

## Chapter 1. Introduction: materials and design

**E1.1 Concepts (working principles).** Even if you shut the windows and doors dust settles in rooms like gentle snow. The need: a device to remove dust from household floors. Here are three established concepts:

- human-powered bellows pump with cloth filter;
- axial-flow electric fan with muslin filter; and
- centrifugal fan with centrifugal dust separation and collection.

What other concepts could fill this need? Nothing, at the concept stage, is too far-fetched; decisions about practicality and cost come later.



**Figure E1.1.** Vacuum cleaners: a hand-powered cleaner from 1880, the Electrolux cylinder cleaner of 1960 and the Dyson centrifugal cleaner of 2010. (Early cleaner courtesy Worcester News.)

**Answer.** Here are some concepts for dust collectors. The list starts with the three in the figure to set the style.

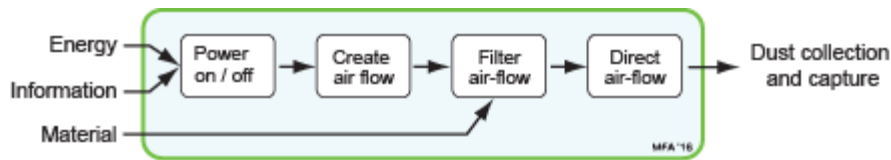
- Entrain dust in airstream created by hand-powered bellows; capture dust from airstream with filter.
- Entrain dust in airstream created by axial electric fan; capture dust from airstream with filter.
- Entrain dust in airstream created by centrifugal fan; capture dust from airstream with centrifugal separator.

*Now some other possibilities.*

- Sweep the floor with a brush; collect dust in dust pan. It has worked for generations and still does.
- Entrain dust in airstream; remove with electrostatic separator. It works for industrial emissions.
- Entrain dust in airstream; remove with fine water spray to create a liquid slurry. It's how oxides of sulphur are removed from power station emissions.
- Entrain dust in airstream; remove by passing airstream across wet fabric. It's what happens when you hang clothes out to dry and a wind stirs up the dust.
- Trap dust on adhesive reel-to-reel adhesive tape rolled across the floor. Adhesive tape works well as a way of trapping mosquitos; why not dust?
- Pay a cleaning agency to come and take the dust out of your house. They have their own methods.

**E1.2 Function structures.** Make a function-structure diagram for the Electrolux cleaner of Figure E1.1.

**Answer.** The diagram is shown below.



**E1.3 Concepts (working principles) for cooling power electronics.** Microchips, particularly those for power electronics, get hot. If they get too hot they cease to function. The need: a scheme for removing heat from power microchips. One is sketched in Figure E1.2. Its working principle is that of thermal conduction to fins from which heat is removed by convection.

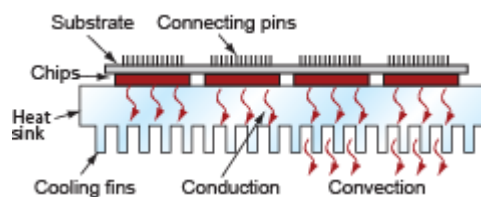


Figure E1.2. A heat sink

Four working principles are listed below: thermal conduction, convection by heat transfer to a fluid medium, evaporation exploiting the latent heat of evaporation of a fluid, and radiation, best achieved with a surface with high emissivity.

The best solutions may be found by combining two or more of these: conduction coupled with convection (as in the sketch – an often-used combination). Consider both the case of long term, continuous-operation heat sinks and the case of short-term, intermittent heat sinks (with pause between operations for system to return to ambient temperature).

**Answer.** Concepts for continuous or long-term operation.

- Transmission of heat by conduction; removal by natural or fan-assisted convection. (Typical system in personal computer.)
- Transmission of heat by conduction; removal by radiation. (Possible system for operation in space.)
- Transmission of heat by conduction, heat exchanger to pass heat to working fluid; heat removed from working fluid by convection (a radiator).
- Transmission of heat by conduction; heat exchanger to pass heat to working fluid (water, for instance) drawn from renewable source (a river, for example); working fluid then rejected.
- Transmission of heat by conduction, active removal by Peltier (thermoelectric) cooling.
- Transmission of heat by conduction, active removal by magnetocaloric (MC) cooling.

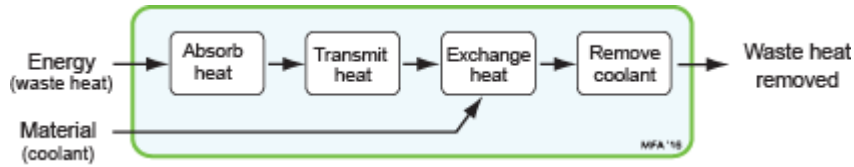
Concepts for pulsed or short-term operation.

- Conduction of heat into heat sink so massive that the temperature rise is acceptably small.
- Conduction of heat to reserve of phase-change material (paraffins, hydrated salts such as  $\text{CaCl}_2 \cdot 6\text{H}_2\text{O}$ ); heat absorbed by the latent heat of melting.

