Chapter 1

Overview of Aerodynamics

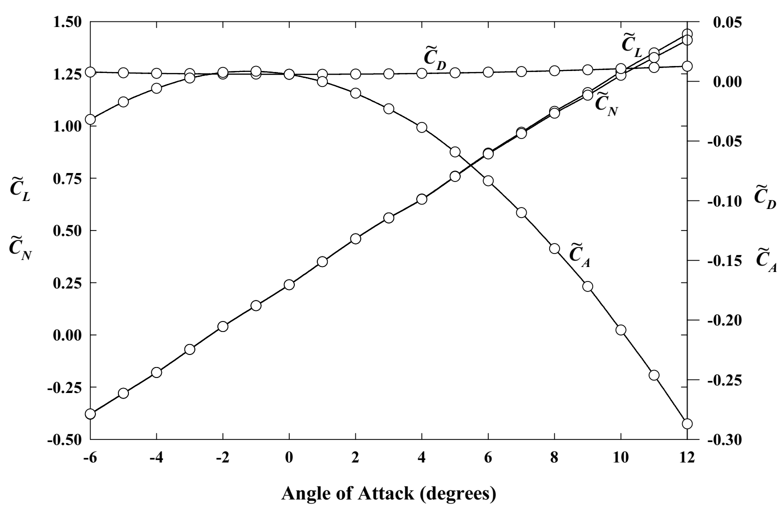
1.1. The given data are a tabulation of the section lift, drag, and quarter-chord moment coefficients taken from test data for a particular airfoil section. On one graph, plot both the section lift coefficient and the section normal force coefficient as a function of angle of attack. On another graph, plot both the section drag coefficient and the section axial force coefficient as a function of angle of attack.

Solution. The normal and axial force coefficients are found from

$$\tilde{C}_N = \tilde{C}_L \cos\alpha + \tilde{C}_D \sin\alpha \qquad \tilde{C}_A = \tilde{C}_D \cos\alpha - \tilde{C}_L \sin\alpha$$

Using these equations in a spread-sheet gives

α (degs)	$\tilde{C}_L$	$\tilde{C}_D$	$\tilde{C}_{m_{c/4}}$	$\tilde{C}_N$	$\tilde{C}_A$
−6.0	−0.38	0.0077	−0.0446	−0.37872	−0.032063
−5.0	−0.28	0.0071	−0.0440	−0.27955	−0.017331
−4.0	−0.18	0.0066	−0.0434	−0.18002	−0.005972
−3.0	−0.07	0.0063	−0.0428	−0.07023	0.002628
−2.0	0.04	0.0060	−0.0422	0.03977	0.007392
−1.0	0.14	0.0059	−0.0416	0.13988	0.008342
0.0	0.24	0.0058	−0.0410	0.24000	0.005800
1.0	0.35	0.0058	−0.0404	0.35005	−0.000309
2.0	0.46	0.0060	−0.0398	0.45993	−0.010057
3.0	0.56	0.0062	−0.0392	0.55956	−0.023117
4.0	0.65	0.0066	−0.0386	0.64888	−0.038758
5.0	0.76	0.0070	−0.0380	0.75772	−0.059265
6.0	0.87	0.0075	−0.0374	0.86602	−0.083481
7.0	0.97	0.0081	−0.0368	0.96376	−0.110174
8.0	1.07	0.0088	−0.0362	1.06081	−0.140201
9.0	1.16	0.0096	−0.0356	1.14722	−0.171982
10.0	1.26	0.0105	−0.0350	1.24268	−0.208456
11.0	1.35	0.0115	−0.0344	1.32739	−0.246303
12.0	1.44	0.0126	−0.0338	1.41115	−0.287068



Notice that there is little difference between the lift and normal force coefficients but the difference between the drag and axial force coefficients is large.

1.2. From the data presented in problem 1.1, on one graph, plot the location of the center of pressure, x_{cp}/c, and the aerodynamic center, x_{ac}/c, as a function of angle of attack.

Solution. The center of pressure is found from

$$\frac{x_{cp}}{c} = \frac{x}{c} - \frac{\widetilde{C}_{m_x}}{\widetilde{C}_N} = 0.25 - \frac{\widetilde{C}_{m_{c/4}}}{\widetilde{C}_N}$$

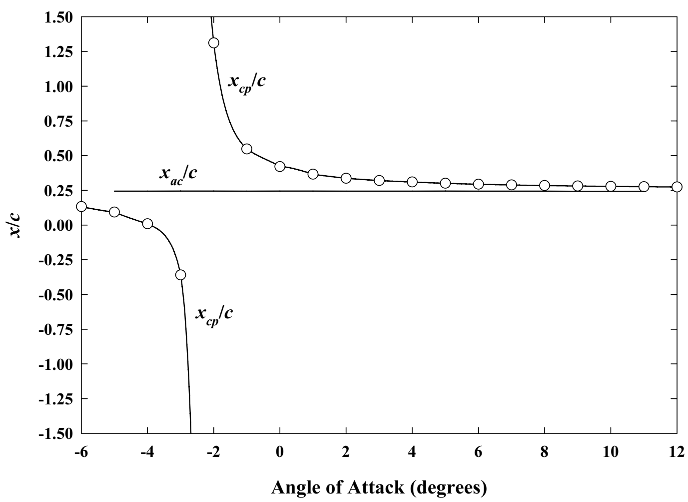
Using a central difference approximation for the aerodynamic derivatives, the aerodynamic center is found from

$$\frac{x_{ac}}{c} = \frac{x}{c} - \frac{\widetilde{C}_{m_x,\alpha}}{\widetilde{C}_{N,\alpha}} = 0.25 - \frac{\widetilde{C}_{m_{c/4},\alpha}}{\widetilde{C}_{N,\alpha}} \cong 0.25 - \frac{\widetilde{C}_{m_{c/4}}(\alpha+\Delta\alpha) - \widetilde{C}_{m_{c/4}}(\alpha-\Delta\alpha)}{2\Delta\alpha} \bigg/ \frac{\widetilde{C}_N(\alpha+\Delta\alpha) - \widetilde{C}_N(\alpha-\Delta\alpha)}{2\Delta\alpha}$$

$$= 0.25 - \frac{\widetilde{C}_{m_{c/4}}(\alpha+\Delta\alpha) - \widetilde{C}_{m_{c/4}}(\alpha-\Delta\alpha)}{\widetilde{C}_N(\alpha+\Delta\alpha) - \widetilde{C}_N(\alpha-\Delta\alpha)}$$

