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1.1 $C_p = C_v + R$ (given)

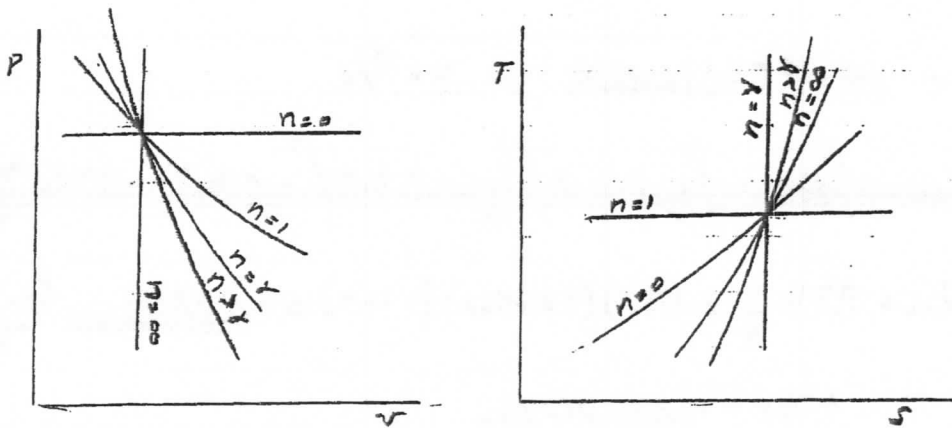
$3.42 \stackrel{?}{=} 2.43 + \frac{766}{778}$ for Hydrogen \rightarrow from table in Appendix

$3.42 \stackrel{?}{=} 2.43 + .985$

$3.42 \stackrel{?}{=} 3.415$ pretty close

1.2 perfect gas $\rightarrow C_p = 0.532 \text{ Btu/lbm} \cdot ^\circ\text{R}$, $C_v = 0.403 \text{ Btu/lbm} \cdot ^\circ\text{R}$
Undergoes a reversible polytropic process, $n = 1.4$

$\gamma = C_p/C_v = .532/.403 = 1.32$ thus process $n > \gamma$

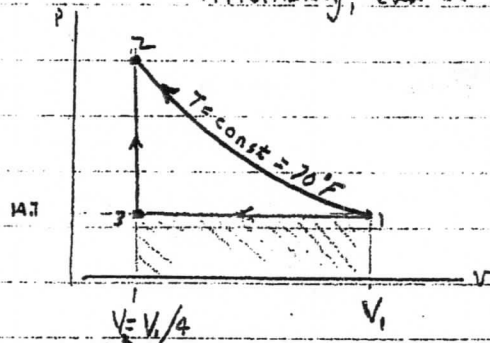


(a) YES, there will be heat transfer in this process

(b) The process would be nearest a vertical line (vice a horizontal line) in either the P-v diagram or the T-s diagram.

1.3 (A) Nitrogen compressed from $70^\circ\text{F} \pm 14.7\text{psia}$ by rev. $T=c$ to $1/4$ original volume.

Alternately, can be comp. to $1/4$ original volume by $p=c$ and then to same end point as above.



(a) since no work from $3 \rightarrow 2$
 W_{1-3} is less than W_{1-2}

(b) $\Delta U_{1-2} = C_v \Delta T_{1-2} = 0$ for either process
 from 1st Law $\rightarrow Q = W + \Delta U = W$

$W = \int p dv \rightarrow$ find W and we also know Q

for $1-2$ by $T=c \rightarrow pV = RT = C(\text{constant}) \rightarrow p = C/v$

$$W_{1-2} = \int p dv = \int \frac{C}{v} dv = C \int \frac{dv}{v} = C \ln v \Big|_1^2 = C \ln \frac{v_2}{v_1} = RT \ln \frac{v_2}{v_1}$$

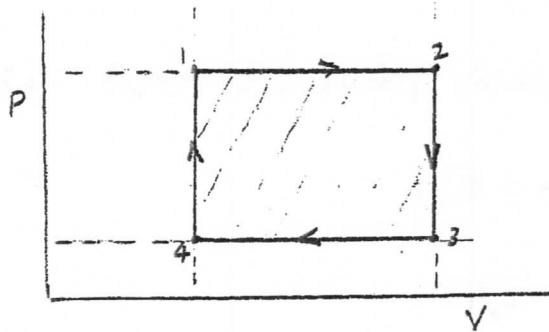
$$W_{1-2} = RT \ln \frac{1}{4} = (55.1)(70+460)(-\ln 4) = -40834 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}$$

$$\div 778 = -52.04 \text{ Btu/lb}_m$$

(- sign indicates work done on the gas)

from above $\rightarrow Q = W = -52.04 \text{ Btu/lb}_m$ (- sign indicates heat removed from gas)