- Conduction of heat to sacrificial evaporative coating; heat removed by latent heat of evaporation.

**E1.4 Function structures.** Make a function-structure diagram for the heat sinks.

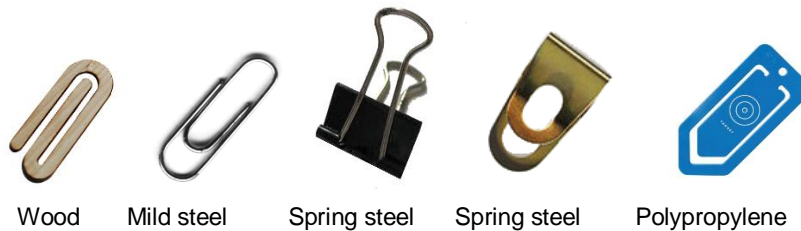
**Answer.** The diagram is shown below.



**E1.5 Attaching paper sheets.** The need: documents consisting of several pages can become confused if the pages are loose. Use the internet to explore alternative concepts for attaching a small number of pages together in a way that still allows them to be read. Record, as far as possible, the materials of the attachment and the property that provides the attachment force.

**Answer.**

Systems that rely on elastic stiffness (thus, elastic modulus) to provide the attachment force.



Systems that rely on strength or plasticity to provide the attachment force.



## Chapter 2. Engineering materials and properties

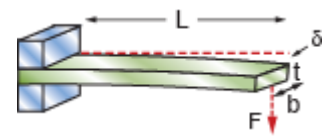
These exercises introduce the reader to two useful resources: the data sheets of Appendix A and the Solutions to Standard Problems of Appendix B. When drawing data from Appendix A use the mean values of the ranges listed there.

**E2.1 Sound velocity.** The speed of sound in pyrex (Borosilicate) glass is 5610 m/s. Find the density of this glass from Appendix A and use it to estimate the modulus  $E$  of pyrex.

**Answer.** The density  $\rho$  of Borosilicate glass, from Table A3 of Appendix A, is 2250 kg/m<sup>3</sup>. The longitudinal wave speed (from the text) is  $\sqrt{E/\rho}$ . Thus the modulus of the glass is

$$E = 2250 \times (5610)^2 = 7.1 \times 10^{10} \text{ Pa} = 71 \text{ GPa}$$

**E2.2 Deflection of beams.** A cantilever beam has a length  $L = 50$  mm. It has a rectangular cross-section width  $b = 5$  mm and a thickness  $t = 1$  mm. It is made of an aluminum alloy. By how much will the end deflect under an end-load of  $F = 5$  N (roughly that exerted by the weight of 5 apples)? Use data from Appendix A4 for the (mean) value of Young's modulus of aluminum alloys, the equation for the elastic deflection of a cantilever from Appendix B3 and for the second moment of a beam from Appendix B2 to find out.

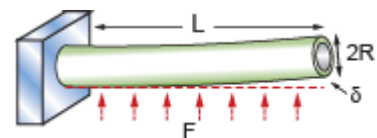


**Answer.** The deflection  $\delta$  of a cantilever under an end load  $F$  is, from Appendix B,

$$\delta = \frac{FL^3}{3EI} \quad \text{with the second moment of area } I = \frac{bt^3}{12}$$

The mean Young's modulus  $E$  for aluminum, from Appendix A3, is 75 GPa. Inserting the data from the question gives a second moment of area  $I = 0.42 \times 10^{-12} \text{ m}^4$  and an end deflection  $\delta = 6.7$  mm.

**E2.3 Deflection of beams.** The wings of a glider are each 4m long. The load-bearing member is the wing spar, a tubular beam running the length of the wing. The wing spar in this glider has a diameter of 140 mm and a wall thickness of 6 mm. It is made of an aluminum alloy. In flight the wing spar is loaded in bending with (we will assume) a uniformly distributed force per unit length. By how much will the wing-tip deflect in calm flight if the loaded glider weighs 1000 kg? Use data from Appendix A3 for the (mean) value of Young's modulus of aluminum alloys, the equation for the elastic deflection of a cantilever from Appendix B3 and for the second moment of a beam from Appendix B2 to find out.



**Answer.** The deflection  $\delta$  of a cantilever under a distributed load  $F$  is, from Appendix B, Table B3

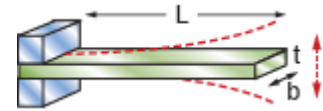
$$\delta = \frac{FL^3}{8EI}$$

The second moment of area of a thin-walled tube of diameter  $2R$  and wall thickness  $t$  (Table B2) is

$$I = \pi R^3 t$$

Each wing spar carries a distributed load of 500kg (4,910 N). The mean Young's modulus  $E$  for aluminum, from Appendix A3, is 75 GPa. Inserting the data from the question results in an end deflection  $\delta = 81$  mm.

