversible adiabatic expansion the entropy does not change.

From eqn. (1.28), with  $ds = 0$ ,

$$Tds = dh - (1/\rho)dp = 0$$

$$\therefore dh = C_p dT = (1/\rho)dp = RT dp/p$$

$$\therefore dT/T = (R/C_p)dp/p = [(\gamma-1)/\gamma] dp/p$$

Integrating this result between limits, denoted by an initial state 1 and a final state 2, yields

$$\ln(T_1/T_2) = [(\gamma-1)/\gamma] \ln(p_1/p_2)$$

$$\therefore T_1/T_2 = (p_1/p_2)^{(\gamma-1)/\gamma} \quad (i)$$

(ii) It has already been shown in the solution of Q. 1.1 for an irreversible adiabatic expansion with small stage efficiency  $\eta_p$ , that

$$T_1/T_2 = (p_1/p_2)^{\eta_p(\gamma-1)/\gamma} \quad (ii)$$

It is a consequence of the second law of thermodynamics that the entropy of a substance (i.e. a system) undergoing an irreversible adiabatic process must increase. The magnitude of the entropy increase can be formulated from eqn. (1.28) as follows:-

$$Tds = dh - dp/\rho = C_p dT - RT dp/p$$

$$\therefore ds = C_p dT/T - R dp/p$$

Integrating and inserting limits, the entropy increase is

$$\begin{aligned} s_2 - s_1 &= C_p \ln(T_2/T_1) - R \ln(p_2/p_1) \\ &= R \left\{ \ln(p_1/p_2) - \left[ \gamma/(\gamma-1) \right] \ln(T_1/T_2) \right\} \end{aligned}$$

Substituting for  $T_1/T_2$  from eqn. (ii), and simplifying

$$s_2 - s_1 = R(1 - \eta_p) \ln(p_1/p_2) \quad (\text{ia})$$

(iii) From the second law of thermodynamics, when an element of heat  $dQ_R$  is transferred reversibly from the surroundings to a unit mass of a substance at an absolute temperature  $T$ , the specific entropy increases by an amount

$$ds = dQ_R/T$$

Thus, a reversible heat transfer from the substance to the surroundings ( $dQ_R < 0$ ) will cause the specific entropy to decrease. In the reversible expansion through the turbine with reversible heat loss the signs of the three elements  $ds$ ,  $dh$  and  $dp$  in the expression  $dQ_R = Tds = dh - dp/\rho$  are all negative. Writing  $dQ_R = kdh$ , eqn. (1.28) gives

$$\begin{aligned} Tds &= kdh = dh - dp/\rho \\ \therefore (1-k)C_p dT &= dp/\rho = RT dp/p \\ \therefore dT/T &= (dp/p)(\gamma-1)/[\gamma(1-k)] \end{aligned}$$

Integrating and inserting limits as before,

$$\therefore T_1/T_2 = (p_1/p_2)^{(\gamma-1)/[\gamma(1-k)]} \quad (\text{iii})$$

Additionally, it is easy to determine the magnitude of the corresponding specific entropy change, as follows:-

$$\begin{aligned} ds &= kC_p dT/T \\ \therefore s_2 - s_1 &= -kC_p \ln(T_1/T_2) = -kC_p(\gamma-1)/[\gamma(1-k)] \ln(p_1/p_2) \\ &= -\ln(p_1/p_2) kR/(1-k) \quad (\text{iiia}) \end{aligned}$$

(iv) The heat loss in each elementary stage is reversible and proportional to  $T$ . This condition is satisfied by

$$dQ_R = Tds = dh - dp/\rho$$

$$\therefore ds = C_p \frac{dT}{T} - R \frac{dp}{p}$$

$$\therefore (s_2 - s_1)/R = \ln(p_1/p_2) - [\gamma/(\gamma-1)] \ln(T_1/T_2)$$

After rearranging and exponentiating,

$$p_1/p_2 = (T_1/T_2)^{\gamma/(\gamma-1)} \cdot \exp[(s_2 - s_1)/R] \quad (\text{iv})$$

The sketch shows the way the entropy changes as the air is expanded from the initial state 1 to the final state 2 corresponding to the first three processes. The final temperatures (and specific entropy changes) are easily determined from the preceding equations.

