Chapter 1

1 (i) Binding energy of a proton and neutron in ^{139}La:

$$^{138}_{57}La + ^{1}_{0}n \rightarrow ^{139}_{57}La + B.E.\ (\text{neutron})$$

$$^{138}_{56}La + ^{1}_{1}H \rightarrow ^{139}_{57}La + BE\ (\text{proton})$$

$$BE\ (\text{neutron}) = \left[^{138}_{57}\Delta + \Delta n - ^{139}_{57}\Delta \right] = \left[-86524.681 + 8071.3171 - (-87231.371) \right]$$
$$= 8778.0071\ \text{keV}$$

$$BE\ (\text{proton}) = \left[^{138}_{56}\Delta + \Delta H - ^{139}_{57}\Delta \right] = \left[-88261.631 + 7288.9705 - (-87231.371) \right]$$
$$= 6257.7105\ \text{keV}$$

∴ The binding energy of the neutron = 8.778(0071) MeV
and the binding energy of the proton 6.257(7105)

} This difference is related to structure → odd # of protons and even # of neutrons (i.e., 82 magic #)

(ii) The mean binding energy per nucleon is:

For $^{139}_{57}La$: $\dfrac{\left[Z\Delta_H + (A-Z)\Delta m_n - ^{A}_{Z}\Delta \right]}{A}$ $\dfrac{\text{keV}}{\text{nucleon}}$

$$= \frac{\left[57 \times 7288.9705 + (139-57)\, 8071.3171 - (-87231.371) \right]}{139} = 8378.0625\ \frac{\text{keV}}{\text{nucleon}}$$

∴ The mean binding energy is 8.378063 $\frac{\text{MeV}}{\text{nucleon}}$ (same number as Appendix B)

This value is slightly less than that given for the neutron but greater than that for the proton since the neutron is paired in La-139 whereas the proton is the odd one.

(iii) $^{139}_{59}Pr$ and $^{139}_{58}Ce$ have β^+ and EC transition energies of 2.11 and 0.28 MeV, respectively.

i.e., $^{139}_{59}Pr \xrightarrow{\beta^+} ^{159}_{58}Ce \xrightarrow{EC} ^{139}_{57}La$

Now $E(\beta^+) = (\text{mass } ^{139}Pr - \text{mass } ^{139}Ce) \times 931.494 - 2 \times 0.511$ MeV and

$E(EC) = (\text{mass } ^{139}Ce - \text{mass } ^{139}La) \times 931.494$ MeV (using CODATA data).

∴ $2.11 = E(\beta^+) = (\text{mass } ^{139}Pr - \text{mass } ^{139}Ce) \times 931.494 - 2 \times 0.511$ MeV (1)

$0.28 = E(EC) = (\text{mass } ^{139}Ce - \text{mass } ^{139}La)\ 931.494$ MeV (2)

Mass excess of $^{139}_{57}La$ is $^{139}_{57}\Delta = -87231.371$ keV or in mass units:

$^{139}_{57}\Delta = \dfrac{-87.231371 \text{ meV}}{931.494 \text{ meV/amu}} = -93646.73\,\mu u$

$\therefore$ ^{139}Mass La $= 139 - 93646.73 \times 10^{-6} = \underline{138.90635}\ u$

Using Eq. (2): Mass ^{139}Ce $= \dfrac{.28}{931.494} + 138.90635 = \underline{138.90665\ u}$

and Eq. (1): Mass ^{139}Pr $= \dfrac{2.11 + 2(0.511)}{931.494} + 138.90665 = \underline{138.91001}\ u$

(iv) Consider the binding energy equation:

$B_{A,Z} = [Z M_H + (A-Z) M_n - M_{A,Z}]\ 931.494$ meV or in terms of mass defe

$\delta M_{A,Z} = \dfrac{B_{A,Z}}{931.494} = [Z M_H + (A-Z) M_n - M_{A,Z}]$

Given the definition of mass excess $M_{A,Z} = \Delta A + A$

$\therefore \delta M_{A,Z} = [Z(\Delta H + 1) + (A-Z)(\Delta n + 1) - (\Delta A + A)]$

$= [Z\Delta H + \cancel{Z} + A\Delta n + \cancel{A} - Z\Delta n - \cancel{Z} - \Delta A - \cancel{A}]$

$\therefore \delta M_{A,Z} = [Z\Delta H + (A-Z)\Delta n - \Delta A]$

(v) For $^{139}_{59}$Pr, A is odd $\Rightarrow$ spin + parity obtained from the 59th proton in the $2d_{5/2}$ level $\therefore$ for d, $l = 2$ $\therefore$ $\underline{+}$ parity and spin $\underline{\frac{5}{2}}$

Similarly for $^{139}_{58}$Ce, the spin and parity are obtained from the 81st neutron in the $1h_{11/2}$ level $\Rightarrow$ for h, $l = 5$ $\therefore$ − parity and spin $\frac{11}{2}$ (actually $\frac{3}{2}+$)

For $^{139}_{57}$La, the spin and parity are obtained from the 57th proton in the $1g_{7/2}$ level $\Rightarrow$ for g, $l = 4$ $\therefore$ + parity and spin $\underline{\frac{7}{2}}$.

