Problem 1.1

The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

Solution 1.1

$$F = C_D \rho V^2 \frac{A}{2}$$

or

$$C_D = \frac{2F}{\rho V^2 A}, \text{ where}$$

$$F \doteq MLT^{-2}, \rho \doteq ML^{-3}, V \doteq LT^{-1}, A \doteq L^2$$

Thus,

$$C_D \doteq \frac{\left(MLT^{-2}\right)}{\left[\left(ML^{-2}\right)\left(LT^{-1}\right)^2\left(L^2\right)\right]} = M^0 L^0 T^0$$

Hence, C_D <u>is dimensionless</u>.

Problem 1.2

The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\mathrm{Ma} = \frac{V}{\sqrt{kRT}},$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

Solution 1.2

We denote the dimension of temperature by θ and use Newton's second law to get $F = \dfrac{ML}{T^2}$.

Then

$$[M] = \frac{\left(\dfrac{L}{T}\right)}{\sqrt{(1)\left(\dfrac{FL}{M\theta}\right)\theta\left(\dfrac{ML}{T^2F}\right)}} = \frac{\left(\dfrac{L}{T}\right)}{\sqrt{\dfrac{L^2}{T^2}}}$$

or

$$[M] = [1].$$

Problem 1.3

Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

Solution 1.3

a) $\text{volume} \doteq \underline{\underline{L^3}}$

b) $\text{acceleration} = \text{time rate of change of velocity} \doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

c) $\text{mass} \doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

$\text{mass} \doteq \underline{\underline{FL^{-1}T^2}}$

d) $\text{moment of inertia}\left(\text{area}\right) = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2}$

$\text{work} \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.4

Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

Solution 1.4

a) $\text{angular velocity} = \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) energy $\sim$ capacity of body to do work

$\text{single work} = \text{force} \times \text{distance} \rightarrow \text{energy} \doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2} \rightarrow \text{energy} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

c) $\text{moment of inertia } (\text{area}) = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

d) $\text{power} = \text{rate of doing work} \doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}} \doteq \left(MLT^{-2}\right)(L)\left(T^{-1}\right) \doteq \underline{\underline{ML^2T^{-3}}}$

e) $\text{pressure} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}} \doteq \left(MLT^{-2}\right)\left(L^{-2}\right) \doteq \underline{\underline{ML^{-1}T^{-2}}}$

Problem 1.5

Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

Solution 1.5

a) $\text{frequency}=\dfrac{\text{cycles}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) $\text{stress}=\dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$$\text{stress} \doteq \frac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

c) $\text{strain}=\dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \underline{\underline{L^0}}\left(\text{dimensionless}\right)$

d) $\text{torque}=\text{force}\times\text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)\left(L\right) \doteq \underline{\underline{ML^2T^{-2}}}$

e) $\text{work}=\text{force}\times\text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)\left(L\right) \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.6

If u is a velocity, x a length, and t a time, what are the dimensions (in the MLT system) of **(a)** $\partial u / \partial t$, **(b)** $\partial^2 u / \partial x \partial t$, and **(c)** $\int (\partial u / \partial t) dx$?

Solution 1.6

a) $\dfrac{\partial u}{\partial t} \doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

b) $\dfrac{\partial^2 u}{\partial x \partial t} \doteq \dfrac{LT^{-1}}{(L)(T)} \doteq \underline{\underline{T^{-2}}}$

c) $\int \dfrac{\partial u}{\partial t} \partial x \doteq \dfrac{\left(LT^{-1}\right)}{T}(\mathrm{L}) \doteq \underline{\underline{L^2 T^{-2}}}$

Problem 1.7

Verify the dimensions, in both the FLT system and the MLT system, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

Solution 1.7

a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$

c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

d) $\text{volume} = (\text{length})^3 \doteq \underline{\underline{L^3}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.8

If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ , **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

Solution 1.8

a) $$\frac{p}{\rho} \doteq \frac{FL^{-2}}{ML^{-3}} = \frac{MLT^{-2}L^{-2}}{ML^{-3}} = \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$$

b) $$pV\rho \doteq \left(ML^{-1}T^{-2}\right)\left(LT^{-1}\right)\left(ML^{-3}\right) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$$

c) $$\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{\left(ML^{-3}\right)\left(LT^{-1}\right)^2} \doteq M^0L^0T^0 \left(\underline{\underline{\text{dimensionless}}}\right)$$

Problem 1.9

If P is a force and x a length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

Solution 1.9

a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \underline{\underline{FL^{-2}}}$

b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

c) $\int P dx \doteq \underline{\underline{FL}}$

Problem 1.10

If V is a velocity, ℓ a length, and ν a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, **(d)** $V/\ell\nu$?