**E2.4 Vibration of beams.** A cantilever beam with length  $L = 200$  mm, width  $b = 12$  mm and thickness  $t = 2$  mm weighs 38 grams and has a natural vibration frequency of  $f = 42$  hertz. What is the modulus of the material of the beam? You will find equations for natural frequencies of vibration and moments of area in Appendix B, Tables B12 and B2.



**Answer.** From Appendix B, the natural frequency of a cantilever beam is

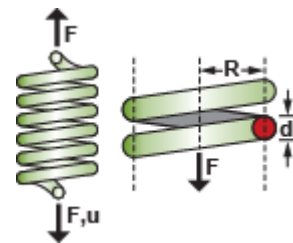
$$f = 0.56 \sqrt{\frac{EI}{m_o L}}$$

where  $m_o = \rho b t$  is the mass per unit length and  $I = \frac{b t^3}{12}$  is the second moment of area of the beam.  $E$  and  $\rho$  are the modulus and density of the beam. In this example  $m_o = 0.19$  kg/m, and  $I = 8 \times 10^{-12}$  m<sup>4</sup>. Inverting the equation for  $f$  and solving for the modulus gives  $E = 214$  GPa.

**E2.5 Forensic materials science.** Exercise E2.4 used a natural vibration frequency to measure the modulus of a beam. Use the information in E2.4 to calculate the density of the beam, then use this and the result of the exercise (the modulus of the beam) as a forensic tool. Scan the Modulus values in Table A3 of Appendix A seeking a match between the calculated modulus and those in the table. What subsets of materials match the calculated modulus? Of these, which subset also matches the calculated density?

**Answer.** The density (mass/volume) of the beam of exercise E2.4 is 7920 kg/m<sup>3</sup>. Its modulus, calculated in exercise E2.4, is 214 GPa. Only one subset of materials – steels – matches both the density and the modulus. Note that this identification has require only very simple measurement: the dimensions of the beam, its weight and its vibration frequency.

**E2.6 Springs.** A spring, wound from stainless steel wire with a wire diameter  $d = 1$  mm, has  $n = 20$  turns of radius  $R = 10$  mm. How much will it extend when loaded with a mass  $M$  of 1 kg? Assume the shear modulus  $G$  of stainless steel to be  $3/8 E$  where  $E$  is Young's modulus, retrieve this from Appendix A3, and use the expression for the extension of springs from Appendix B6 to find out.

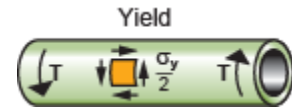


**Answer.** The extension  $u$  of a spring under a force  $F = Mg = 9.81$  N (here  $g$  is the acceleration due to gravity) is

$$u = \frac{64 F R^3 n}{G d^4}$$

Young's modulus for stainless steel is 200 GPa, so shear modulus  $G \approx 76$  GPa. Inserting the data gives a deflection  $u = 10.4$  mm.

**E2.7 Torsion of tubes.** A thick-walled tube has an inner radius  $r_i = 10$  mm and an outer radius  $r_o = 15$  mm. It is made from polycarbonate, PC. What is the maximum torque  $T_f$  that the tube can carry without the onset of yield? Retrieve the (mean) yield strength  $\sigma_y$  of PC from Appendix A3, the expression for the torque at onset of yield from Appendix B6 and that for the polar moment of a thick walled tube from Appendix B2 to find out.

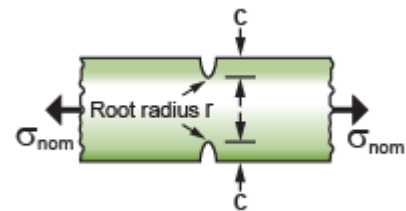


**Answer.** The torque at the onset of yield for a thick walled tube is

$$T_f = \frac{K\sigma_y}{2r_o} \quad \text{with} \quad K = \frac{\pi}{2}(r_o^4 - r_i^4)$$

The mean yield strength of  $\sigma_y$  of PC from Appendix A3 is 65 MPa. Inserting the data from the question gives a torque at the onset of yield of  $T_f = 138$  N.m.

**E2.8 Stress concentrations.** A round bar, 20 mm in diameter, has a shallow circumferential notch with a depth  $c = 1$  mm with a root radius  $r = 10$  microns. The bar is made of a low carbon steel with a yield strength of  $\sigma_y = 250$  MPa. It is loaded axially with a nominal stress,  $\sigma_{nom}$  (the axial load divided by the un-notched area). At what value of  $\sigma_{nom}$  will yield first commence at the root of the notch? Use the stress concentration estimate of Appendix B9 to find out.



**Answer.** The stress concentration caused by notch of depth  $c$  and root radius  $r$  is

$$\frac{\sigma_{max}}{\sigma_{nom}} = 1 + \alpha \left( \frac{c}{r} \right)^{1/2} \quad \text{with} \quad \alpha \approx 2 \quad \text{for tension}$$

Yield first starts when  $\sigma_{max} = \sigma_y$ . Inserting the data from the question gives a nominal stress for first yield of 11.9 MPa. Stress concentrations can be very damaging – in this example, a cyclic stress of only  $\pm 12$  MPa will ultimately initiate a fatigue crack at the notch root.