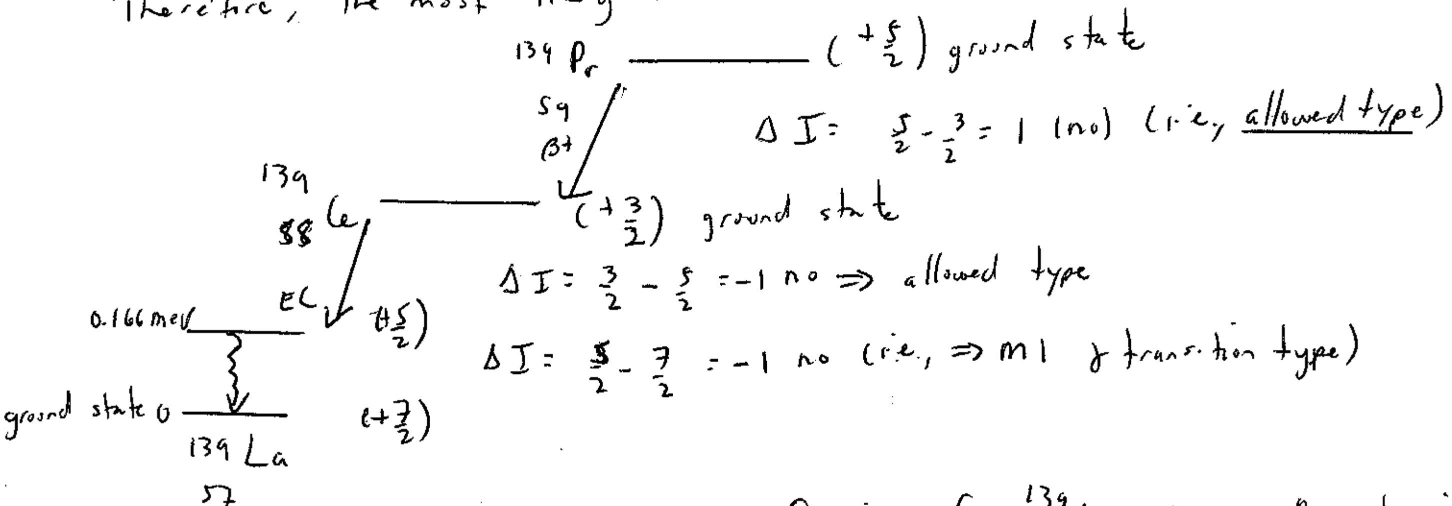
However, the ΔI for the positron transition from the ground states for

$^{139}_{59}\text{Pr}(+\frac{5}{2}) \xrightarrow{\beta^+} {}^{139}_{58}\text{Ce}(-\frac{11}{2})$ is $\Delta I = \frac{5}{2} - \frac{11}{2} = -3$ yes (there is a parity change

Hence the beta selection rules suggest a 3rd forbidden type.

However, since we are near a magic number for ^{139}Ce (81st neutron) and the fact that the spin prediction suggest a 3rd forbidden type, it is reasonable that a lower energy state may exist by promoting a nucleon from a level of lower angular momentum into the higher angular momentum level so that a pair can be formed. Hence, for ^{139}Ce, it appears that a neutron removed from a pair in the $2d_{3/2}$ level to form a pair with the $1h_{11/2}$ neutron; the odd particle is therefore a $2d_{3/2}$ neutron (implying a $+3/2$ parity and spin ground state. (Alternatively, a neutron could have been promoted from the $3s_{1/2}$ state for ^{139}Ce, however this would have implied a greater angular momentum change for the positron transition ($\Delta I = \frac{5}{2} - \frac{1}{2} = 2$ no $\Rightarrow$ 2nd forbidden type), as well as for the EC transition.

Therefore, the most likely transitions are:



Therefore, with reference to the above figure, for $^{139}_{57}La$ the 57th neutron is in the $1g_{7/2}$ level (ground state $+\frac{7}{2}$) and in the energy level diagram the next state is $2d_{5/2}$ (i.e., 1st excited state $+\frac{5}{2}$). Hence, the ΔI change for the EC transition is less between the ground state of ^{139}Ce and the 1st excited state of ^{139}La, then between the two ground states implying a greater decay to the excited state. Note that the ΔI change for this E.C. transition is $\Delta I = \frac{3}{2} - \frac{5}{2} = -1$ no, which according to β decay systematics is an allowed type hence this transition is likely by E.C. decay since the theory of EC is similar to β decay (i.e., it only involves the further wave function of the atomic electrons). Further, ^{139}Ce undergoes an E.C. decay rather than β+ decay because the mass which equals 0.28 meV is less than the threshold 1.022 MeV for β+ decay.

