Chapter 2
Electrical Components and Circuits

Node/Mesh Method

Q2.1 Determine the voltage, v across R_5 in Figure 2.18 when

$V_{S1} = 4\,V; \quad V_{S2} = 2\,V;$

$R_1 = 2k\Omega; \quad R_2 = 4k\Omega; \quad R_3 = 4k\Omega; \quad R_4 = 2k\Omega; \quad R_5 = 6k\Omega; \quad R_6 = 2k\Omega;,$

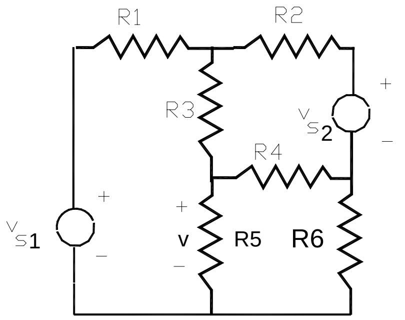
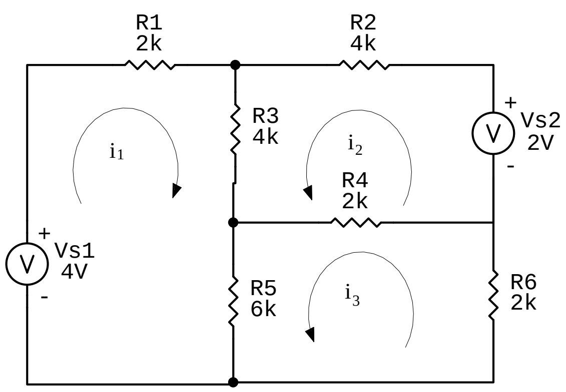


Figure 2.18

Solution: Mesh Analysis



Mesh 1

$Vs_1 = i_1 (R_1 + R_3 + R_5) + i_2 (R_3) + i_3 (R_5)$
$4 = i_1 (2 + 4 + 6) + i_2 (4) + i_3 (6)$
$4 = i_1 (12) + i_2 (4) + i_3 (6)$

Mesh 2

$Vs_2 = i_2 (R_2 + R_3 + R_4) + i_1 (R_3) - i_3 (R_4)$

2 = i_2 (4 + 4 + 2) + i_1 (4) - i_3 (2)
2 = i_2 (10) + i_1 (4) - i_3 (2)

Mesh 3

0 = i_3 (R_4 + R_5 + R_6) - i_2 (R_4) + i_1 (R_5)
0 = i_3 (2 + 6 +2) - i_2 (2) + i_1 (6)
0 = i_3 (10) - i_2 (2) + i_1 (6)

Solving for i_1 ,i_2 ,i_3 using matrices, we have

$$\begin{bmatrix} 12 & 4 & 6 \\ 4 & 10 & -2 \\ 6 & -2 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

where i_1 ,i_2 ,i_3 equate to

(all values are $\times 10^3$, hence answers in $\times 10^{-3}$)

i_1 = 0.52238mA
i_2 = 0.07466mA
i_3 = 0.328mA

V_{R6} = i_1(6000) + i_3(6000)
V_{R6} = 6000 (i_3 + i_1)
V_{R6} = 6000 (0.328×10^{-3} + 0.52238×10^{-3})
V_{R6} = 1.166V (Solution)

Q2.2 For Figure 2.19, V_{S2} and R_S model a temperature sensor, and the voltage R_3 indicates the temperature. Determine the temperature.

$V_{S1} = 24\ V;\ \ V_{S2} = kT;\ k = 15V/^o\ C$

$R_1 = R_S = 15k\Omega;\ \ R_2 = 5k\Omega;\ \ R_3 = 10k\Omega;\ \ R_4 = 24k\Omega;\ \ \ V_{R3} = -4V$,