Using these equations in a spread-sheet gives

α (degs)	$\widetilde{C}_L$	$\widetilde{C}_D$	$\widetilde{C}_{m_{c/4}}$	$\widetilde{C}_N$	x_{cp}/c	x_{ac}/c
-6.0	-0.38	0.0077	-0.0446	-0.37872	0.13224	
-5.0	-0.28	0.0071	-0.0440	-0.27955	0.09261	0.24396
-4.0	-0.18	0.0066	-0.0434	-0.18002	0.00892	0.24427
-3.0	-0.07	0.0063	-0.0428	-0.07023	-0.35939	0.24454
-2.0	0.04	0.0060	-0.0422	0.03977	1.31120	0.24429
-1.0	0.14	0.0059	-0.0416	0.13988	0.54741	0.24401
0.0	0.24	0.0058	-0.0410	0.24000	0.42083	0.24429
1.0	0.35	0.0058	-0.0404	0.35005	0.36541	0.24454
2.0	0.46	0.0060	-0.0398	0.45993	0.33654	0.24427
3.0	0.56	0.0062	-0.0392	0.55956	0.32006	0.24365
4.0	0.65	0.0066	-0.0386	0.64888	0.30949	0.24394
5.0	0.76	0.0070	-0.0380	0.75772	0.30015	0.24447
6.0	0.87	0.0075	-0.0374	0.86602	0.29319	0.24418
7.0	0.97	0.0081	-0.0368	0.96376	0.28818	0.24384
8.0	1.07	0.0088	-0.0362	1.06081	0.28412	0.24346
9.0	1.16	0.0096	-0.0356	1.14722	0.28103	0.24340
10.0	1.26	0.0105	-0.0350	1.24268	0.27816	0.24334
11.0	1.35	0.0115	-0.0344	1.32739	0.27592	0.24288
12.0	1.44	0.0126	-0.0338	1.41115	0.27395	



1.3. As predicted from thin airfoil theory, the section lift and leading-edge moment coefficients for a NACA 2412 airfoil section are given by

$$\widetilde{C}_L = 2\pi(\alpha + 0.03625) \text{ and } \widetilde{C}_{m_{le}} = -\frac{\pi}{2}(\alpha + 0.07007)$$

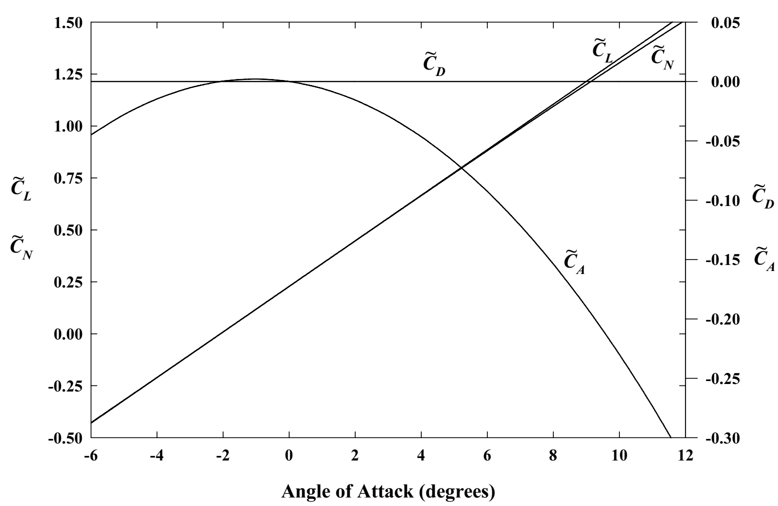
where α is in radians. On one graph, plot both the section lift and normal force coefficients as a function of angle of attack, from −6 to +12 degrees. On another graph, plot both the section drag and axial force coefficients as a function of angle of attack, from −6 to +12 degrees.

Solution. Thin airfoil theory predicts zero drag and the normal and axial force coefficients are found from

$$\widetilde{C}_N = \widetilde{C}_L \cos\alpha + \widetilde{C}_D \sin\alpha = 2\pi(\alpha + 0.03625)\cos\alpha$$
$$\widetilde{C}_A = \widetilde{C}_D \cos\alpha - \widetilde{C}_L \sin\alpha = -2\pi(\alpha + 0.03625)\sin\alpha$$

Using these equations in a spread-sheet gives

α (degs)	α (rad)	$\widetilde{C}_L$	$\widetilde{C}_D$	$\widetilde{C}_N$	$\widetilde{C}_A$
−6.0	−0.10472	−0.43021	0.000000	−0.42785	−0.044969
−5.0	−0.08727	−0.32055	0.000000	−0.31933	−0.027937
−4.0	−0.06981	−0.21088	0.000000	−0.21037	−0.014710
−3.0	−0.05236	−0.10122	0.000000	−0.10108	−0.005298
−2.0	−0.03491	0.00844	0.000000	0.00844	0.000295
−1.0	−0.01745	0.11810	0.000000	0.11809	0.002061
0.0	0.00000	0.22777	0.000000	0.22777	0.000000
1.0	0.01745	0.33743	0.000000	0.33738	−0.005889
2.0	0.03491	0.44709	0.000000	0.44682	−0.015603
3.0	0.05236	0.55675	0.000000	0.55599	−0.029138
4.0	0.06981	0.66641	0.000000	0.66479	−0.046487
5.0	0.08727	0.77608	0.000000	0.77312	−0.067640
6.0	0.10472	0.88574	0.000000	0.88089	−0.092585
7.0	0.12217	0.99540	0.000000	0.98798	−0.121309
8.0	0.13963	1.10506	0.000000	1.09431	−0.153795
9.0	0.15708	1.21473	0.000000	1.19977	−0.190025
10.0	0.17453	1.32439	0.000000	1.30427	−0.229978
11.0	0.19199	1.43405	0.000000	1.40770	−0.273630
12.0	0.20944	1.54371	0.000000	1.50998	−0.320956



1.4. From the thin airfoil relations for the airfoil section presented in problem 1.3, on one graph, plot the location of the center of pressure, x_{cp}/c, and the aerodynamic center, x_{ac}/c, as a function of angle of attack, from −6 to +12 degrees.