**E2.9 Thermal stress.** An acrylic (PMMA) window is clamped in a thick low carbon steel frame at  $T = 20$  C. The temperature falls to  $T = -20$  C, putting the window under tension because the thermal expansion coefficient of PMMA is larger than that of steel. If the window was stress-free at 20C, what stress does it carry at -20 C? Use the result that the bi-axial stress caused by a bi-axial strain difference  $\Delta\varepsilon$  is

$$\sigma = \frac{E\Delta\varepsilon}{1-\nu}$$



where  $E$  is Young's modulus for PMMA and Poisson's ratio  $\nu = 0.33$ . You will find data for expansion coefficients in Table A6 and for moduli in Table A3, Appendix A. Use mean values.

**Answer.** The strain difference caused by difference in thermal expansion,  $\alpha$ , when the temperature changes by  $\Delta T$  is

$$\Delta\varepsilon = (\alpha_{PMMA} - \alpha_{Low\ C\ steel})\Delta T$$

From Appendix A6,

$$\alpha_{PMMA} = 117 \times 10^{-6}/C \quad \text{and} \quad \alpha_{Low\ C\ steel} = 12.3 \times 10^{-6}/C$$

giving  $\Delta\varepsilon = 4.2 \times 10^{-3}$ . The modulus of PMMA, from Appendix A3, is  $E = 3.0$  GPa. The equation given in the question then predicts a tensile stress in the window of  $\sigma = 19$  MPa.

**E2.10 Unstable cracks.** The PMMA window described in Exercise 2.9 has a contained crack of length  $2a = 0.5$  mm. If the maximum tensile stress that is anticipated in the window is  $\sigma = 20$  MPa, will the crack propagate? Choose an appropriate equation for crack propagation from Appendix B10 and data for the fracture toughness  $K_{1c}$  of PMMA from Appendix A6 to calculate the length of crack that is just unstable under this tensile stress.

**Answer.** The crack length is small compared with the width of the window, so the appropriate choice of equation describing crack instability is

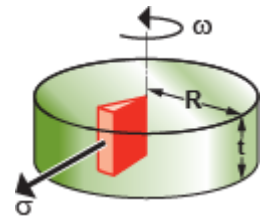
$$C\sigma\sqrt{\pi a} \geq K_{1c} \quad \text{with} \quad C = 1.0$$

Inserting the data we find the length of the shortest crack that is just unstable:

$$2a = \frac{2}{\pi} \left( \frac{K_{1c}}{\sigma} \right)^2 = 2.1 \text{ mm, using } (K_{1c})_{PMMA} = 1.15 \text{ MPa}\cdot\text{m}^{1/2}$$

Thus the 0.5mm crack will not propagate.

**E2.11 Centrifugal stress.** A flywheel with a radius  $R = 200$  mm is designed to spin up to 8000 rpm. It is proposed to make it out of ductile (nodular) cast iron, but the casting shop can guarantee only that it will have no crack-like flaws greater than  $2a = 2$  mm in length. Use the expression for the maximum stress in a spinning disk in Appendix B7, that for the stress intensity at a small enclosed crack from Appendix B10 and data for cast iron from Appendix A2 and A4 to establish if the flywheel is safe. Take Poisson's ratio  $\nu$  for cast iron to be 0.33.

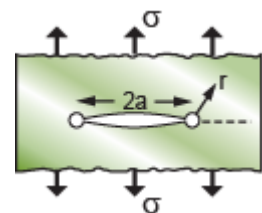


**Answer.** The maximum tensile stress in a spinning disk is

$$\sigma_{max} = \frac{(3+\nu)}{8} \rho \omega^2 R^2, \quad \text{and} \quad K_1 = \sigma_{max} \sqrt{\pi a} \leq K_{1c}$$

for a contained crack. Here  $\omega = 2\pi W/60$  radians/sec when  $W$  is the rotational velocity in rpm. Inserting the data from the question and the mean values for density  $\rho = 7200$  kg/m<sup>3</sup> and  $K_{1c} = 38$  MPa·m<sup>1/2</sup> from Appendix A, we find the maximum rotational velocity that will just cause the cracks to propagate is 4350 radians/s, or 41,600 rpm. The flywheel is safe.

**E2.12 Blunting cracks.** Liberty ships, thousands of which were constructed during World War II, were welded rather than riveted. Many were mistakenly made from a steel that became brittle at the temperatures of the Atlantic in winter. They had square hatches with sharp corners from which a crack could start and propagate across the deck which, being welded, allowed a continuous path not interrupted by riveted plates. It is said that alert seamen, observing such a crack to start, would seize a power drill and drill a hole at the crack tips, effectively blunting it and reducing the stress concentration. If the crack when the seaman first saw it was 1 m long, and the largest drill he had was  $2r = 25$  mm in diameter, what is the stress concentration at the end of the crack once the hole was drilled? You will find the equation for the stress at a distance  $r$  from the tip of a crack with a length  $2a$  Appendix B, Table B10.