With  $T_1 = 1100$  K,  $p_1/p_2 = 6$ ,  $\eta_p = 0.85$ ,  
 $\gamma = 1.333$  and  $k = 0.1$ ,

From eqn. (i),

$$T_{2i} = 1100/6^{0.2498} = \underline{703.1 \text{ K}}$$

From eqn. (ii)

$$T_{2ii} = 1100/6^{0.2123} = \underline{751.9 \text{ K}}$$

and the corresponding entropy increase is, from eqn. (iia),

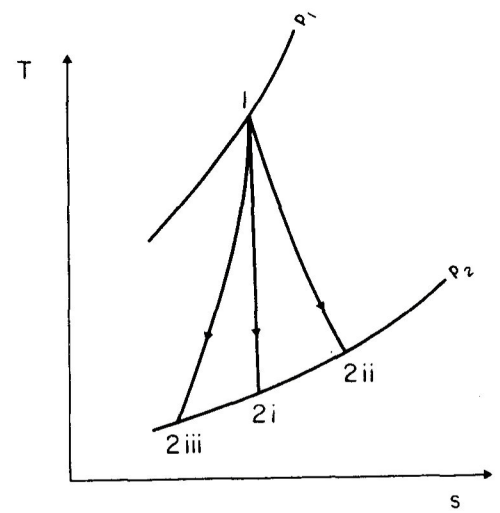
$$(s_{2ii} - s_1)/R = 0.15 \ln 6 = 0.2688$$

From eqn. (iii)

$$T_{2iii} = 1100/6^{0.2776} = \underline{669.0 \text{ K}}$$

and the corresponding entropy change is from eqn. (iiia)

$$(s_{2iii} - s_1)/R = -(0.1/0.9) \ln 6 = -0.2$$



1.7 A multi-stage high-pressure steam turbine is supplied with steam at a stagnation pressure of 7 MPa and a stagnation temperature of  $500^{\circ}\text{C}$ . The corresponding specific enthalpy is 3410 kJ/kg. The steam exhausts from the turbine at a stagnation pressure of 0.7 MPa abs., the steam having been in a super-

heated condition throughout the expansion. It can be assumed that the steam behaves like a perfect gas over the range of the expansion and that  $\gamma = 1.3$ . Given that the turbine flow process has a small-stage efficiency of 0.82, determine

- (i) the temperature and specific volume at the end of the expansion;
- (ii) the reheat factor.

The specific volume of superheated steam is represented by  $pv = 0.231(h - 1943)$ , where  $p$  is in kPa,  $v$  is in  $\text{m}^3/\text{kg}$  and  $h$  is in kJ/kg.

Solution. (i) In the notation of Q. 1.1 the actual temperature ratio across the turbine, from eqn. (1.54), is

$$T_{o1}/T_{o2} = (p_{o1}/p_{o2})^{\eta_p(\gamma-1)/\gamma}$$

where  $\eta_p(\gamma-1)/\gamma = 0.82 \times 0.3/1.3 = 0.1892$  and  $p_{o1}/p_{o2} = 10$

$$\therefore T_{o1}/T_{o2} = 10^{0.1892} = 1.5461$$

The inlet stagnation temperature  $T_{o1} = 500 + 273 = 773$  K, hence the outlet stagnation temperature is

$$T_{o2} = 773/1.5461 = \underline{500 \text{ K}}$$

The specific volume  $v_{o2}$  corresponding to stagnation conditions at outlet is determined with the superheated steam relation  $pv = 0.231(h - 1943)$  and the perfect gas law  $pv = RT$ . Combining these two equations,

$$T_{o2}/T_{o1} = (h_{o2} - 1943)/(h_{o1} - 1943)$$

$$\therefore h_{o2} - 1943 = (h_{o1} - 1943)T_{o2}/T_{o1} = (3410 - 1943)500/773$$

$$= 948.9$$

$$\therefore h_{o2} = 2891.9 \text{ kJ/kg}$$

$$\therefore v_{o2} = 0.231 \times 948.9/700$$

$$= \underline{0.3131 \text{ m}^3/\text{kg}}$$

(ii) The reheat factor is defined, eqn. (1.56), as

$$R_H = \eta_t/\eta_p$$