Chapter 4
Consumer Choice

Solutions to Review Questions

1. If the consumer has a positive marginal utility for each of two goods, why will the consumer always choose a basket on the budget line?

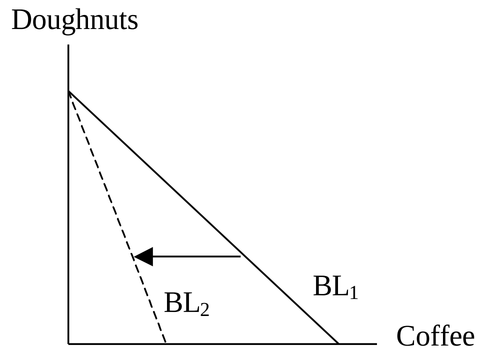
Relative to any point on the budget line, when the consumer has a positive marginal utility for all goods she could increase her utility by consuming some basket outside the budget line. However, baskets outside the budget line are unaffordable to her, so she is constrained (as in "constrained optimization") to choosing the most preferred basket that lies along the budget line.

2. How will a change in income affect the location of the budget line?

An increase in income will shift the budget line away from the origin in a parallel fashion expanding the set of possible baskets from which a consumer may choose. A decrease in income will shift the budget line in toward the origin in a parallel fashion, reducing the set of possible baskets from which a consumer may choose.

3. How will an increase in the price of one of the goods purchased by a consumer affect the location of the budget line?

If the price of one of the goods increases, the budget line will rotate inward on the axis for the good with the price increase. The budget line will continue to have the same intercept on the other axis. For example, suppose someone buys two goods, cups of coffee and doughnuts, and suppose the price of a cup of coffee increases. Then the budget line will rotate as in the following diagram:



4. What is the difference between an interior optimum and a corner point optimum in the theory of consumer choice?

With an interior optimum the consumer is choosing a basket that contains positive quantities of all goods, while with a corner point optimum the consumer is choosing a basket with a zero quantity for one of the goods. The tangency condition usually does not apply at corner optima.

5. At an optimal interior basket, why must the slope of the budget line be equal to the slope of the indifference curve?

If the optimum is an interior solution, the slope of the budget line must equal the slope of the indifference curve. If these slopes are not equal at the chosen interior basket then the "bang for the buck" condition will not hold. This condition states that at the optimum the extra utility gained per dollar spent on good x must be equal to the extra utility gained per dollar spent on good y. If this condition does not hold at the chosen basket, then the consumer could reallocate his income to purchase more of the good with the higher "bang for the buck" and increase his total utility while remaining within the given budget. Thus, if these slopes are not equal the basket cannot be optimal assuming an interior solution.

6. At an optimal interior basket, why must the marginal utility per dollar spent on all goods be the same?

At an interior optimum, the slope of the budget line must equal the slope of the indifference curve. This implies

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

This can be rewritten as

$$\frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

which is known as the "bang for the buck" condition. If this condition does not hold at the chosen interior basket, then the consumer can increase total utility by reallocating his spending to purchase more of the good with the higher "bang for the buck" and less of the other good.

7. Why will the marginal utility per dollar spent not necessarily be equal for all goods at a corner point?

The "bang for the buck" condition will not necessarily hold at a corner solution optimum. The consumer could theoretically increase total utility by reallocating his spending to purchase more of the good with the higher "bang for the buck" and less of the other good. Since the basket is a corner point, however, he is already purchasing zero of one of the goods. This implies that he cannot purchase less of the good with a zero quantity (since negative quantities make no sense) and therefore cannot reallocate spending.

8. Suppose that a consumer with an income of $1,000 finds that basket *A* maximizes utility subject to his budget constraint and realizes a level of utility *U*1. Why will this basket also minimize the consumer's expenditures necessary to realize a level of utility *U*1?

In the utility maximization problem, the consumer maximizes utility subject to a fixed budget constraint. At the optimum the slope of the budget line will equal the slope of the indifference curve. If we now hold that indifference curve fixed, we can solve an expenditure minimization problem in which we ask what is the minimum expenditure necessary to achieve that fixed level of utility. Since the slope of the budget line and indifference curve have not changed, when the expenditure is minimized the budget line and indifference curve will be tangent at the same point as in the utility maximization problem. The same basket is optimal in both problems.

9. What is a composite good?

First, consumers typically allocate income to more than two goods. Second, economists often want to focus on the consumer's response to purchases of a single good or service. In this case it is useful to present the consumer choice problem using a two-dimensional graph. Since there are more than two goods the consumer is purchasing, however, an economist would need more than two dimensions to show the problem graphically. To reduce the problem to two dimensions, economists often group the expenditures on all other goods besides the one in question into a single good termed a "composite good." When the problem is shown graphically, one axis represents the composite good while the other axis represents the single good in question. By creating this composite good, the problem can be illustrated using a two-dimensional graph.