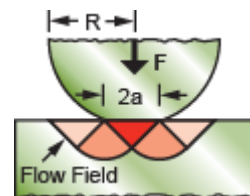


**Answer.** A crack concentrates stress locally. This local stress  $\sigma_\ell$  falls off with distance  $r$  from the tip of a crack as

$$\sigma_{\ell} = \frac{C\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}$$

where  $C$  is a constant, equal to 1 and . By drilling out the material at the crack tip the largest stress is that evaluated at the surface of the whole, where  $r = 12.5$  mm. Inserting this and the crack length  $2a = 1$  m gives a local stress  $\sigma_{\ell} = 4.5 \sigma$ .

**E2.13. Contact forces.** A clamp has a hemispherical contact faces of radius  $R = 10$  mm to allow for misorientation. It is used to clamp a heavy lead archaeological treasure for X-ray examination. How much clamping force,  $F$ , can be applied without damage (meaning plastic deformation) to the treasured object? You will find the both the modulus  $E$  and yield strength  $\sigma_y$  of lead alloys in Appendix A, Table A3 (use mean values), and the equation for force required to trigger plastic deformation beneath a spherical indenter in Appendix B, Table B8.



**Answer.** From Table A3, the yield strength of lead alloys is 11 MPa and the modulus is 14 GPa. The force to cause yielding below a spherical indenter, from Table B8, is

$$\sigma_s = \frac{F}{\pi a^2} > 3\sigma_y$$

Immediately before yielding starts, the contact is elastic and  $a$  is the radius of the contact-patch,  $a$ , is

$$a = 0.7 \left( \frac{FR}{E} \right)^{1/3}$$

Combining these equations and solving for the clamping force  $F$  gives

$$F = 126 \left( \frac{R}{E} \right)^2 (\sigma_y)^3$$

Entering values for the parameters gives a permissible clamping force of 0.085 N. This is an impossibly low clamping force so clamping the object in this way is impractical. A softer clamping system, distributing load over a much larger area, is needed.

**E2.14 Estimating thermal properties.** You wish to measure, approximately, the thermal conductivity  $\lambda$  of polyethylene (PE). To do so you block one end of a PE pipe with a wall thickness of  $x = 3$  mm and diameter of 30 mm and fill it with boiling water while clutching the outside with your other hand. You note that the outer surface of the pipe first becomes appreciably hot at a time  $t \approx 18$  seconds after filling the inside with water. Use this information, plus data for specific heat  $C_p$  and density  $\rho$  of PE from Appendix A, Tables A2 and A5, to estimate  $\lambda$  for PE. How does your result compare with the listed value in Table A7?

**Answer.** The distance  $x$  that heat diffuses in a time  $t$  is approximately

$$x = \sqrt{2at} \quad \text{with} \quad a = \frac{\lambda}{\rho C_p}$$

( $\alpha$  is the thermal diffusivity). Inserting the data from the question and the mean values  $\rho = 950 \text{ kg/m}^3$  and  $C_p = 1850 \text{ J/kg/K}$  from the Appendix, we find  $\lambda \approx 0.44 \text{ W/m.K}$ . The result given in Appendix A6 for the thermal conductivity of PE is  $0.40 - 0.44 \text{ W/m.K}$ .

**E2.15 Heat exchangers.** A power transistor with an area of  $A$  of  $1 \text{ mm}^2$  dissipates 10 watts. It is attached to a heat sink 2 mm thick, effectively cooled to 30 C on the back face by forced air flow. If the heat sink is made of aluminium nitride, how hot will the microchip be at steady state? You will find equations of heat flow in Appendix B, Table B15, and the thermal properties of aluminium nitride in Appendix A, Tables A5 and A6.

**Answer.** The thermal conductivity of aluminium nitride, from Table A6, is  $170 \text{ W/m.K}$ . The basic equation of steady-state heat flux  $J$  ( $\text{J/m}^2\cdot\text{sec}$ ) (Table B15), is

$$J = -\lambda \frac{dT}{dx}$$

where  $\lambda$  is the thermal conductivity and  $dT/dx$  is the temperature gradient. The heat flow,  $q$ , away from the transistor is

$$q = AJ = -A \lambda \frac{dT}{dx} = 1 \times 10^{-6} \times 170 \times \frac{\Delta T}{2 \times 10^{-3}} = 8.5 \times 10^{-2} \Delta T \text{ watts}$$

Equating this to the dissipated power, 10 watts, and solving for  $\Delta T$  gives

$$\Delta T = 118^\circ\text{C}$$

Thus the temperature of the transistor is  $148^\circ\text{C}$ .

**E2.16 Choosing dielectrics.** The capacitance  $C$  of a condenser with two plates each of area  $A$  separated by a dielectric of thickness  $t$  is

$$C = \epsilon_r \epsilon_0 \frac{A}{t}$$

where  $\epsilon_0$  is the permittivity of free space and  $\epsilon_r$  is the dielectric constant of the material between the plates. Select a dielectric by scanning data in Appendix A, Tables A7 and Table A8, first to maximize  $C$  and second to minimize it, for a given  $A$  and  $t$ .