Solution 1.10

a) $V\ell\nu \doteq \left(LT^{-1}\right)(L)\left(L^2T^{-1}\right) \doteq L^4T^{-2} \left(\underline{\underline{\text{not dimensionless}}}\right)$

b) $\dfrac{V\ell}{\nu} \doteq \dfrac{\left(LT^{-1}\right)(L)}{\left(L^2T^{-1}\right)} \doteq L^0T^0 \left(\underline{\underline{\text{dimensionless}}}\right)$

c) $V^2\nu \doteq \left(LT^{-1}\right)^2\left(L^2T^{-1}\right) \doteq L^4T^{-3} \left(\underline{\underline{\text{not dimensionless}}}\right)$

d) $\dfrac{V}{\ell\nu} \doteq \dfrac{\left(LT^{-1}\right)}{(L)\left(L^2T^{-1}\right)} \doteq L^{-2} \left(\underline{\underline{\text{not dimensionless}}}\right)$

Problem 1.11

The momentum flux is given by the product $\dot{m}V$, where $\dot{m}$ is mass flow rate and V is velocity. If mass flow rate is given in units of mass per unit time, show that the momentum flux can be expressed in units of force.

Solution 1.11

$$[\dot{m}V] = \left(\frac{M}{T}\right)\left(\frac{L}{T}\right) = M\frac{L}{T^2} = \left(\frac{FT^2}{L}\right)\frac{L}{T^2} = \underline{\underline{F}}$$

Problem 1.12

An equation for the frictional pressure loss Δp (inches H_2O) in a circular duct of inside diameter $d\left(\text{in.}\right)$ and length $L\left(\text{ft}\right)$ for air flowing with velocity $V\left(\text{ft/min}\right)$ is

$$\Delta p = 0.027\left(\frac{L}{d^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82},$$

where V_0 is a reference velocity equal to $1000\,\text{ft/min}$. Find the units of the "constant" 0.027.

Solution 1.12

Solving for the constant gives

$$0.027 = \frac{\Delta p_L}{\left(\frac{L}{D^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82}}.$$

The units give

$$[0.027] = \frac{\left(\text{in. H}_2\text{O}\right)}{\left(\frac{\text{ft}}{\text{in.}^{1.22}}\right)\left(\frac{\frac{\text{ft}}{\text{min}}}{\frac{\text{ft}}{\text{min}}}\right)^{1.82}}$$

$$\boxed{[0.027] = \frac{\text{in. H}_2\text{O}\cdot\text{in.}^{1.22}}{\text{ft}}}$$

Problem 1.13

The volume rate of flow, Q , through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $\left(FL^{-2}T\right)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi / 8$? Would you classify this equation as a general homogeneous equation? Explain.

Solution 1.13

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\frac{\left[L^4\right]\left[FL^2\right]}{\left[FL^{-2}T\right]\left[L\right]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

The constant is $\frac{\pi}{8}$ is dimensionless.

Yes. This is a general homogeneous equation
because it is valid in any consistent units system.

Problem 1.14

Show that each term in the following equation has units of lb/ft^3. Consider u a velocity, y a length, x a length, p a pressure, and μ an absolute viscosity.

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}.$$

Solution 1.14

$$\left[\frac{\partial p}{\partial x}\right] = \frac{\left[\dfrac{\text{lb}}{\text{ft}^2}\right]}{[\text{ft}]} \quad \text{or} \quad \boxed{\left[\frac{\partial p}{\partial x}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]},$$

and

$$\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb} \cdot \text{sec}}{\text{ft}^2}\right] \frac{\left[\dfrac{\text{ft}}{\text{sec}}\right]}{\left[\text{ft}^2\right]} \quad \text{or} \quad \boxed{\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]}.$$

Problem 1.15

The pressure difference, Δp , across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density $\left(ML^{-3}\right)$, D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_υ and K_u . Would this equation be valid in any system of units?

Solution 1.15

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1 \right]^2 \rho V^2$$

$$FL^{-2} \,\square\, [K_v] \frac{FT}{L^2} \frac{L}{T} \frac{1}{L} \quad + [K_u] \left(\frac{L^2}{L^2} - 1 \right)^2 \left(\frac{FT^2}{L} \frac{1}{L^3} \right) \left(\frac{L}{T} \right)^2$$

$$FL^{-2} \,\square\, [K_v]\left(FL^{-2}\right) \quad + [K_u]\left(FL^{-2}\right)$$

K_υ and K_u are dimensionless because each term in the equation must have the same dimensions,. Yes. The equation would be valid in any consistent system of units.

Problem 1.16

Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_υ, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_\upsilon)^a (\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See the equation $c = \sqrt{\frac{E_\upsilon}{\rho}}$.)

Solution 1.16

Substituting $[c] = LT^{-1}$ $[E_v] = FL^{-2}$ $[\rho] = FL^{-4}T^2$ into the equation provided yields:

$$\left[LT^{-1}\right] = \left[\left(FL^{-2}\right)^a\right]\left[\left(FL^{-4}T^2\right)^b\right] = F^{a+b}L^{-2a-4b}T^{2b}$$

Dimensional homogeneity requires that the exponent of each dimension on both sides of the equal sign be the same.

