1 Problems Chapter 1: Introduction to Nanoelectronics

2 Problems Chapter 2: Classical Particles, Classical Waves, and Quantum Particles

2.1. What is the energy (in Joules and eV) of a photon having wavelength 650 nm? Repeat for an electron having the same wavelength and only kinetic energy.

Solution: For the photon,

$$\lambda_p = \frac{hc}{E}, \quad E = \frac{hc}{\lambda} = \frac{hc}{650 \times 10^{-9}} = 3.058 \times 10^{-19} \text{ J} \tag{1}$$
$$E_{eV} = \frac{E_J}{|q_e|} = 1.91 \text{ eV}.$$

For the electron,

$$E = \frac{h^2}{2m_e\lambda_e^2} = \frac{h^2}{|q_e|\, 2m_e \left(650 \times 10^{-9}\right)^2} = 5.704 \times 10^{-25} \text{ J} \tag{2}$$
$$= 3.56 \times 10^{-6} \text{ eV}.$$

2.2. For light (photons), in classical physics the relation

$$c = \lambda f \tag{3}$$

is often used, where c is the speed of light, f is the frequency, and λ is the wavelength. For photons, is the de Broglie wavelength the same as the wavelength in (3)? Explain your reasoning. Hint: use Einstein's formula

$$E = mc^2 = \sqrt{p^2c^2 + m_0c^4}, \tag{4}$$

where m_0 is the particle's rest mass (which, for a photon, is zero).

Solution: Yes, these wavelengths are the same. From Einstein's formula, $E = pc$ for photons, and using $E = hf$ we have

$$c = \lambda f = \lambda \frac{E}{h} = \lambda \frac{pc}{h}, \tag{5}$$

so that we must have $\lambda = h/p$.

2.3. Common household electricity in the United States is 60 Hz, a typical microwave oven operates at 2.4×10^9 Hz, and ultraviolet light occurs at 30×10^{15} Hz. In each case, determine the energy of the associated photons in joules and eV.

Solution:

$$E = hf, \tag{6}$$
$$E_{elec} = h60 = 3.98 \times 10^{-32} \text{ J } = 2.48 \times 10^{-13} \text{ eV}$$
$$E_{oven} = h\left(2.4 \times 10^9\right) = 1.59 \times 10^{-24} \text{ J } = 9.92 \times 10^{-6} \text{ eV}$$
$$E_{uv} = h\left(30 \times 10^{15}\right) = 1.99 \times 10^{-17} \text{ J } = 124.2 \text{ eV}.$$

2.4. Assume that a HeNe laser pointer outputs 1 mW of power at 632 nm.

(a) Determine the energy per photon

Solution: Each photon carries

$$E_p = \hbar\omega = \hbar\frac{2\pi c}{\lambda} = \hbar\frac{(2\pi)\left(3 \times 10^8\right)}{632 \times 10^{-9}} = 3.145 \times 10^{-19} \text{ J} = 1.963 \text{ eV}. \tag{7}$$

(b) Determine the number of photons per second, N.

Solution: The sum of all N photons has power

$$P = NE_p \ (1/\text{s}) \ \text{J} = 10^{-3} \ \text{J/s} \tag{8}$$
$$\rightarrow N = \frac{10^{-3}}{3.145 \times 10^{-19}} = 3.1797 \times 10^{15} \ \text{photons/s.}$$

2.5. Repeat 2.4 if the laser outputs 10 mW of power. How does the number of photons per second scale with power?

Solution: The sum of all N photons has power

$$P = NE_p \ (1/\text{s}) \ \text{J} = 10 \times 10^{-3} \ \text{J/s} \tag{9}$$
$$\rightarrow N = \frac{10 \times 10^{-3}}{3.145 \times 10^{-19}} = 3.1797 \times 10^{16} \ \text{photons/s.}$$

The number of photons scales linearly with power.

2.6. Calculate the de Broglie wavelength of

(a) a proton moving at 437, 000 m/s,
(b) a proton with kinetic energy 1, 100 eV,
(c) an electron travelling at 10, 000 m/s.
(d) a 800 kg car moving at 60 km/h.

