CHAPTER II

PROBLEM #1

$$\nabla \times \overline{\mathcal{H}} = \frac{\partial \overline{\mathcal{D}}}{\partial t} \qquad (2\text{-}15)$$

$$\nabla \times \nabla \times \overline{\mathcal{H}} = \nabla \times \frac{\partial \overline{\mathcal{D}}}{\partial t}$$

$$\nabla(\nabla \cdot \overline{\mathcal{H}}) - \nabla^2 \overline{\mathcal{H}} = \nabla \times \frac{\partial \overline{\mathcal{D}}}{\partial t}$$

$$\nabla(\nabla \cdot \overline{\mathcal{H}}) - \nabla^2 \overline{\mathcal{H}} = \nabla \times \frac{\partial}{\partial t} \varepsilon \overline{\mathcal{E}}$$

$$= \frac{\partial}{\partial t} \varepsilon \nabla \times \overline{\mathcal{E}}$$

$$= \frac{-\partial^2}{\partial t^2} \varepsilon \mu_o \overline{\mathcal{H}} \qquad \text{(using Eq. 2-14)}$$

now $\nabla \cdot \mu_o \overline{\mathcal{H}} = \mu_o \nabla \cdot \overline{\mathcal{H}} = 0$ (from 2-17)

so $-\nabla^2 \overline{\mathcal{H}} = \frac{-\partial^2}{\partial t^2} \varepsilon \mu_o \overline{\mathcal{H}}$

or
$$\nabla^2 \overline{\mathcal{H}} - \varepsilon \mu_o \frac{\partial^2}{\partial t^2} \overline{\mathcal{H}} = 0 \qquad (2\text{-}27)$$

PROBLEM #2

$$\nabla \times \overline{E} = -j\omega\mu\overline{H}$$

$$\nabla \times \nabla \times \overline{E} = \nabla \times -j\omega\mu\overline{H}$$

$$\nabla \times \nabla \times \overline{E} = \omega^2 \mu\varepsilon\overline{E}$$

$$\nabla(\nabla \cdot \overline{E}) - \nabla^2 \overline{E} = \omega^2 \mu\varepsilon\overline{E}$$

$$-\nabla^2 \overline{E} = \omega^2 \mu\varepsilon\overline{E}$$

$$\nabla^2 \overline{E} + \omega^2 \mu\varepsilon\overline{E} = 0$$

$$\nabla^2 \overline{E} + k^2 \overline{E} = 0$$

PROBLEM #3

$$v = \frac{c}{n} = \frac{3\text{x}10^8 \text{m/sec}}{1.530} = 1.96 \times 10^8 \text{m/sec}$$

$$\lambda = \frac{\lambda_o}{n} = 0.65\ \mu\text{m}$$

$$k = \frac{2\pi}{\lambda_o} n = \frac{2\pi}{1\mu\text{m}}(1.530) = 9.61 \times 10^6/\text{m}$$

$$\eta = \sqrt{\frac{\mu_o}{\varepsilon}} = \frac{1}{n}\sqrt{\frac{\mu_o}{\varepsilon_o}} = \frac{1}{1.530}(377)$$

$$= 246\Omega$$

Free space $v = c = 3 \times 10^8 \text{m/sec}$

$$\lambda = 1\ \mu\text{m}$$

$$k = \frac{2\pi}{\lambda_o} = 6.28 \times 10^6/\text{m}$$

PROBLEM #4

I.

$$E_T = \tau_\perp E_i$$

$$\tau_\perp = \frac{2n_2\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2} \qquad (2\text{-}128)$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\sin\theta_2 = \frac{1.5 \sin 30°}{1.4}$$

$$\theta_2 = 32.39°$$

$$\tau_\perp = \frac{2\,\frac{1}{1.4}\cos 30°}{\frac{1}{1.4}\cos 30° + \frac{1}{1.5}\cos 32.39°}$$

$$= 1.05$$

$$E_T = 1.05\left[5e^{-j\beta_2\hat{n}_2\cdot\bar{r}}\right]\hat{a}_x$$

a) $|E_T| = |5.25|$ independent of z position

b)

$$\bar{S}_2 = \frac{1}{2\eta_2}\left|E_T\right|^2 (\hat{n}_2)$$

$$= \frac{1}{2\sqrt{\frac{\mu_o}{\varepsilon_o\varepsilon_r}}}|5.25|^2\left((\sin 32.39°(-\hat{a}_y) + \cos 32.39°\,\hat{a}_z)\right)$$

$$= 5.12 \times 10^{-2}(-0.54\,\hat{a}_y + 0.84\,\hat{a}_z)$$

$$= -2.74 \times 10^{-2}\,\hat{a}_y + 4.32 \times 10^{-2}\,\hat{a}_z$$

The z component is 4.32×10^{-2} watts.

PROBLEM #4 (Cont'd)

II.

$$\theta_c = \sin^{-1} n_2/n_1$$

$$= 68.96°$$

PROBLEM #5

θ_1 is greater than the critical angle. Total internal reflection occurs and the wave in medium 2 is an evanescent wave.

$$E_T = \bar{E}_2\, e^{-jk_2\hat{n}_2\cdot\bar{r}}$$

For an evanescent wave this becomes

$$E_T = \bar{E}_2\, e^{-\alpha z}\, e^{j\beta y} \qquad (2\text{-}151)$$

$$\alpha = \omega\sqrt{\mu_o\varepsilon_2}\sqrt{\frac{n_1^2}{n_2^2}\sin^2\theta_1 - 1}$$

$$= 2.35 \times 10^6/\text{m}$$

$$|E_T| = |E_2|\, e^{-\alpha z}$$

at $z = 10\ \mu$m

$$|E_T| = |E_2|\, e^{-23.5}$$

$$= 6.48 \times 10^{-11}\,|E_2| \quad \text{V/m}$$

The power transmitted in the z direction is reactive. The z component of the Poynting vector is zero $(\hat{a}_z\cdot\bar{S}_T)$ is imaginary.