Solution. From problem 1.3

$$\widetilde{C}_L = 2\pi(\alpha + 0.03625) \text{ and } \widetilde{C}_{m_{le}} = -\frac{\pi}{2}(\alpha + 0.07007)$$
$$\widetilde{C}_N = \widetilde{C}_L \cos\alpha + \widetilde{C}_D \sin\alpha = 2\pi(\alpha + 0.03625)\cos\alpha$$

The center of pressure is found from

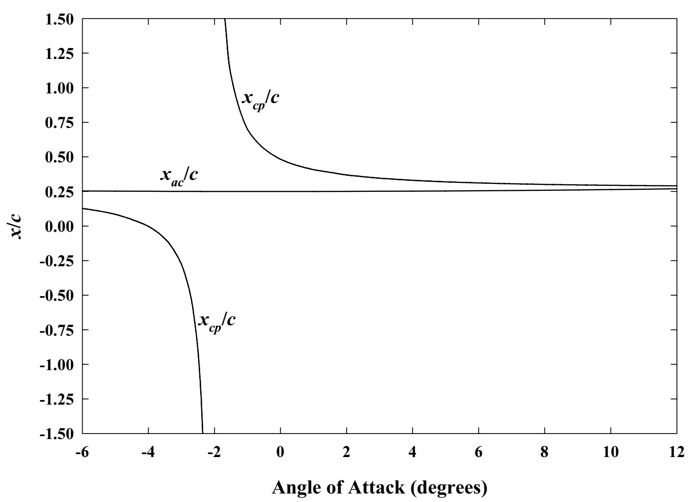
$$\frac{x_{cp}}{c} = \frac{x}{c} - \frac{\widetilde{C}_{m_x}}{\widetilde{C}_N} = 0.0 - \frac{\widetilde{C}_{m_{le}}}{\widetilde{C}_N} = -\frac{-(\pi/2)(\alpha + 0.07007)}{2\pi(\alpha + 0.03625)\cos\alpha} = \frac{(\alpha + 0.07007)}{4(\alpha + 0.03625)\cos\alpha}$$

The aerodynamic center is found from

$$\frac{x_{ac}}{c} = \frac{x}{c} - \frac{\widetilde{C}_{m_x,\alpha}}{\widetilde{C}_{N,\alpha}} = 0.0 - \frac{\widetilde{C}_{m_{le},\alpha}}{\widetilde{C}_{N,\alpha}} = -\frac{-(\pi/2)}{2\pi\cos\alpha - 2\pi(\alpha + 0.03625)\sin\alpha} = \frac{1}{4[\cos\alpha - (\alpha + 0.03625)\sin\alpha]}$$

Using these equations in a spread-sheet gives

α (degs)	α (rad)	x_{cp}/c	x_{ac}/c
-6.00	-0.10472	0.12721	0.25320
-5.00	-0.08727	0.08459	0.25208
-4.00	-0.06981	-0.00192	0.25120
-3.00	-0.05236	-0.27521	0.25055
-2.00	-0.03491	6.54765	0.25014
-1.00	-0.01745	0.69992	0.24996
0.00	0.00000	0.48324	0.25000
1.00	0.01745	0.40750	0.25027
2.00	0.03491	0.36905	0.25078
3.00	0.05236	0.34589	0.25151
4.00	0.06981	0.33052	0.25248
⋮	⋮	⋮	⋮
10.00	0.17453	0.29459	0.26366
11.00	0.19199	0.29242	0.26650
12.00	0.20944	0.29077	0.26967



1.5. Write a computer subroutine to compute the geopotential altitude, temperature, pressure, and air density as a function of geometric altitude for the standard atmosphere that is defined in Table 1.2.1. The only input variable should be the geometric altitude in meters. All output variables should be in standard SI units. Test this subroutine by comparing its output with the results that were obtained in Example 1.2.1 and those presented in Appendix A.

Solution. The following Fortran subroutine provides one possible solution. Beyond 79,000 m this subroutine assumes constant temperature, which is not according to the actual definition beyond 90,000 m.

```
      Subroutine statsi(h,z,t,p,d)
!     h = geometric altitude, specified by user (m)
!     z = geopotential altitude, returned by subroutine (m)
!     t = temperature, returned by subroutine (K)
!     p = pressure, returned by subroutine (N/m**2)
!     d = density, returned by subroutine (kg/m**3)
      real Lt,zsa(0:8),Tsa(0:8),Psa(0:8)
      data zsa/    0.,11000.,20000.,32000.,47000.,52000.,61000.,79000.,9.9e20/
      data Tsa/288.15,216.65,216.65,228.65,270.65,270.65,252.65,180.65,180.65/
      data g0,R,Re,Psa(0)/9.80665,287.0528,6356766.,101325./
      z=Re*h/(Re+h)
      do i=1,8
         Lt=-(Tsa(i)-Tsa(i-1))/(zsa(i)-zsa(i-1))
         if(Lt.eq.0.)then
            if(z.le.zsa(i))then
               t=Tsa(i-1)
               p=Psa(i-1)*exp(-g0*(z-zsa(i-1))/R/Tsa(i-1))
               d=p/R/t
               return
            else
               Psa(i)=Psa(i-1)*exp(-g0*(zsa(i)-zsa(i-1))/R/Tsa(i-1))
            endif
         else
            ex=g0/R/Lt
            if(z.lt.zsa(i))then
               t=Tsa(i-1)-Lt*(z-zsa(i-1))
               p=Psa(i-1)*(t/Tsa(i-1))**ex
               d=p/R/t
               return
            else
               Psa(i)=Psa(i-1)*(Tsa(i)/Tsa(i-1))**ex
            endif
         endif
      end do
      t=Tsa(8)
      p=0.0
      d=0.0
      return
      end
```

1.6. Write a computer subroutine to compute the geopotential altitude, temperature, pressure, and air density as a function of geometric altitude for the standard atmosphere that is defined in Table 1.2.1. The only input variable should be the geometric altitude in feet. All output variables should be in English engineering units. Test the subroutine by comparison with Example 1.2.2 and the results presented in Appendix B. Since this standard atmosphere is defined in SI units, this subroutine could be written using simple unit conversion and a single call to the subroutine written for problem 1.5.