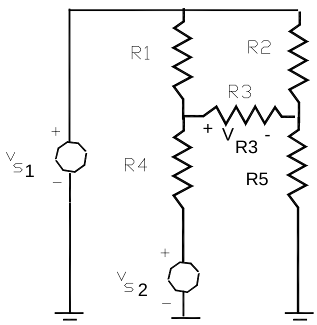
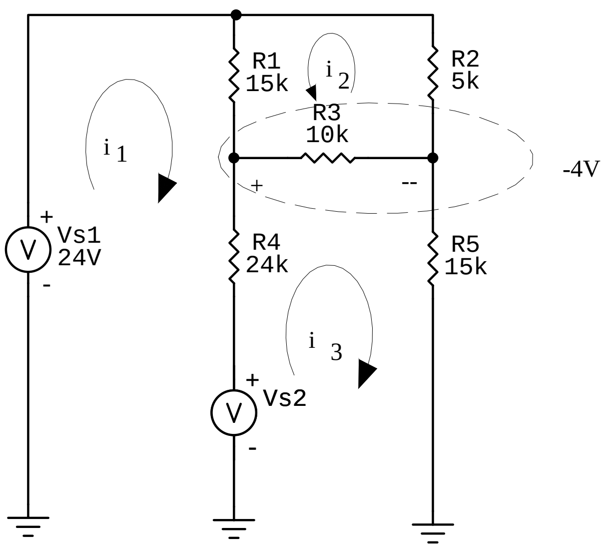


Figure 2.19

Solution: Mesh Analysis



Mesh 1

$V_{s1} - V_{s2} = i_1 (R_1+R_4) - i_3(R_4) + i_2 (R_1)$
$24 - V_{s2} = i_1 (15+24) - i_3(24) + i_2 (15)$
$24 - V_{s2} = i_1 (39) - i_3(24) + i_2 (15)$ [1]

Mesh 2

$0 = i_2 (R_1+R_2+ R_3)+ i_1 (R_1)+ i_3(R_3)$
$0 = i_2 (15+10+ 5)+ i_1 (15)+ i_3(10)$
$0 = i_2 (30)+ i_1 (15)+ i_3(10)$ [2]

Mesh 3

$V_{s2} = i_3 (R_3+R_4+ R_5) - i_2(R_3) + i_1 (R_4)$
$V_{s2} = i_3 (10+24+15) - i_2(10) + i_1 (24)$
$V_{s2} = i_3 (49) - i_2(10) + i_1 (24)$ [3]

Also we have,

$-4 = 10 (i_3 + i_2)$

$i_2 = - i_3 - 0.4mA$ [4]

Substituting Equation [4] in Equation [2],

$0 = (- i_3 - 0.4) (30)+ i_1 (15)+ i_3(10)$
$12= (- i_3) (30)+ i_1 (15)+ i_3(10)$
$12= (- i_3) (20)+ i_1 (15)$ [5]

Adding Eq*[1]* and Eq.*[3]*

$$\begin{bmatrix} 24 - V_{s2} & 39 & 15 & -24 \\ V_{s2} & -24 & 10 & 49 \end{bmatrix}$$

gives us

$$\begin{bmatrix} 24 & 15 & 25 & 25 \end{bmatrix} \quad [6]$$

which is the same as

$24 = 15 i_1 +25 i_2 +25 i_3$ [6]

Substituting Eq. [4] in Eq. [6],

$24 = 15 i_1 +25 (- i_3 - 0.4) +25 i_3$
$24 = 15 i_1 -25 i_3 - 10 +25 i_3$
$24 = 15 i_1 - 10$
$i_1 = 2.267mA$

From Eq.*[5]*, we now have

$i_3 = 1.10mA$

and from Eq.*[4]*, we have

$i_2 = -1.5mA$

Substituting in Eq.*[3]*, V_{s2} is

$V_{s2} = -15.508V.$

$V_{s2} = kT$ *where k = 15V/°C*

$$T = \frac{V_{s2}}{15}$$

T = -1.0338°C (solution)

Q2.3 For the circuit in Figure 2.20 having:

$V_S = 10\ V;\quad A_V = 50\ R_1 = 3k\Omega;\quad R_2 = 8k\Omega;\quad R_3 = 2k\Omega;\quad R_4 = 0.3k\Omega$,

determine the voltage across R4 using KCL and node analysis.