(vi) Using the Binding energy per nucleon (Appendix B) for $^{139}_{59}Pr$, $^{139}_{58}Ce$, $^{139}_{57}La$, the corresponding binding energies B are:

B_{Pr} = 8349.482 (keV/nucleon) × 139 nucleons = 1160.578 MeV

B_{Ce} = 8370.428 (keV/nucleon) × 139 nucleons = 1163.489 MeV

B_{La} = 8378.063 (keV/nucleon) × 139 nucleons = 1164.551 MeV

Given the Weissaker formula for odd A nuclides:

$$B(A,Z) = 14.1A - 13.1A^{2/3} - 0.58Z(Z-1)A^{-1/3} - 18(N-Z)^2A^{-1}$$

Hence, the binding energies are:

	$^{139}_{59}Pr$	$^{139}_{58}Ce$	$^{139}_{57}La$
−B (mass data) (MeV)	−1160.578	−1163.489	−1164.551
−B(A,Z) (Weissaker) (MeV)	−1168.12	−1169.72	−1170.05
Z	59	58	57

The discrepancy in binding energies is due to the fact that the constants in the Weissaker formula are derived over a large range of A values. Thus, extrapolating the binding energies by fitting the constants to the specific mass data where:

$$B = aZ^2 + bZ + c$$

for the matrix problem $A\vec{x} = \vec{b}$

where $A = \begin{bmatrix} 59^2 & 59 & 1 \\ 58^2 & 58 & 1 \\ 57^2 & 57 & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $\vec{b} = [-1160.578, -1163.489, -1164.551]$

can be solved by Maple (see attached) yielding the solution

a = 0.924499555, b = −105.2454489, c = 1831.310530

See attached plot of −B (MeV) versus Z

```
> with(linalg):
> b:= vector([-1160.578,-1163.489,-1164.551]);
```

$$b := [-1160.578, -1163.489, -1164.551]$$

```
> A:= matrix(3,3,[59^2,59,1,58^2,58,1,57^2,57,1]);
```

$$A := \begin{bmatrix} 3481 & 59 & 1 \\ 3364 & 58 & 1 \\ 3249 & 57 & 1 \end{bmatrix}$$

```
> x:= linsolve(A,b);
```

$$x := [0.9244995555, -105.2554489, 1831.310530]$$

```
> a:=.9244995555; b:=-105.2554489;c:=1831.310530;
```