Solution. The following Fortran subroutine provides one possible solution.

```
      Subroutine statee(h,z,t,p,d)
c     h = geometric altitude, specified by user (ft)
c     z = geopotential altitude, returned by subroutine (ft)
c     t = temperature, returned by subroutine (R)
c     p = pressure, returned by subroutine (lbf/ft**2)
c     d = density, returned by subroutine (slugs/ft**3)
      hsi=h*0.3048
      call statsi(hsi,zsi,tsi,psi,dsi)
      z=zsi/0.3048
      t=tsi*1.8
      p=psi*0.02088543
      d=dsi*0.001940320
      return
      end
```

1.7. Show that the velocity distribution for the 2-D source flow, which is expressed in Eq. (1.5.4), is irrotational.

Solution. For this flow

$$\mathbf{V} = \frac{\Lambda}{2\pi r}\mathbf{i}_r$$

In cylindrical coordinates the vorticity is given by

$$\nabla\times\mathbf{V} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ \frac{\Lambda}{2\pi r} & 0 & 0 \end{vmatrix} = 0$$

1.8. Show that the velocity distribution for the 2-D vortex flow, which is expressed in Eq. (1.5.6), is irrotational.

Solution. For this flow

$$\mathbf{V} = -\frac{\Gamma}{2\pi r}\mathbf{i}_\theta$$

In cylindrical coordinates the vorticity is given by

$$\nabla\times\mathbf{V} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ 0 & -\frac{\Gamma}{2\pi} & 0 \end{vmatrix} = 0$$

1.9. Show that the velocity distribution for the 2-D doublet flow, which is expressed in Eq. (1.5.8), is irrotational.

Solution. For this flow

$$\mathbf{V} = -\frac{\kappa\cos\theta}{2\pi r^2}\mathbf{i}_r - \frac{\kappa\sin\theta}{2\pi r^2}\mathbf{i}_\theta$$

In cylindrical coordinates the vorticity is given by

$$\nabla\times\mathbf{V} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix} = \begin{vmatrix} \frac{1}{r}\mathbf{i}_r & \mathbf{i}_\theta & \frac{1}{r}\mathbf{i}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ -\frac{\kappa\cos\theta}{2\pi r^2} & -\frac{\kappa\sin\theta}{2\pi r} & 0 \end{vmatrix} = \left(\frac{\kappa\sin\theta}{2\pi r^2} - \frac{\kappa\sin\theta}{2\pi r^2}\right)\frac{1}{r}\mathbf{i}_z = 0$$

1.10. Show that the velocity distribution for the 2-D stagnation flow, which is expressed in Eq. (1.5.10), is irrotational.

Solution. For this flow

$$\mathbf{V} = Bx\mathbf{i}_x - By\mathbf{i}_y$$

In Cartesian coordinates the vorticity is given by

$$\nabla\times\mathbf{V} = \begin{vmatrix} \mathbf{i}_x & \mathbf{i}_y & \mathbf{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} = \begin{vmatrix} \mathbf{i}_x & \mathbf{i}_y & \mathbf{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Bx & -By & 0 \end{vmatrix} = 0$$

1.11. Show that the velocity distribution for the 2-D vortex flow, which is expressed in Eq. (1.5.6), satisfies the incompressible continuity equation.

Solution. For this flow

$$\mathbf{V} = -\frac{\Gamma}{2\pi r}\mathbf{i}_\theta$$

The incompressible continuity equation is given by

$$\nabla\cdot\mathbf{V} = 0$$

In cylindrical coordinates the divergence is given by

$$\nabla\cdot\mathbf{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(V_\theta) + \frac{\partial}{\partial z}(V_z) = \frac{1}{r}\frac{\partial}{\partial r}(0) + \frac{1}{r}\frac{\partial}{\partial\theta}\left(-\frac{\Gamma}{2\pi r}\right) + \frac{\partial}{\partial z}(0) = 0$$

1.12. The section geometry for a NACA 4-digit series airfoil is completely fixed by the maximum camber, the location of maximum camber, the maximum thickness, and the chord length. The first digit indicates the maximum camber in percent of chord. The second digit indicates the distance from the leading edge to the point of maximum camber in tenths of the chord. The last two digits indicate the maximum thickness in percent of chord. The camber line is given by

$$y_c(x) = \begin{cases} y_{mc}\left[2\left(\dfrac{x}{x_{mc}}\right)-\left(\dfrac{x}{x_{mc}}\right)^2\right], & 0 \le x \le x_{mc} \\ y_{mc}\left[2\left(\dfrac{c-x}{c-x_{mc}}\right)-\left(\dfrac{c-x}{c-x_{mc}}\right)^2\right], & x_{mc} \le x \le c \end{cases}$$

where y_{mc} is the maximum camber and x_{mc} is the position of maximum camber. The thickness about the camber line, measured perpendicular to the camber line itself, is given by

$$t(x) = t_m\left[2.969\sqrt{\frac{x}{c}} - 1.260\left(\frac{x}{c}\right) - 3.516\left(\frac{x}{c}\right)^2 + 2.843\left(\frac{x}{c}\right)^3 - 1.015\left(\frac{x}{c}\right)^4\right]$$

where t_m is the maximum thickness. For the NACA 2412 airfoil section, determine x/c and y/c for the camber line, upper surface, and lower surface at chordwise positions of $x/c = 0.10$.

Solution. The NACA 2412 airfoil section has 2% maximum camber located at 40% chord. We wish to determine the dimensionless coordinates of the camber line at 10% chord. Thus, we use the camber line definition for $0 \le x \le x_{mc}$, which gives

$$\frac{x_c}{c} = \frac{x}{c} = 0.10$$

$$\frac{y_c}{c} = \frac{y_{mc}}{c}\left[2\left(\frac{x/c}{x_{mc}/c}\right)-\left(\frac{x/c}{x_{mc}/c}\right)^2\right] = 0.02\left[2\left(\frac{0.10}{0.40}\right)-\left(\frac{0.10}{0.40}\right)^2\right] = 0.00875$$

Similarly, the dimensionless thickness at 10% chord is

$$\begin{aligned}\frac{t}{c} &= \frac{t_m}{c}\left[2.969\sqrt{\frac{x}{c}} - 1.260\left(\frac{x}{c}\right) - 3.516\left(\frac{x}{c}\right)^2 + 2.843\left(\frac{x}{c}\right)^3 - 1.015\left(\frac{x}{c}\right)^4\right] \\ &= 0.12\left[2.969\sqrt{0.1} - 1.260(0.1) - 3.516(0.1)^2 + 2.843(0.1)^3 - 1.015(0.1)^4\right] \\ &= 0.0936554\end{aligned}$$

From the camber line definition, the camber line slope at 10% chord is

$$\frac{dy_c}{dx} = y_{mc}\left[2\left(\frac{1}{x_{mc}}\right)-2\left(\frac{x}{x_{mc}^2}\right)\right] = 2\frac{y_{mc}/c}{x_{mc}/c}\left[1-\left(\frac{x/c}{x_{mc}/c}\right)\right] = 2\frac{0.02}{0.40}\left[1-\left(\frac{0.10}{0.40}\right)\right] = 0.075$$