**Answer.** (a) Capacitance is maximized by selecting materials with high  $\epsilon_r$ . Appendix A shows that most polymers and ceramics have dielectric constants in the range 2 – 10, the ferroelectric ceramics barium titanate, lead zirconium titanate (PZT) and lead lanthanum zirconium titanate (PLZT) all have dielectric constants greater than 1000.

(b) Capacitance is minimized by materials with low  $\epsilon_r$ : the non-polar polymers polyethylene, polypropylene and teflon (PTFE) are the lowest among solid materials. Polymer foams have low dielectric constants because they are mainly air (dielectric constant 1)

**E2.17 Piezoelectric actuators.** Piezoelectric materials respond to an electric field by change of shape. The strain  $\varepsilon$  induced by a field  $E$  is

$$\varepsilon = d_{33}E$$

where  $d_{33}$  is the piezoelectric charge coefficient, listed in Table A8. Its units are pm/V. A sample of soft PZT (lead zirconium titanate) is exposed to a field of 10 MV/m. How large is the resulting strain? You will find the piezoelectric charge coefficient for soft PZT in Table A8. Use a mean value.

**Answer.** The piezoelectric charge coefficient  $d_{33}$  for soft PZT is  $675 \times 10^{-12}$  m/V. Thus a field of  $10 \times 10^6$  V/m will induce a strain of  $\varepsilon = 6.75 \times 10^{-3} = 0.675\%$ .

**E2.18 Pyroelectric temperature sensing.** Remote, non-contact, temperature sensing is possible lby focussing radiation from the source onto a pyroelectric material, using the charge that appears on its surface as a measure of temperature. A sensor is required to measure temperature in the range 100 – 200 C. To work in this range the material must have a ferro-electric Curie temperature greater than 200 C, since otherwise it ceases to be pyroelectric in the sensing range. The signal is maximized by selecting a material with the largest possible pyroelectric coefficient. Use the data for pyroelectrics in Appendix A, Section A8, to make a selection.

**Answer.** Lithium niobate has the largest pyroelectric coefficient of those listed in the table but its ferro-electric Curie temperature is too low to allow its use in the range 100 – 200 C. Lithium tantalite best meets the criteria for the sensor described in the question.

**E2.19 Magnetostrictive actuation.** A magnetostrictive actuator has and active core of length  $L = 20$  mm. It is made of Terfenol D, a giant magnetostrictive alloy of terbium, dysprosium and iron. If a field sufficiently large to saturate the core is applied, by how much will the core change in length? Use the mean value of the data for Terfenol D listed in Appendix A, Section A9, to find out.

**Answer.** The mean saturation magnetostrictive strain  $\varepsilon_{sat}$  for Terfenol D, from Appendix A, is  $1,700 \times 10^{-6}$ . Thus the core will change in length by

$$\delta = \varepsilon_{sat} L = 1,700 \times 10^{-6} \times 20 \times 10^{-3} = 3.4 \times 10^{-5} \text{m} = 34 \mu\text{m}$$

**E2.20 Material price.** It is proposed to replace the cast iron casing of a power tool with one with precisely the same dimension molded from nylon. Will the material cost of the nylon casing be greater or less than that made of cast iron? Use data from Appendix A3 and A11 to find out.

**Answer.** If the dimensions of the cast iron and nylon cases are the same, the volume of material required to make them are equal. Thus the cheaper option is the one with the lower material cost per unit volume  $C_v$ , where

$$C_v = \rho C_m$$

and  $\rho$  is the material density and  $C_m$  the material cost per kg. Data from Appendix A3 and A11 are assembled below, using the means of the ranges.

	Density kg/m <sup>3</sup>	Price \$/kg	Cost per unit vol \$/m <sup>3</sup>
Cast iron	7150	0.63	4500
Nylon	1130	3.45	3900

Surprisingly, the nylon casing has a lower material cost than that made of cast iron.

**E2.21. Carbon footprint.** Plastics are sometimes portrayed as environmentally poor because most are synthesized from oil or natural gas, implying a larger embodied energy and carbon footprint than other materials. Is this an accurate portrait? Use the data from Appendix A, Table A10 and A2 to compare the carbon footprint of three of the materials most widely used in household products: polypropylene, aluminium and carbon steel. Use mean values of the values in the tables.

**Answer.** Carbon steel has the lowest carbon footprint per unit weight, polypropylene the next lowest. If ranked instead by carbon footprint per unit volume, polypropylene has by far the lowest value. Any serious comparison of this type needs a more detailed examination: materials are used in products because they perform a function. If these three materials are candidates, the best choice is that with the lowest carbon footprint per unit of function. We explore ways to do this in later chapters of this book.