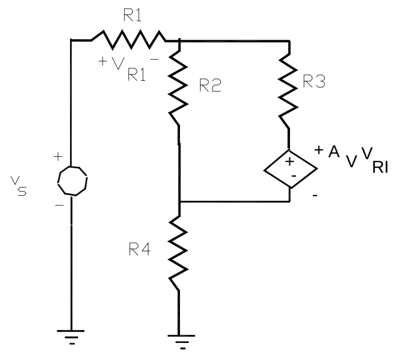
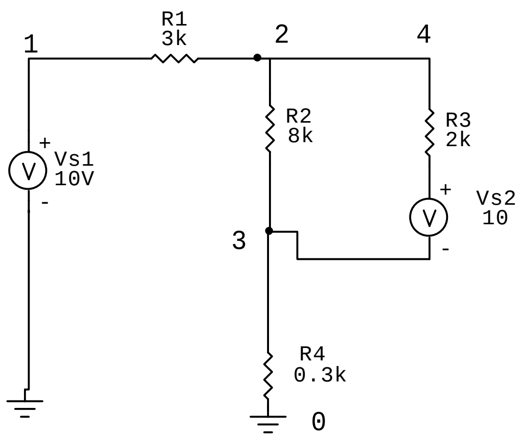


Figure 2.20

Solutions -Nodal Analysis



We shall look first look at Node 3 since it is the node closest to reference.

$$\frac{v_2 - v_3}{8k} = \frac{v_3 + 50_{VR1} - V_2}{2k} + \frac{V_3}{0.3k} \qquad (1)$$

$V_1 - V_2 = V_{R1}$ (Node 2) (2)

$V_1 = 10V$ (Node 1) (3)

Simplifying Eq.1, we have,

$40k = 30.5k(v_3) + 203.5k(v_1) - 203.5k(v_2)$ (4)

Looking again at Node 2 we have,

$$\frac{v_2 - v_3}{8k} = \frac{v_3 + 50_{VR1} - V_2}{2k} + \frac{V_3}{0.3k} \qquad (5)$$

Simplified, it equals

$-201.5k(V_2) + 4.5k(v_3) = -1977k$ (6)

Solving for the two using matrices,

$$\begin{bmatrix} -201.5 & 4.5 \\ -203.5 & 30.5 \\ & \end{bmatrix} \begin{bmatrix} -1976 \\ -1995 \end{bmatrix}$$

Thus, v_2 = 9.8 and hence from equation 2, V_{R1} =0.2. This thus means that 50(0.2) is 10V.

V_3 from matrix analysis is 49.6mV. Thus, the current across R4 is 0.16mA.

Q2.4 Determine (i) the current, I; (ii) the voltage at node A, in Figure 2.21, where

$V_1 = 5\ V;\quad V_2 = 10\ V;\ R_1 = 1k\Omega;\quad R_2 = 8k\Omega;\quad R_3 = 10k\Omega;\ R_4 = 2k\Omega;\ R_5 = 2k\Omega,$

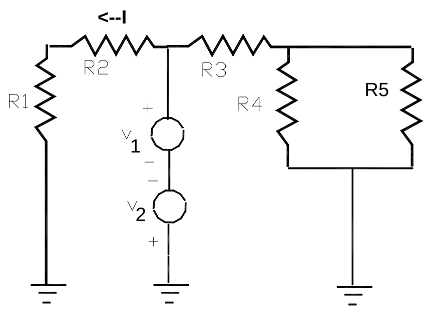
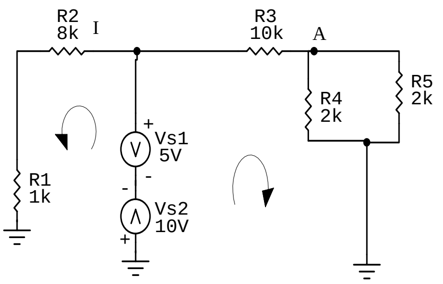


Figure 2.21

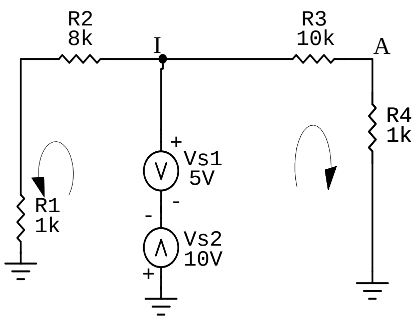
Solution: Mesh Analysis

R4 and R5 are in parallel and can be combined into one single resistor.



$$R_S = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{2 \times 2}{2 + 2} = 1k$$

The equivalent circuit is



Vs1 and Vs2 are in opposite directions hence the cumulative voltage across the two is (10-5) = 5V.