Using Eqs. (1.6.1) through (1.6.4), the dimensionless coordinates of the upper and lower surfaces at 10% chord are

$$\frac{x_u}{c} = \frac{x}{c} - \frac{t/c}{2\sqrt{1+(dy_c/dx)^2}}\frac{dy_c}{dx} = 0.10 - \frac{0.0936554}{2\sqrt{1+(0.075)^2}}0.075 = 0.096498$$

$$\frac{y_u}{c} = \frac{y_c}{c} + \frac{t/c}{2\sqrt{1+(dy_c/dx)^2}} = 0.00875 + \frac{0.0936554}{2\sqrt{1+(0.075)^2}} = 0.055447$$

$$\frac{x_\ell}{c} = \frac{x}{c} + \frac{t/c}{2\sqrt{1+(dy_c/dx)^2}}\frac{dy_c}{dx} = 0.10 + \frac{0.0936554}{2\sqrt{1+(0.075)^2}}0.075 = 0.103502$$

$$\frac{y_\ell}{c} = \frac{y_c}{c} - \frac{t/c}{2\sqrt{1+(dy_c/dx)^2}} = 0.00875 - \frac{0.0936554}{2\sqrt{1+(0.075)^2}} = -0.037947$$

1.13. Using the formulas given in problem 1.12, for the NACA 0012 airfoil section, determine x/c and y/c of the upper and lower surfaces at chordwise positions of x/c = 0.00, 0.001, 0.10, 0.40, 0.50, 0.90, and 1.00.

Solution. Following the procedure used in the solution to problem 1.12 results in

x_c	y_c	x_u	y_u	x_ℓ	y_ℓ
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.001000	0.000000	0.001000	0.005557	0.001000	−0.005557
0.100000	0.000000	0.100000	0.046828	0.100000	−0.046828
0.400000	0.000000	0.400000	0.058030	0.400000	−0.058030
0.500000	0.000000	0.500000	0.052940	0.500000	−0.052940
0.900000	0.000000	0.900000	0.014477	0.900000	−0.014477
1.000000	0.000000	1.000000	0.001260	1.000000	−0.001260

1.14. Using the formulas given in problem 1.12, for the NACA 4421 airfoil section, determine x/c and y/c of the upper and lower surfaces at chordwise positions of x/c = 0.00, 0.001, 0.10, 0.40, 0.50, 0.90, and 1.00.

Solution. Following the procedure used in the solution to problem 1.12 results in

x_c	y_c	x_u	y_u	x_ℓ	y_ℓ
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.001000	0.000200	−0.000903	0.009737	0.002903	−0.009338
0.100000	0.017500	0.087844	0.098542	0.112156	−0.063542
0.400000	0.040000	0.400000	0.141553	0.400000	−0.061553
0.500000	0.038889	0.502058	0.131511	0.497942	−0.053734
0.900000	0.012222	0.902798	0.037402	0.897202	−0.012958
1.000000	0.000000	1.000291	0.002186	0.999709	−0.002186

1.15. Using the formulas given in problem 1.12, for the NACA 0021 airfoil section, determine the angle of the upper surface relative to the chord line in degrees at a chordwise position of $x/c = 0.10$.

Solution. The slope of the upper surface is

$$\frac{dy_u}{dx_u} = \frac{dy_u}{dx} \bigg/ \frac{dx_u}{dx} \tag{1}$$

From Eqs. (1.6.1) and (1.6.2), for this symmetric airfoil with $y_c = 0$ we have

$$\frac{dx_u}{dx} = \frac{d}{dx}\left[x - \frac{t}{2\sqrt{1+(dy_c/dx)^2}}\frac{dy_c}{dx}\right] = \frac{dx}{dx} = 1 \tag{2}$$

$$\frac{dy_u}{dx} = \frac{d}{dx}\left[y_c + \frac{t}{2\sqrt{1+(dy_c/dx)^2}}\right] = \frac{1}{2}\frac{dt}{dx} \tag{3}$$

From the thickness definition in problem 1.12, with $x/c = 0.1$ and $t_m/c = 0.21$

$$\begin{aligned}
\frac{dt}{dx} &= t_m \frac{d}{dx}\left[2.969\sqrt{\frac{x}{c}} - 1.260\left(\frac{x}{c}\right) - 3.516\left(\frac{x}{c}\right)^2 + 2.843\left(\frac{x}{c}\right)^3 - 1.015\left(\frac{x}{c}\right)^4\right] \\
&= \frac{t_m}{c}\left[\frac{2.969}{2}\frac{1}{\sqrt{x/c}} - 1.260 - 2(3.516)\frac{x}{c} + 3(2.843)\left(\frac{x}{c}\right)^2 - 4(1.015)\left(\frac{x}{c}\right)^3\right] \\
&= 0.21\left[\frac{2.969}{2}\frac{1}{\sqrt{0.1}} - 1.260 - 2(3.516)\,0.1 + 3(2.843)(0.1)^2 - 4(1.015)(0.1)^3\right] = 0.59061
\end{aligned}$$

Using Eqs. (2) and (3) in Eq. (1) gives

$$\frac{dy_u}{dx_u} = \frac{1}{2}\frac{dt}{dx}$$

and

$$\theta_u = \tan^{-1}\left(\frac{dy_u}{dx_u}\right) = \tan^{-1}\left(\frac{1}{2}\frac{dt}{dx}\right) = \tan^{-1}\left(\frac{0.59061}{2}\right) = 0.28714 = 16.45^\circ$$

1.16. Using the formulas given in problem 1.12, for the NACA 4421 airfoil section, determine the angle of the upper surface relative to the chord line in degrees at a chordwise position of $x/c = 0.10$.