	Density kg/m <sup>3</sup>	Carbon footprint kg/kg	Carbon footprint kg/m <sup>3</sup>
Polypropylene	900	3.15	2840
Aluminum	2700	12.5	33,800
Carbon steel	7850	1.8	14,100

## Chapter 3. Material property charts

The 32 exercises in this section involve the simple use of the charts of Chapter 3 to find materials with give property profiles. They are answered by placing selection lines on the appropriate chart and reading off the materials that lie on the appropriate side of the lines. It is a good idea to present the results as a table. All can be solved by using the printed charts, which you are free to copy for your personal use.

If the CES Edu Materials Selection software is available, the same exercises can be solved by its use. This involves first creating the chart, then applying the appropriate box or line selections. The results, at Level 2, are the same as those read from the hard copy charts (most of which were made using the Level 2 database). The software allows more detailed charts using the Level 3 database.

**E3.1** Use Figure 3.3 to identify one metal and one polymer with a longitudinal wave speed close to 1000 m/s.

**Answer.** Lead and polypropylene both lie close to the contour of longitudinal wave speed of 1000 m/s.

**E3.2** A component is at present made from a brass, a copper alloy. Use the Young's modulus – Density ( $E - \rho$ ) chart of Figure 3.3 to suggest three other metals that, in the same shape, would be stiffer. "Stiffer" means a higher value of Young's modulus.

**Answer.** Metals that are stiffer than brass are listed in the table.

Material	Comment
Steels	The cheapest stiff, strong structural metal, widely used.
Nickel alloys	More expensive than steel
Tungsten alloys	Refractory (high-melting) and relatively expensive

**E3.3** Use the Young's modulus – Density ( $E - \rho$ ) chart of Figure 3.3 to identify materials with both a modulus  $E > 50$  GPa and a density  $\rho < 2000$  kg/m<sup>3</sup>.

**Answer.** There are only two materials on the chart with modulus  $E > 50$  GPa and density  $\rho < 2000$  kg/m<sup>3</sup>.

Material	Comment
Magnesium alloys	Magnesium is the lightest of all common structural metals – only beryllium is lighter, but it is very expensive and its oxide is toxic.
CFRP – carbon-fiber reinforced plastic	CFRP is both lighter and stiffer than magnesium. That is one reason it is used for competition cars and bikes.

**E3.4** Use the Young's Modulus-Density ( $E - \rho$ ) chart of Figure 3.3 to find (a) metals that are stiffer and less dense than steels and (b) materials (not just metals) that are both stiffer and less dense than steel.

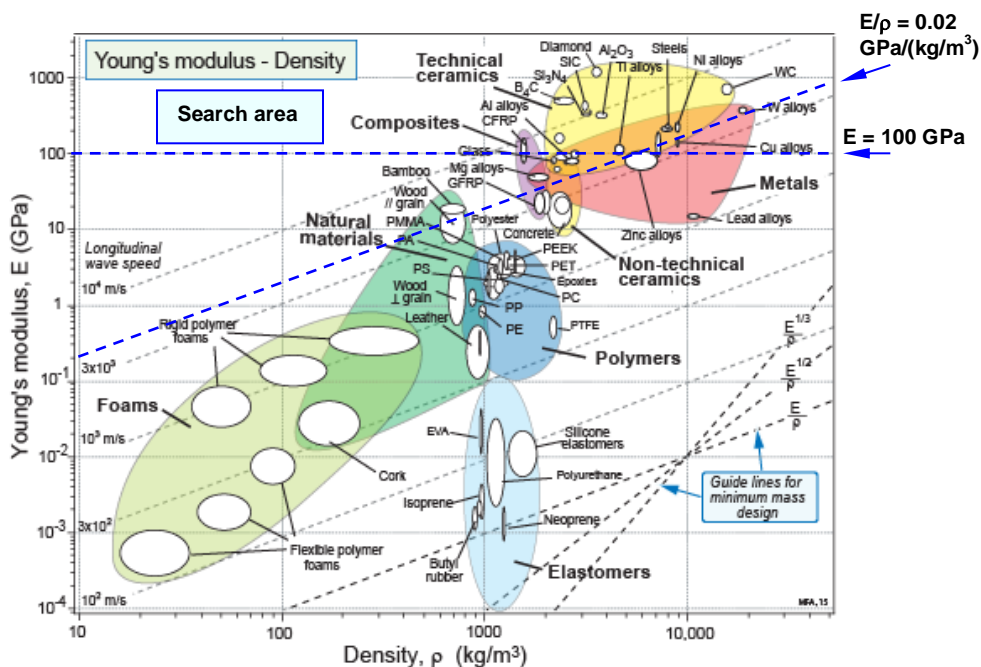
**Answer.** (a) No metals in Figure 3.3 are both stiffer and less dense than steel, though nickel alloys come close. (b) Several ceramics qualify: Boron carbide,  $B_4C$ , silicon carbide, SiC, silicon nitride  $Si_3N_4$ , alumina  $Al_2O_3$  and (inevitably) diamond.