Solution: (a)

$$\lambda = \frac{h}{p} = \frac{h}{m_p v} = \frac{h}{(1.673 \times 10^{-27})(437000)} = 9.065 \times 10^{-13} \ \text{m} \tag{10}$$

(b)

$$E = \frac{1}{2} m_p v^2 = 1100 \times |q_e| \rightarrow v = \sqrt{\frac{(2)(1100 \times |q_e|)}{1.673 \times 10^{-27}}} = 4.59 \times 10^5 \ \text{m/s} \tag{11}$$
$$\lambda = \frac{h}{p} = \frac{h}{m_p v} = \frac{h}{(1.673 \times 10^{-27})(4.59 \times 10^5)} = 8.631 \times 10^{-13} \ \text{m}$$

(c)

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{(9.1095 \times 10^{-31})(10000)} = 7.274 \times 10^{-8} \ \text{m} \tag{12}$$

(d)

$$60 \ \frac{\text{km}}{\text{hour}} \times \frac{1 \ \text{m}}{10^{-3} \ \text{km}} \times \frac{1 \ \text{hour}}{60 \ \text{min}} \times \frac{1 \ \text{min}}{60 \ \text{s}} = \frac{60}{10^{-3}(60^2)} = 16.67 \ \text{m/s} \tag{13}$$
$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{(800)(16.67)} = 4.969 \times 10^{-38} \ \text{m}$$

2.7. Determine the wavelength of a 150 gram baseball traveling 90 miles/hour. Use this result to explain why baseballs do not seem to diffract around baseball bats.

Solution:

$$\frac{90 \ \text{miles}}{\text{hour}} \times \frac{1 \ \text{km}}{0.6214 \ \text{miles}} \times \frac{1 \ \text{m}}{10^{-3} \ \text{km}} \times \frac{1 \ \text{hour}}{60 \ \text{min}} \times \frac{1 \ \text{min}}{60 \ \text{s}} \tag{14}$$
$$= \frac{90}{0.6214(10^{-3})(60^2)} = 40.23 \ \text{m/s} \tag{15}$$
$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{(150 \times 10^{-3})(40.23)} = 1.098 \times 10^{-34} \ \text{m}$$

The de Broglie wavelength is too small to observe diffraction, since one would observe diffraction on size scales of the order of λ. The size scale of the bat is far to large.

2.8. How much would the mass of a ball need to be in order for it to have a de Broglie wavelength of 1 m (at which point its wave properties would be clearly observable)? Assume that the ball is travelling 90 miles/hour.

Solution:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\,(40.23)} = 1 \text{ m} \rightarrow m = \frac{h}{40.23} = 1.647 \times 10^{-35} \text{ kg} \tag{16}$$

2.9. Determine the momentum carried by a 640 nm photon. Since a photon is massless, does this momentum have the same meaning as the momentum carried by a particle with mass?

Solution:

$$\lambda = \frac{h}{p} = 640 \times 10^{-9} \rightarrow p = \frac{h}{640 \times 10^{-9}} = 1.035 \times 10^{-27} \text{ Js/m=kg m/s} \tag{17}$$

The momentum has essentially the same meaning as for a particle having mass: the photon momentum exerts a force on objects (in general, force multiplied by time equals momentum) that can be used to, for example, move objects.

2.10. Consider a 4 eV electron, a 4 eV proton, and a 4 eV photon. For each, compute the de Broglie wavelength, the frequency, and the momentum.

Solution: For the photon,

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{hc}{E} = \frac{hc}{4\,|q_e|} = 310.17 \text{ nm}, \\ E &= hf \rightarrow f = \frac{E}{h} = \frac{4\,|q_e|}{h} = 9.672 \text{ Hz}, \\ p &= \frac{E}{c} = \frac{4\,|q_e|}{c} = 2.136 \times 10^{-27} \text{ kg m/s}. \end{aligned} \tag{18}$$

For the electron,

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m_e E}} = \frac{h}{\sqrt{2m_e 4\,|q_e|}} = 0.613 \text{ nm}, \\ E &= hf \rightarrow f = \frac{E}{h} = \frac{4\,|q_e|}{h} = 9.672 \text{ Hz}, \\ p &= m_e v = (m_e)\left(1.186 \times 10^6\right) = 1.080 \times 10^{-24} \text{ kg m/s, since} \\ E &= \frac{1}{2} m_e v^2 = 4\,|q_e| \rightarrow v = \sqrt{\frac{(2)\,(4\,|q_e|)}{m_e}} = 1.186 \times 10^6 \text{ m/s} \end{aligned} \tag{19}$$

For the proton,

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m_p E}} = \frac{h}{\sqrt{2m_p 4\,|q_e|}} = 0.0143 \text{ nm}, \\ E &= hf \rightarrow f = \frac{E}{h} = \frac{4\,|q_e|}{h} = 9.672 \text{ Hz}, \\ p &= m_p v = m_p\,(27683) = 4.630 \times 10^{-23} \text{ kg m/s, since} \\ E &= \frac{1}{2} m v^2 = 4\,|q_e| \rightarrow v = \sqrt{\frac{(2)\,(4\,|q_e|)}{m_p}} = 27,683 \text{ m/s} \end{aligned} \tag{20}$$

Obviously, f is the same for all particles since $E = hf$. The momentum values are very small, but smallest for the photon. The wavelength is far larger for the photon than for the electron, which itself has a far larger wavelength than for the proton (the proton has far greater mass than the electron).

2.11. Determine the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 1.5 volts.