Solution. The slope of the upper surface is

$$\frac{dy_u}{dx_u} = \frac{dy_u}{dx} \bigg/ \frac{dx_u}{dx} \tag{1}$$

From Eqs. (1.6.1) and (1.6.2), for this cambered airfoil we have

$$
\begin{aligned}
\frac{\partial x_u}{\partial x} &= \frac{\partial}{\partial x}\left[x - \frac{t}{2\sqrt{1+(dy_c/dx)^2}}\frac{dy_c}{dx}\right]\\
&= 1 - \frac{1}{2\sqrt{1+(dy_c/dx)^2}}\left(\frac{dy_c}{dx}\frac{dt}{dx} + t\frac{d^2y_c}{dx^2}\right) + \frac{1}{2}\frac{t}{2\left[1+(dy_c/dx)^2\right]^{3/2}}2\left(\frac{dy_c}{dx}\right)^2\frac{d^2y_c}{dx^2}\\
&= \frac{1}{2\sqrt{1+(dy_c/dx)^2}}\left[2\sqrt{1+(dy_c/dx)^2} - \frac{dy_c}{dx}\frac{dt}{dx} - \frac{1-(dy_c/dx)^2}{1+(dy_c/dx)^2}t\frac{d^2y_c}{dx^2}\right]
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
\frac{\partial y_u}{\partial x} &= \frac{\partial}{\partial x}\left[y_c + \frac{t}{2\sqrt{1+(dy_c/dx)^2}}\right] = \frac{\partial y_c}{\partial x} + \frac{1}{2\sqrt{1+(dy_c/dx)^2}}\frac{\partial t}{\partial x} - \frac{1}{2}\frac{t}{2\left[1+(dy_c/dx)^2\right]^{3/2}}2\frac{\partial y_c}{\partial x}\frac{\partial^2 y_c}{\partial x^2}\\
&= \frac{1}{2\sqrt{1+(dy_c/dx)^2}}\left[\frac{\partial y_c}{\partial x}2\sqrt{1+(dy_c/dx)^2} + \frac{\partial t}{\partial x} - \frac{dy_c/dx}{1+(dy_c/dx)^2}t\frac{\partial^2 y_c}{\partial x^2}\right]
\end{aligned}
\tag{3}
$$

From the camber line definition in problem 1.12, with $x/c = 0.1$, $x_{mc}/c = 0.4$, and $y_{mc}/c = 0.04$

$$
\frac{dy_c}{dx} = y_{mc}\frac{d}{dx}\left[2\left(\frac{x}{x_{mc}}\right) - \left(\frac{x}{x_{mc}}\right)^2\right] = \frac{y_{mc}}{x_{mc}}\left(2 - 2\frac{x}{x_{mc}}\right) = 2\frac{y_{mc}/c}{x_{mc}/c}\left(1 - \frac{x/c}{x_{mc}/c}\right) = 2\frac{0.04}{0.4}\left(1 - \frac{0.1}{0.4}\right) = 0.15
$$

$$
\frac{d^2y_c}{dx^2} = y_{mc}\frac{d^2}{dx^2}\left[2\left(\frac{x}{x_{mc}}\right) - \left(\frac{x}{x_{mc}}\right)^2\right] = -2\frac{y_{mc}}{x_{mc}^2} = -2\frac{y_{mc}/c}{(x_{mc}/c)^2}\frac{1}{c} = -2\frac{0.04}{(0.4)^2}\frac{1}{c} = -0.5\frac{1}{c}
$$

From the thickness definition in problem 1.12, with $x/c = 0.1$ and $t_m/c = 0.21$

$$
t = \frac{t_m}{c}\left[2.969\sqrt{\frac{x}{c}} - 1.260\left(\frac{x}{c}\right) - 3.516\left(\frac{x}{c}\right)^2 + 2.843\left(\frac{x}{c}\right)^3 - 1.015\left(\frac{x}{c}\right)^4\right]c = 0.16390\,c
$$

$$
\frac{dt}{dx} = \frac{t_m}{c}\left[\frac{2.969}{2}\frac{1}{\sqrt{x/c}} - 1.260 - 2(3.516)\frac{x}{c} + 3(2.843)\left(\frac{x}{c}\right)^2 - 4(1.015)\left(\frac{x}{c}\right)^3\right] = 0.59061
$$

Using Eqs. (2) and (3) in Eq. (1) gives

$$
\frac{dy_u}{dx_u} = \frac{\dfrac{\partial y_c}{\partial x}2\sqrt{1+(dy_c/dx)^2} + \dfrac{\partial t}{\partial x} - \dfrac{dy_c/dx}{1+(dy_c/dx)^2}t\dfrac{\partial^2 y_c}{\partial x^2}}{2\sqrt{1+(dy_c/dx)^2} - \dfrac{dy_c}{dx}\dfrac{dt}{dx} - \dfrac{1-(dy_c/dx)^2}{1+(dy_c/dx)^2}t\dfrac{d^2y_c}{dx^2}} = 0.45026
$$

$$
\theta_u = \tan^{-1}\left(\frac{dy_u}{dx_u}\right) = \tan^{-1}(0.45026) = 0.42307 = 24.24°
$$

This can be closely approximated as

$$
\theta_u \cong \tan^{-1}\left(\frac{1}{2}\frac{dt}{dx} + \frac{dy_c}{dx}\right) = \tan^{-1}\left(\frac{0.59061}{2} + 0.15\right) = 0.41984 = 24.00°
$$

1.17. The formulas for the geometry of a NACA 4-digit series airfoil are given in problem 1.12. For the NACA 2412 airfoil section, use thin airfoil theory to determine the zero-lift angle of attack, the lift coefficients at 0- and 5-degree angles of attack, and the quarter-chord moment coefficient. Also plot the center of pressure as a function of angle of attack from −5 to 10 degrees.

Solution. From the camber line definition in problem 1.12, with $x_{mc}/c = 0.4$ and $y_{mc}/c = 0.02$

$$\frac{y_c}{c} = \begin{cases} \dfrac{1}{10}\left(\dfrac{x}{c}\right) - \dfrac{1}{8}\left(\dfrac{x}{c}\right)^2, & 0.0 \le \dfrac{x}{c} \le 0.4 \\ \dfrac{1}{15}\left(1-\dfrac{x}{c}\right) - \dfrac{1}{18}\left(1-\dfrac{x}{c}\right)^2, & 0.4 \le \dfrac{x}{c} \le 1.0 \end{cases}$$

Differentiating the given camber line equation, the camber line slope is

$$\frac{dy_c}{dx} = \begin{cases} \dfrac{1}{10} - \dfrac{1}{4}\dfrac{x}{c}, & 0.0 \le \dfrac{x}{c} \le 0.4 \\ \dfrac{2}{45} - \dfrac{1}{9}\dfrac{x}{c}, & 0.4 \le \dfrac{x}{c} \le 1.0 \end{cases}$$

Using the change of variables

$$\frac{x}{c} \equiv \frac{1}{2}(1-\cos\theta)$$

we have

$$\frac{dy_c}{dx} = \begin{cases} \dfrac{1}{10} - \dfrac{1}{8}(1-\cos\theta), & 0.0 \le (1-\cos\theta) \le 0.8 \\ \dfrac{2}{45} - \dfrac{1}{18}(1-\cos\theta), & 0.8 \le (1-\cos\theta) \le 2.0 \end{cases}$$