Material	Comment
Alumina $Al_2O_3$	Alumina is the most widely used of all technical ceramics (spark plugs, circuit boards...) All ceramics are brittle – they have low values of fracture toughness $K_{IC}$ and toughness $G_{IC}$ . They are, however, exceedingly hard, so are used as cutting tools.
Silicon nitride $Si_3N_4$	
Boron carbide, $B_4C$	
Silicon carbide, SiC	
Diamond	

**E3.5** Use the ( $E - \rho$ ) chart of Figure 3.3 to identify metals with both  $E > 100$  GPa and  $E/\rho > 0.02$  GPa/(kg/m<sup>3</sup>).

**Answer.** The chart shows the selection. The metals that lie in the search area are listed in the table.

Material	Comment
Steels	Cheap, widely used. Stiff structural material.
Nickel alloys	More expensive than steel
Titanium alloys	Titanium alloys are very expensive.



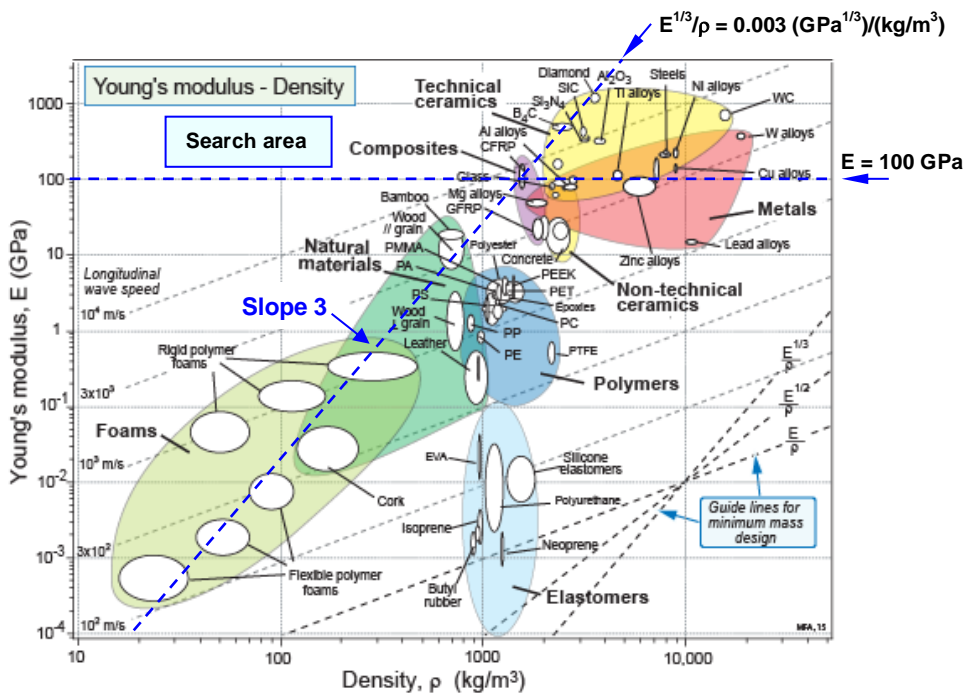
**E3.6.** Use the  $E/\rho$  chart of Figure 3.3 to identify materials with both  $E > 100$  GPa and  $E^{1/3}/\rho > 0.003$  (GPa) $^{1/3}/(\text{kg}/\text{m}^3)$ . Remember that, on taking logs, the index  $M = E^{1/3}/\rho$  becomes

$$\log(E) = 3 \log(\rho) + 3 \log(M)$$

and that this plots as a line of slope 3 on the chart, passing through the point  $E = 27$  when  $\rho = 1000$  in the units on the chart.

**Answer.** The chart shows the selection. The materials that lie in the search area are listed in the table. No metals survive.

Material	Comment
CFRP	Carbon-fiber composites excel in stiffness at low weight.
Boron carbide, $B_4C$	Boron carbide is exceptionally stiff, hard and light; it is used for body armour.



**E3.7** Use the  $E-\rho$  chart of Figure 3.3 to establish whether woods have a higher specific stiffness  $E/\rho$  than epoxies.

**Answer.** Parallel to the grain, woods have much higher specific stiffness than epoxies. Perpendicular to the grain, woods have about the same value as epoxies.

Material	$E/\rho$ (GPa)/(kg/m³) $\times 10^3$
Woods parallel to the grain	8 - 29
Woods transverse to the grain	0.7 - 4.0
Epoxies	1.8 - 2.5

**E3.8** Do titanium alloys have a higher or lower specific strength (strength/density,  $\sigma_f / \rho$ ) than the best steels? This is important when you want strength at low weight (landing gear of aircraft, mountain bikes). Use the  $\sigma_f / \rho$  chart of Figure 3.4 to decide.

**Answer.** The guide line for specific strength,  $\sigma_f / \rho$ , on the strength – density chart of Figure 3.4, if drawn through the middle of the bubble for titanium alloys, passes above the elongated bubble for steels. Thus titanium alloys have a higher specific strength than steels, even though the best steels are as strong.

Material	$\sigma_f / \rho$ (MPa/(kg/m <sup>3</sup> )) x 10 <sup>3</sup>
Titanium alloys	54 - 270
Steels	25 - 170