Next, we combine the resistors in series on either side into one, such that R1 and R2 combine with 9k and R4 and R4 combine with 11k. Now the circuit has the 9k resistor and the 11k resistor in parallel. From theory, voltage across parallel resistors is same and thus +5V. With this, I can be computed as follows

$$I = \frac{(V_{s2} - V_{s1})}{R_1 + R_2} = 5/9k = 0.56mA$$

Voltage at Node A can be calculated using the same convention but splitting the 11k resistor now into a 10k and a 1k as shown by the earlier figure. Note that current across resistors in series remain the same.

$$I_r = \frac{(V_{s2} - V_{s1})}{R_3 + R'_4} = \frac{5}{11k} = 0.455mA$$

thus, voltage at node A is the voltage across R4 and R5.

R'_4 refers to the 1k resistor as shown in the second figure.

$$V_A = I_r R_4^{\ 3} = 0.455 \times 1 = 0.455V$$

Q2.5 In Figure 2.22, F_1 and F_2 are fuses. Under normal conditions they are modeled as short circuits. However, if excess current flows through a fuse, it melts and consequently blows (becoming an open circuit). For the following parameter values:
$V_{s1} = 1105V;\ V_{s2} = 110\ V;\ \ ;\ R_1 = 100\Omega;\ R_2 = 25\Omega;\ \ R_3 = 75\Omega;\ \ R_4 = R_5 = 15\Omega$.

If fuse F1 now blows or open, determine, using KVL, and mesh analysis, the voltages across R_1, R_2 and R_3 under normal condition.

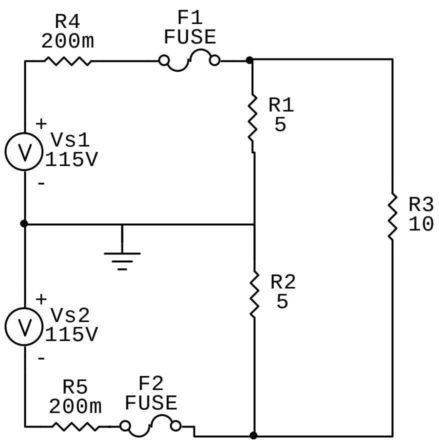


Figure 2.22: Fused circuit

Solution

Using KVL:

$$I_1(R_1 + R_4) - I_3 R_1 = V_{s1}$$
$$I_2(R_2 + R_5) - I_3 R_2 = V_{s2}$$
$$-I_1 R_1 - I_2 R_2 + I_3(R_1 + R_2 + R_3) = 0$$

Substituting resistor values and rearranging:

$$115I_1 \quad 0 \quad -75I_3 = 110$$
$$0 \quad 40I_2 \quad 75I_3 = 110$$
$$100I_1 + \quad 25I_2 - 200I_3 = 0$$

When fuse F1 is open then

$$I_1 = 0; V_{s1} = 0;$$

leading to

$$40I_2 \quad 75I_3 = 110$$
$$25I_2 - 200I_3 = 0$$

from which

$$I_2 = 3.6A; \ I_3 = 0.45A$$

$$V_{R1} = 0V$$
$$V_{R2} = R_2(I_3 - I_2) = 25(0.45 - 3.6) = -78.75\ V$$
$$V_{R3} = I_3R_3 = 0.45 \times 75 = 33.75\ V$$

Thevenin/Norton Equivalent

Q2.6. In Figure 2.23, $V_S = 12\ V$; $R_1 = 7k\Omega$; $R_2 = 3k\Omega$; $R_3 = 8k\Omega$; $R_3 = 6k\Omega$,

Determine:

(i) the Thevenin equivalent of the circuit to the left of a-b

(ii) the voltage between a-b

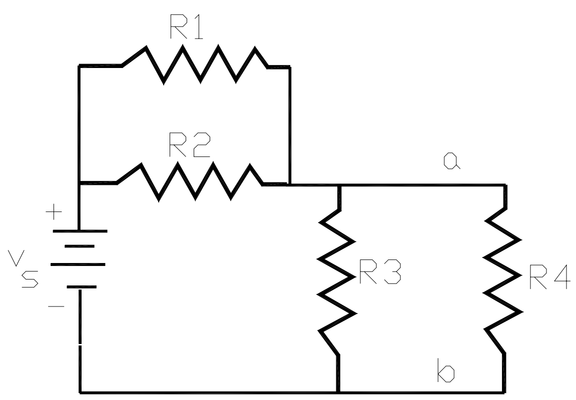
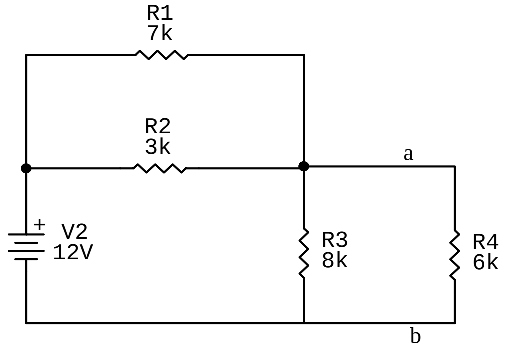


Figure 2.23

Solution: Thevenin's Voltages

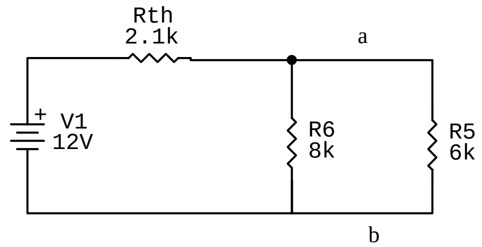


a) Combining R1 and R2 in parallel into one resistor

$$R_{12} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{7 \times 3}{7+3} = \frac{21}{10}\,\text{k}\Omega$$

Thus the Thevenin equivalent circuit is shown below.

b) Voltage between *a* and *b* is given by



Thevnins Equivalent Circuit

$$V_{TH} = \frac{R_{TH} + R_3}{R_{TH}\, R_3} V_s$$

$$V_{TH} = \frac{2.1+8}{2.1 \times 8} 12$$

$$V_{TH} = 7.214V$$

Q2.7. In the circuit shown in Figure 2.24,

R_1 = 10kΩ	R_4 = 5kΩ	R_7 =3kΩ	C = 20μF
R_2 = 2kΩ	R_5 = 8kΩ	V_1 = 15V	I = 20mA
R_3 = 4kΩ,	R_6 = 6kΩ	V_2 = 12V	

Determine the Norton Equivalent with respect to the 20μF capacitor.

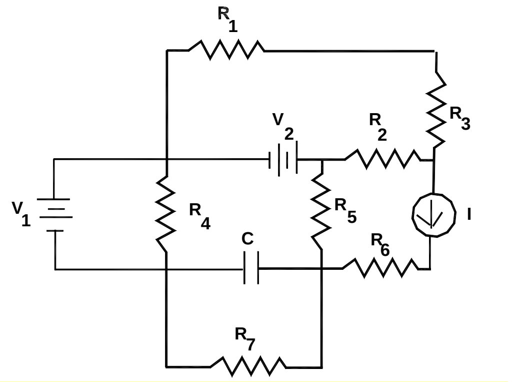
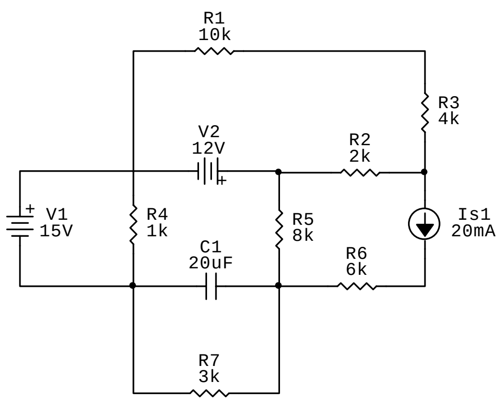
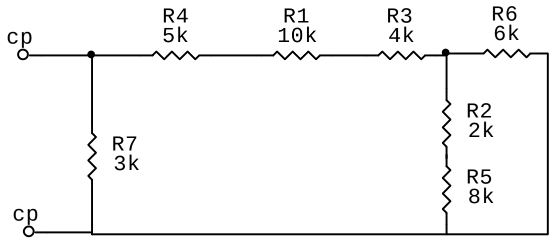


Figure 2.24

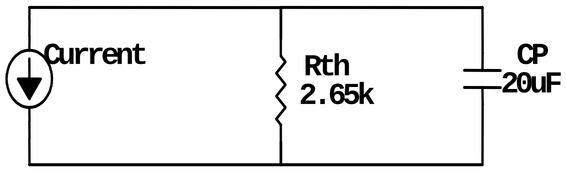
Solution: Norton's Equivalent Circuit



Equivalent Circuits with respect to C.



a) Combining R2, R5 and R6 gives us (8+2)//6 = 3.75k. Combining R4,R1, R3 and 3.75k gives us 22.75k. R7 and 22.75k then give 2.65k, which is the Thevenin's equivalent resistance of the circuit.