1.4



$$P_1 = P_2 = 1.0 \text{ MPa} \quad M = 10^6$$

$$P_3 = P_4 = 0.4 \text{ MPa}$$

$$V_1 = V_4 = 0.6 \text{ m}^3$$

$$V_2 = V_3 = 1.0 \text{ m}^3$$

$$\oint dE = \oint dH = \oint dS = 0 \quad \text{cyclic integral of any property} = 0$$

$$\oint \delta W = \text{area enclosed by diagram (if processes are reversible)}$$

$$\oint \delta W = (V_3 - V_4)(P_2 - P_3) = (1.0 - 0.6)(1.0 - 0.4)(10^6)$$

$$= (0.4)(0.6)10^6 = 0.24 \times 10^6 \frac{\text{N} \cdot \text{m}^3}{\text{m}^3} = \underline{\underline{0.24 \times 10^6 \text{ N} \cdot \text{m}}}$$

(is positive because cycle is clockwise \rightarrow Net work Done By Medium)

1.5

METHANE \rightarrow Rev. polytropic process with $n=1.4 \rightarrow$ Perfect gas.

$$Q = W + \Delta U = \int p \, dv + C_v \Delta T$$

$$\text{from } p v^n = c, \quad p = c/v^n$$

$$Q_{1-2} = \int_1^2 \frac{c}{v^n} \, dv + C_v (T_2 - T_1) = \frac{c (v^{-n+1})}{-n+1} \Big|_1^2 + C_v (T_2 - T_1), \quad \text{but } c = p v^n$$

$$Q_{1-2} = \frac{p v}{1-n} \Big|_1^2 + C_v (T_2 - T_1) = \frac{p_2 v_2 - p_1 v_1}{(1-n)} + C_v (T_2 - T_1), \quad \text{but } p v = R T$$

$$Q_{1-2} = \frac{R}{1-n} (T_2 - T_1) + C_v (T_2 - T_1) = \left[\frac{R}{1-n} + C_v \right] (T_2 - T_1)$$

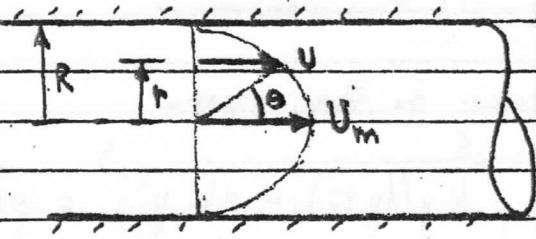
$$Q_{1-2} = \left[\frac{519}{1-1.4} + 1690 \right] (T_2 - T_1) = (-1297.5 + 1690) (T_2 - T_1)$$

$$Q_{1-2} = \underline{392.5} (T_2 - T_1) \quad \frac{N-m}{kg \cdot K} \quad \text{or} \quad \frac{\text{Joules}}{kg}$$

$$C_p - C_v = R, \quad C_p = C_v + R, \quad \Delta h = C_p \Delta T$$

No. Q is not equal to Δh or Δu for same ΔT

2.1

HEMISPHERICAL VELOCITY
DISTRIBUTION

$$U = U_m \cos \theta$$

$$\dot{m} = \int \rho u dA = \rho \int U_m \cos \theta (2\pi r dr)$$

$$\dot{m} = 2\pi \rho U_m \int_0^R \cos \theta r dr$$

$$r = R \sin \theta$$

$$dr = R \cos \theta d\theta$$

$$\dot{m} = 2\pi \rho U_m \int \cos \theta (R \sin \theta) (R \cos \theta d\theta)$$

$$= 2\pi R^2 \rho U_m \int_0^{\pi/2} \sin \theta \cos^2 \theta d\theta$$

$$= 2\pi R^2 \rho U_m \left[-\frac{1}{3} \cos^3 \theta \right]_0^{\pi/2} = -\frac{2\pi R^2 \rho U_m}{3} (0 - 1)$$

$$\dot{m} = \underline{\underline{\frac{2}{3} \pi R^2 \rho U_m}}$$

But $\dot{m} = \rho A V_{av} = \rho \pi R^2 V_{av}$

THUS $\rho \pi R^2 V_{av} = \frac{2}{3} \pi R^2 \rho U_m$

$$\underline{\underline{V_{av} = \frac{2}{3} U_m}}$$