$$a := 0.9244995555$$

$$b := -105.2554489$$

$$c := 1831.310530$$

```
> plot((a*Z^2+b*Z+c), Z = 53..59);#(-BE (MeV) vs Z)
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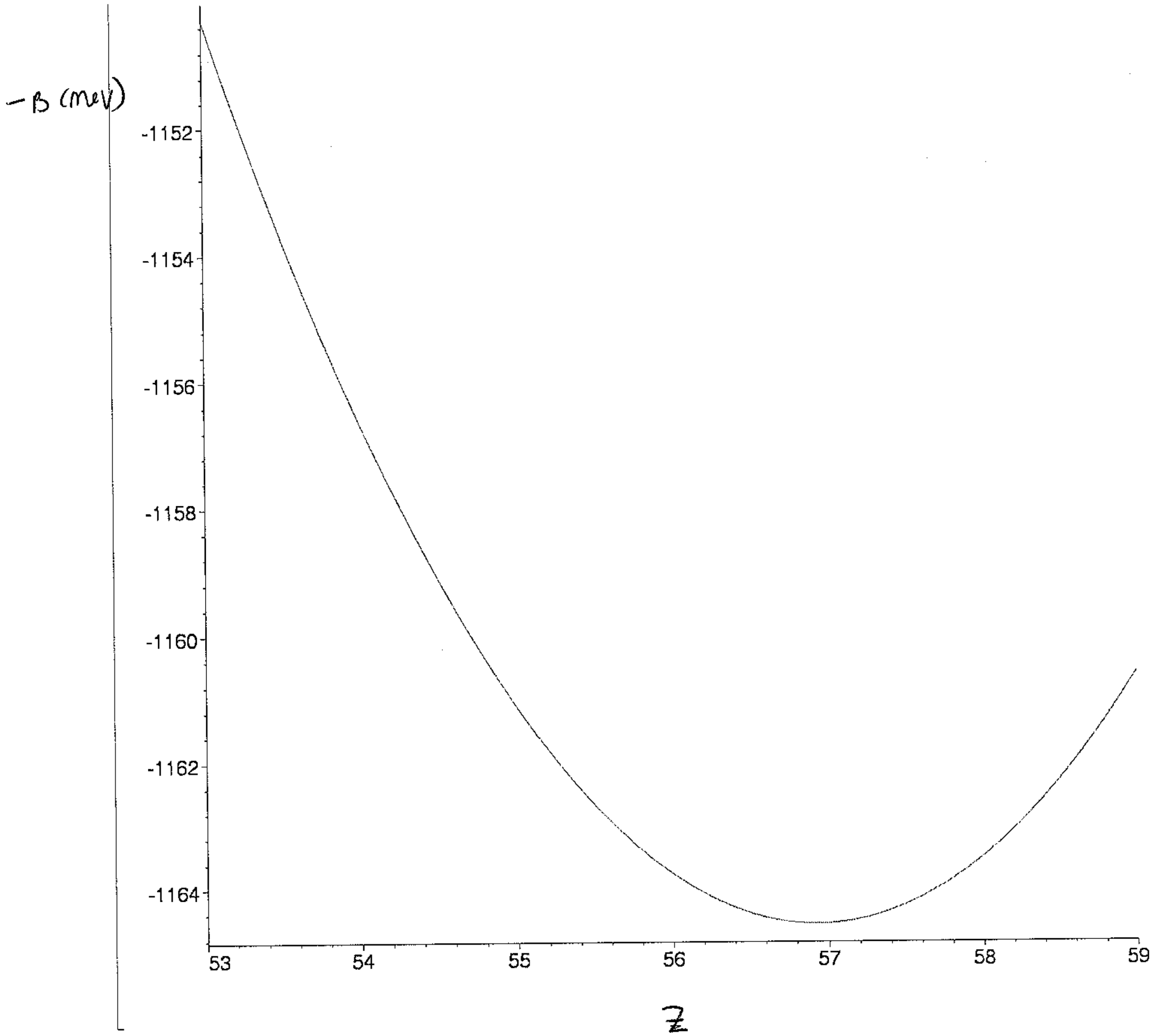
-B (meV)
-1152
-1154
-1156
-1158
-1160
-1162
-1164
53
54
55
56
57
58
59
Z

[>
[>

2. The threshold energy T_m for a particle of mass m striking a nucleus of mass M with a reaction energy Q is:

$$T_m = -Q\frac{(M+m)}{M}$$

(i) $^{204}_{82}Pb + \gamma \rightarrow {}^{203}_{82}Pb + {}^{1}_{0}n$

$\therefore Q = (\Delta^{204}Pb - \Delta^{203}Pb - \Delta n)$ where Δ is the mass excess in keV:

$Q = (-25109.735 - (-24786.57) - 8071.3171) = -8394.482$ keV

$= -\underline{8.394\ meV}$

This reaction is endothermic and the threshold energy necessary to make this reaction go is (since $m=0$ for a γ-ray) $\Rightarrow T_m = -Q = \underline{8.394\ meV}$

(ii) $^{109}_{47}Ag + {}^{4}_{2}\alpha \rightarrow {}^{1}_{1}p + {}^{112}_{48}Cd$

$\therefore Q = (\Delta^{109}Ag + \Delta^{4}He - \Delta^{112}Cd - \Delta^{1}H)$ keV (when 2 electron masses were add to each side of the equation)

$\therefore Q = (-88722.669 + 2424.91565 - (-90580.518) - 7288.9705)$ keV $= -3006.206$ keV

$= \underline{-3.006\ meV}$

This reaction is also endothermic. Therefore, the threshold energy is:

$$T_m = -Q\frac{(M+m)}{M} = +3.006206\,\frac{(108.904752 + 4.001506)}{108.904752} = \underline{3.117\ meV}$$

Here $m = 4.00260325\,u - 2 \times \frac{.511006\ meV}{931.494\ meV/amu} = 4.001506\,u$

The atomic masses are taken from Appendix B and m is the mass of the He nucleus (α particles).

(iii) $^{138}_{57}La + ^{1}_{0}n \rightarrow ^{139}_{57}La + \gamma$

$$Q = [\Delta^{138}La + \Delta n - \Delta^{139}La] \text{ keV}$$

$$\therefore Q = [-86524.681 + 8071.3171 - (-87231.371)] \text{ keV} = 8778.0071$$

$$= \underline{8.778 \text{ MeV}}$$

Hence this reaction is exoergic (Q positive) and therefore the reaction does not require a threshold energy for this reaction to occur (same for all (n,γ) and some (n,fission))