Sinusoidal sources

Q2.8 For the circuit shown in Figure 2.25, determine, for the values given,

(i) the equivalent impedance of the circuit the source current
(i) the source current

$$R_1 = 200\Omega; \quad R_2 = 100\Omega; \quad L = 50mH; \quad C = 50\mu F; \quad v_s(t) = 5\cos\left(5000t + \frac{\pi}{4}\right).$$

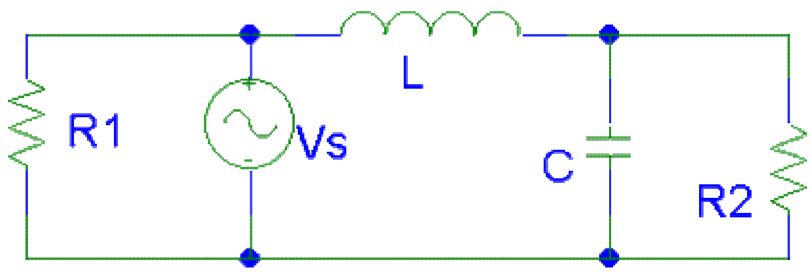
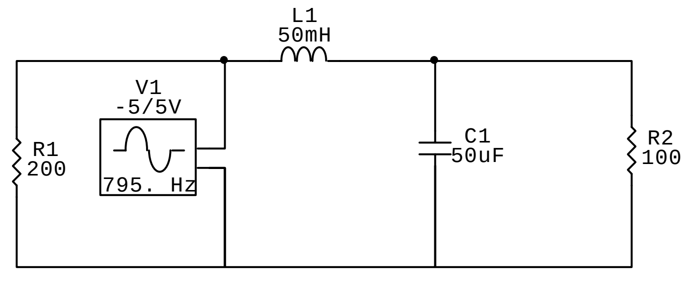
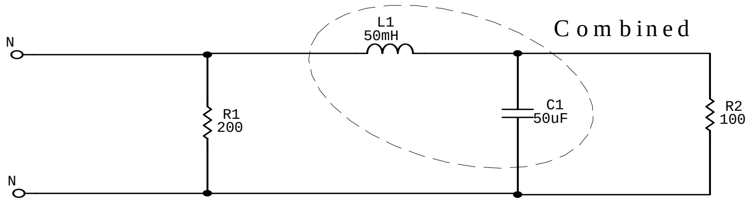


Figure 2.25

Solution:





The impedance Z_c of the capacitor and Z_L of the inductor are calculated.

$$Z_c = \frac{-j}{\omega C} = \frac{-j}{(5000)(50\times 10^{-6})} = -4j$$

$$Z_L = j\omega L = j(5000)(50\times 10^{-3}) = 250j$$

Combining Z_c and Z_L together since they are in parallel,

$$Z = \frac{Z_c \times Z_L}{Z_c + Z_L} = \frac{(-4j)(250j)}{(-4j)+(250j)} = -4j$$

Combining R_1 with Z since they are parallel to make a series impedance with R_2.

$$Z_1 = \frac{(200)(-4j)}{(200)+(-4j)} = \frac{800\angle -90}{200\angle -1.145} = 4\angle -88.55 = 0.1 - 4j$$

Now Z_1 is in series with R_2 and hence can be added together

$$Z_{TH} = (0.1-4j)+(100) = 100.1-4j = 100.18\angle -2.29$$

With the voltage source having $5\angle 45$ V, using Ohms Law, the current flowing is given by

$$I_{source} = \frac{5\angle 45}{100.18\angle -2.29} = 0.0499\angle 47.29 = 0.0338 + 0.0367j$$

The source current is therefore $50\angle 47.29$ mA.