F: 0 =a+b

L: 1 =-2a-4b

T: -1 =2b

Therefore:

T: -1 =2b → b= -1/2

F: a =-b → a= 1/2

L: 1 =-2a-4b = -2(1/2) -4(-1/2)= 1✓

$$a = \frac{1}{2}; \quad b = -\frac{1}{2}$$

Yes, this is consistent with the standard formula for the speed of sound.

Problem 1.17

A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09\,BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in ft^3/s when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

Solution 1.17

$$Q = 3.09\,BH^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L][L]^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L]^{\frac{5}{2}}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{\frac{1}{2}}T^{-1}$ and is therefore not dimensionless. No.

Since the constant has dimensions its value will change with a change in units. No.

Problem 1.18

A commercial advertisement shows a pearl falling in a bottle of shampoo. If the diameter D of the pearl is quite small and the shampoo sufficiently viscous, the drag $\mathcal{D}$ on the pearl is given by Stokes's law,

$$\mathcal{D} = 3\pi\mu VD,$$

where V is the speed of the pearl and μ is the fluid viscosity. Show that the term on the right side of Stokes's law has units of force.

Solution 1.18

$$[\mathcal{D}] = [3\pi\mu VD] = \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)L = M\frac{L}{T^2} = \frac{FT^2}{L}\frac{L}{T^2} = \underline{\underline{F}}$$

Problem 1.20

Express the following quantities in SI units: **(a)** $10.2\ \text{in./min}$, **(b)** $4.81\ \text{slugs}$, **(c)** $3.02\ \text{lb}$, **(d)** $73.1\ \text{ft/s}^2$, **(e)** $0.0234\ \text{lb}\cdot\text{s/ft}^2$.

Solution 1.20

a) $$10.2\,\frac{\text{in.}}{\text{min}} = \left(10.2\,\frac{\text{in.}}{\text{min}}\right)\left(2.540\times 10^{-2}\,\frac{\text{m}}{\text{in.}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 4.32\times 10^{-3}\,\frac{\text{m}}{\text{s}} = \underline{\underline{4.32\,\frac{\text{mm}}{\text{s}}}}$$

b) $$4.81\,\text{slugs} = (4.81\,\text{slugs})\left(1.459\times 10\,\frac{\text{kg}}{\text{slug}}\right) = \underline{\underline{70.2\,\text{kg}}}$$

c) $$3.02\,\text{lb} = (3.02\,\text{lb})\left(4.448\,\frac{\text{N}}{\text{lb}}\right) = \underline{\underline{13.4\,\text{N}}}$$

d) $$73.1\,\frac{\text{ft}}{\text{s}^2} = \left(73.1\,\frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times 10^{-1}\,\frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right) = \underline{\underline{22.3\,\frac{\text{m}}{\text{s}^2}}}$$

e) $$0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \left(0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times 10\,\frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right) = \underline{\underline{1.12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

Problem 1.21

Express the following quantities in BG units: **(a)** $14.2\ \text{km}$, **(b)** $8.14\,\text{N/m}^3$, **(c)** $1.61\,\text{kg}/\text{m}^3$, **(d)** $0.0320\,\text{N}\cdot\text{m/s}$, **(e)** $5.67\,\text{mm/hr}$.

Solution 1.21

a) $$14.2\,\text{km} = \left(14.2\times10^3\ \text{m}\right)\left(3.281\frac{\text{ft}}{\text{m}}\right) = \underline{\underline{4.66\times10^4\ \text{ft}}}$$

b) $$8.14\frac{\text{N}}{\text{m}^3} = \left(8.14\frac{\text{N}}{\text{m}^3}\right)\left(6.366\times10^{-3}\ \frac{\frac{\text{lb}}{\text{ft}^3}}{\frac{\text{N}}{\text{m}^3}}\right) = \underline{\underline{5.18\times10^{-2}\ \frac{\text{lb}}{\text{ft}^3}}}$$

c) $$1.61\frac{\text{kg}}{\text{m}^3} = \left(1.61\frac{\text{kg}}{\text{m}^3}\right)\left(1.940\times10^{-3}\ \frac{\frac{\text{slugs}}{\text{ft}^3}}{\frac{\text{kg}}{\text{m}^3}}\right) = \underline{\underline{3.12\times10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}}$$

d) $$0.0320\,\frac{\text{N}\cdot\text{m}}{\text{s}} = \left(0.0320\,\frac{\text{N}\cdot\text{m}}{\text{s}}\right)\left(7.376\times10^{-1}\ \frac{\frac{\text{ft}\cdot\text{lb}}{\text{s}}}{\frac{\text{N}\cdot\text{m}}{\text{s}}}\right) = \underline{\underline{2.36\times10^{-2}\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}}$$

e) $$5.67\,\frac{\text{mm}}{\text{hr}} = \left(5.67\times10^{-3}\ \frac{\text{m}}{\text{hr}}\right)\left(3.281\frac{\text{ft}}{\text{m}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right) = \underline{\underline{5.17\times10^{-6}\ \frac{\text{ft}}{\text{s}}}}$$