From the definition of θ, we can write

$$\theta \equiv \cos^{-1}\left(1 - 2\frac{x}{c}\right)$$

$$\theta_{mc} = \cos^{-1}\left(1 - 2\frac{x_{mc}}{c}\right) = \cos^{-1}(1-0.8) = 1.3694384$$

$$\frac{dy_c}{dx} = \begin{cases} \dfrac{1}{8}\cos\theta - \dfrac{1}{40}, & 0 \le \theta \le 1.3694384 \\ \dfrac{1}{18}\cos\theta - \dfrac{1}{90}, & 1.3694384 \le \theta \le \pi \end{cases}$$

The first three coefficients in the series solution are then

$$\begin{aligned} A_0 &= \alpha - \frac{1}{\pi}\int_{\theta=0}^{\pi} \frac{dy_c}{dx}\,d\theta = \alpha - \frac{1}{\pi}\left[\int_0^{1.3694384}\left(\frac{1}{8}\cos\theta - \frac{1}{40}\right)d\theta + \int_{1.3694384}^{\pi}\left(\frac{1}{18}\cos\theta - \frac{1}{90}\right)d\theta\right] \\ &= \alpha - \frac{1}{\pi}\left[\left(\frac{\sin\theta}{8} - \frac{\theta}{40}\right)_0^{1.3694384} + \left(\frac{\sin\theta}{18} - \frac{\theta}{90}\right)_{1.3694384}^{\pi}\right] \\ &= \alpha - \frac{1}{\pi}\left[\frac{\sin(1.3694384)}{8} - \frac{1.3694384}{40} - \frac{\pi}{90} - \frac{\sin(1.3694384)}{18} + \frac{1.3694384}{90}\right] \\ &= \alpha - 0.004492886 \end{aligned}$$

$$
\begin{aligned}
A_1 &= \frac{2}{\pi}\int_{\theta=0}^{\pi}\frac{dy_c}{dx}\cos\theta\,d\theta\\
&= \frac{2}{\pi}\left[\int_0^{1.3694384}\left(\frac{\cos^2\theta}{8}-\frac{\cos\theta}{40}\right)d\theta+\int_{1.3694384}^{\pi}\left(\frac{\cos^2\theta}{18}-\frac{\cos\theta}{90}\right)d\theta\right]\\
&= \frac{2}{\pi}\left[\left(\frac{\sin\theta\cos\theta+\theta}{16}-\frac{\sin\theta}{40}\right)_0^{1.3694384}+\left(\frac{\sin\theta\cos\theta+\theta}{36}-\frac{\sin\theta}{90}\right)_{1.3694384}^{\pi}\right]\\
&= \frac{2}{\pi}\left[\frac{\sin(1.3694384)\cos(1.3694384)+1.3694384}{16}-\frac{\sin(1.3694384)}{40}\right.\\
&\qquad\left.+\frac{\pi}{36}-\frac{\sin(1.3694384)\cos(1.3694384)+1.3694384}{36}+\frac{\sin(1.3694384)}{90}\right]\\
&= 0.08149514
\end{aligned}
$$

$$
\begin{aligned}
A_2 &= \frac{2}{\pi}\int_{\theta=0}^{\pi}\frac{dy_c}{dx}\cos(2\theta)\,d\theta\\
&= \frac{2}{\pi}\left[\int_0^{1.3694384}\left(\frac{1}{8}\cos\theta-\frac{1}{40}\right)\cos(2\theta)\,d\theta+\int_{1.3694384}^{\pi}\left(\frac{1}{18}\cos\theta-\frac{1}{90}\right)\cos(2\theta)\,d\theta\right]\\
&= \frac{2}{\pi}\left[\int_0^{1.3694384}\left(\frac{1}{8}\cos\theta-\frac{1}{40}\right)(2\cos^2\theta-1)\,d\theta+\int_{1.3694384}^{\pi}\left(\frac{1}{18}\cos\theta-\frac{1}{90}\right)(2\cos^2\theta-1)\,d\theta\right]\\
&= \frac{2}{\pi}\left[\int_0^{1.3694384}\left(\frac{\cos^3\theta}{4}-\frac{\cos^2\theta}{20}-\frac{\cos\theta}{8}+\frac{1}{40}\right)d\theta\right.\\
&\qquad\left.+\int_{1.3694384}^{\pi}\left(\frac{\cos^3\theta}{9}-\frac{\cos^2\theta}{45}-\frac{\cos\theta}{18}+\frac{1}{90}\right)d\theta\right]\\
&= \frac{2}{\pi}\left[\left(\frac{\sin\theta(\cos^2\theta+2)}{12}-\frac{\sin\theta\cos\theta+\theta}{40}-\frac{\sin\theta}{8}+\frac{\theta}{40}\right)_0^{1.3694384}\right.\\
&\qquad\left.+\left(\frac{\sin\theta(\cos^2\theta+2)}{27}-\frac{\sin\theta\cos\theta+\theta}{90}-\frac{\sin\theta}{18}+\frac{\theta}{90}\right)_{1.3694384}^{\pi}\right]\\
&= \frac{2}{\pi}\left[\frac{\sin(1.3694384)[\cos^2(1.3694384)+2]}{12}\right.\\
&\qquad-\frac{\sin(1.3694384)\cos(1.3694384)}{40}-\frac{\sin(1.3694384)}{8}\\
&\qquad-\frac{\sin(1.3694384)[\cos^2(1.3694384)+2]}{27}\\
&\qquad\left.+\frac{\sin(1.3694384)\cos(1.3694384)}{90}+\frac{\sin(1.3694384)}{18}\right] = 0.01386128
\end{aligned}
$$

The section lift coefficient is

$$\widetilde{C}_L = 2\pi(A_0+\tfrac{1}{2}A_1) = 2\pi(\alpha+0.036254684) = 2\pi(\alpha-\alpha_{L0})$$

Thus, the zero-lift angle of attack for this airfoil is

$$\alpha_{L0} \;=\; -0.036254684 \text{ rad} \;=\; -2.077°$$

The section lift coefficient at a 0-degree angle of attack is

$$\widetilde{C}_L \;=\; 2\pi(\alpha-\alpha_{L0}) \;=\; 2\pi[0-(-0.036254684)] \;=\; 0.2278$$

The section lift coefficient at a 5-degree angle of attack is

$$\widetilde{C}_L \;=\; 2\pi(\alpha-\alpha_{L0}) \;=\; 2\pi[5\pi/180-(-0.036254684)] \;=\; 0.7761$$

The quarter-chord moment coefficient at any angle of attack is

$$\widetilde{C}_{m_{c/4}} \;=\; \frac{\pi}{4}(A_2-A_1) \;=\; \frac{\pi}{4}(0.01386128-0.08149514) \;=\; -0.05312$$

The center of pressure is determined from

$$\begin{aligned}\frac{x_{cp}}{c} &= \frac{1}{4}+\frac{\pi}{4\widetilde{C}_L}(A_1-A_2) \;=\; \frac{1}{4}+\frac{\pi}{8\pi(\alpha-\alpha_{L0})}(A_1-A_2)\\ &= \frac{1}{4}+\frac{0.08149514-0.01386128}{8[\alpha-(-0.036254684)]}\\ &= \frac{1}{4}+\frac{0.0084542325}{\alpha+0.036254684}\end{aligned}$$

From this equation, the center of pressure as a function of angle of attack is shown in Fig. 1.17. Notice that, for this cambered airfoil, the center of pressure moves dramatically with angle of attack, particularly in the region near the zero lift angle of attack. The result shown in this figure is typical of center of pressure variation with angle of attack for all cambered airfoils. For this reason, center of pressure is almost never used when working with cambered airfoils. For symmetric airfoils, however, the center of pressure remains fixed at the quarter-chord, independent of angle of attack.

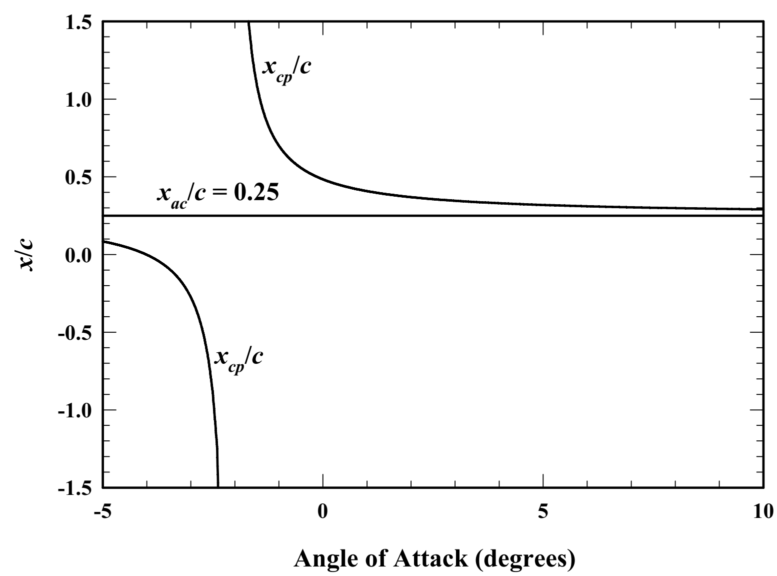


Figure 1.17 Variation in center of pressure with angle of attack for a NACA 2412 airfoil, as predicted by thin airfoil theory.

1.18. The formulas for the geometry of a NACA 4-digit series airfoil are given in problem 1.12. Using thin airfoil theory, for the NACA 4412 airfoil section, determine the zero-lift angle of attack, the lift coefficients at 0- and 5-degree angles of attack, the quarter-chord moment coefficient, and the center of pressure at the 0- and 5-degree angles of attack.

Solution. From the camber line definition in problem 1.12, with $x_{mc}/c = 0.4$ and $y_{mc}/c = 0.04$

$$\frac{y_c}{c} = \begin{cases} \frac{1}{5}\left(\frac{x}{c}\right) - \frac{1}{4}\left(\frac{x}{c}\right)^2, & 0.0 \le \frac{x}{c} \le 0.4 \\ \frac{2}{15}\left(1-\frac{x}{c}\right) - \frac{1}{9}\left(1-\frac{x}{c}\right)^2, & 0.4 \le \frac{x}{c} \le 1.0 \end{cases}$$

Differentiating the given camber line equation, the camber line slope is

$$\frac{dy_c}{dx} = \begin{cases} \frac{1}{5} - \frac{1}{2}\frac{x}{c}, & 0.0 \le \frac{x}{c} \le 0.4 \\ \frac{4}{45} - \frac{2}{9}\frac{x}{c}, & 0.4 \le \frac{x}{c} \le 1.0 \end{cases}$$

Using the change of variables

$$\frac{x}{c} \equiv \frac{1}{2}(1-\cos\theta)$$

we have

$$\frac{dy_c}{dx} = \begin{cases} \frac{1}{5} - \frac{1}{4}(1-\cos\theta), & 0.0 \le (1-\cos\theta) \le 0.8 \\ \frac{4}{45} - \frac{1}{9}(1-\cos\theta), & 0.8 \le (1-\cos\theta) \le 2.0 \end{cases}$$

From the definition of θ, we can write

$$\theta \equiv \cos^{-1}\left(1 - 2\frac{x}{c}\right)$$

$$\theta_{mc} = \cos^{-1}\left(1 - 2\frac{x_{mc}}{c}\right) = \cos^{-1}(1-0.8) = 1.3694384$$

$$\frac{dy_c}{dx} = \begin{cases} \frac{1}{4}\cos\theta - \frac{1}{20}, & 0 \le \theta \le 1.3694384 \\ \frac{1}{9}\cos\theta - \frac{1}{45}, & 1.3694384 \le \theta \le \pi \end{cases}$$

The first three coefficients in the series solution are then

$$\begin{aligned} A_0 &= \alpha - \frac{1}{\pi}\int_{\theta=0}^{\pi} \frac{dy_c}{dx}\, d\theta = \alpha - \frac{1}{\pi}\left[\int_0^{1.3694384}\left(\frac{1}{4}\cos\theta - \frac{1}{20}\right)d\theta + \int_{1.3694384}^{\pi}\left(\frac{1}{9}\cos\theta - \frac{1}{45}\right)d\theta\right] \\ &= \alpha - \frac{1}{\pi}\left[\left(\frac{\sin\theta}{4} - \frac{\theta}{20}\right)_0^{1.3694384} + \left(\frac{\sin\theta}{9} - \frac{\theta}{45}\right)_{1.3694384}^{\pi}\right] \\ &= \alpha - \frac{1}{\pi}\left[\frac{\sin(1.3694384)}{4} - \frac{1.3694384}{20} - \frac{\pi}{45} - \frac{\sin(1.3694384)}{9} + \frac{1.3694384}{45}\right] \\ &= \alpha - 0.00898577 \end{aligned}$$