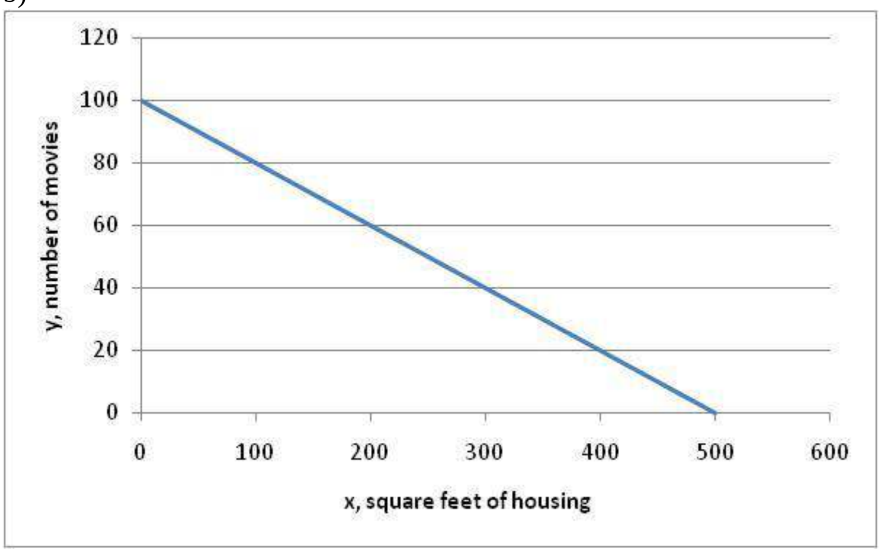
10. How can revealed preference analysis help us learn about a consumer's preferences without knowing the consumer's utility function?

By employing revealed preference analysis one can make inferences regarding a consumer's preferences without knowing what the consumer's indifference map looks like. For example, if a consumer chooses basket A over basket B when basket B costs at least as much as basket A, we know that basket A is at least as preferred as basket B. If the consumer chooses basket C, which is more expensive than basket D, then we know the consumer strictly prefers basket C to basket D. By observing enough of these choices, one can determine how the consumer ranks baskets even without knowing the exact shape of the consumer's indifference map.

Solutions to Problems

4.1 Pedro is a college student who receives a monthly stipend from his parents of $1,000. He uses this stipend to pay rent for housing and to go to the movies (you can assume that all of Pedro's other expenses, such as food and clothing have already been paid for). In the town where Pedro goes to college, each square foot of rental housing costs $2 per month. The price of a movie ticket is $10 per ticket. Let *x* denote the square feet of housing, and let *y* denote the number of movie tickets he purchases per month.
a) What is the expression for Pedro's budget constraint?
b) Draw a graph of Pedro's budget line.
c) What is the maximum number of square feet of housing he can purchase given his monthly stipend?
d) What is the maximum number of movie tickets he can purchase given his monthly stipend?
e) Suppose Pedro's parents increase his stipend by 10 percent. At the same time, suppose that in the college town he lives in, all prices, including housing rental rates and movie ticket prices, increase by 10 percent. What happens to the graph of Pedro's budget line?

a) $2x + 10y \leq 1000$
b)



c) The maximum amount of housing Pedro can purchase is his budget divided by the price of housing: $1,000/$2 per square feet = 500 square feet.
d) The maximum number of movie tickets Pedro can purchase is his budget divided by the price of a movie ticket: $1,000/$10 per tickets = 100 tickets.
e) His budget line does not change at all.

Initially, the budget line (with x on the horizontal axis and y on the vertical axis) has a horizontal intercept equal to 1000/2 = 500 and a vertical intercept equal to 1000/10 = 100. The slope of the budget line is -2/10 = - 0.20 (the price of housing divided by the price of movie tickets).

With the increase in Pedro's stipend and the increases in prices we have:

- Horizontal intercept of budget line: 1000(1.10)/(2(1.10)) = 500
- Vertical intercept of budget line: 1000(1.10)/(10(1.10)) = 100
- Slope of budget line: -2(1.10)/(10(1.10)) = - 0.20.

These are the same as before and thus the budget line does not change.

4.2 Sarah consumes apples and oranges (these are the only fruits she eats). She has decides that her monthly budget for fruit will be $50. Suppose that one apple costs $0.25, while one orange costs $0.50. Let *x* denote the quantity of apples and *y* denote the quantity of oranges that Sarah purchases.

a. What is the expression for Sarah's budget constraint?

b. Draw a graph of Sarah's budget line.

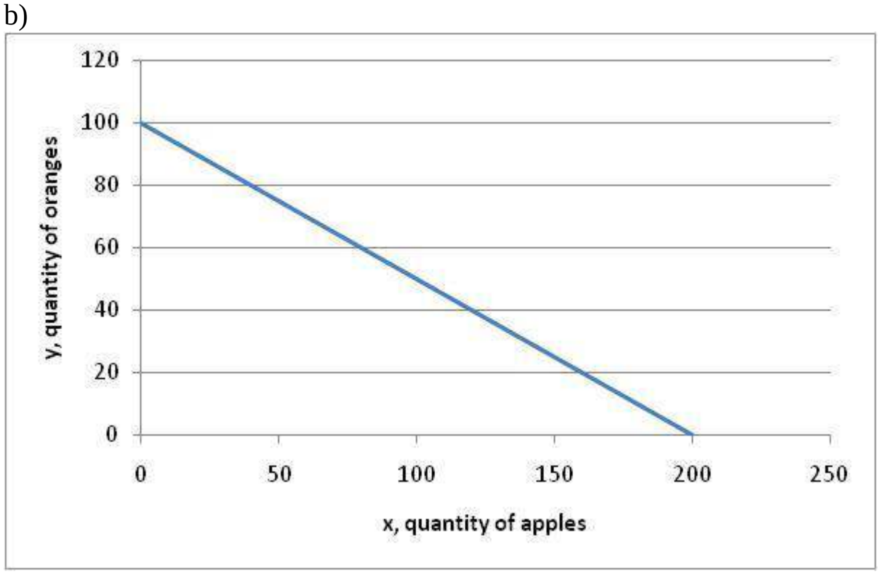
c. Show graphically how Sarah's budget line changes if the price of apples increases to $0.50.

d. Show graphically how Sarah's budget line changes if the price of oranges decreases to $0.25.

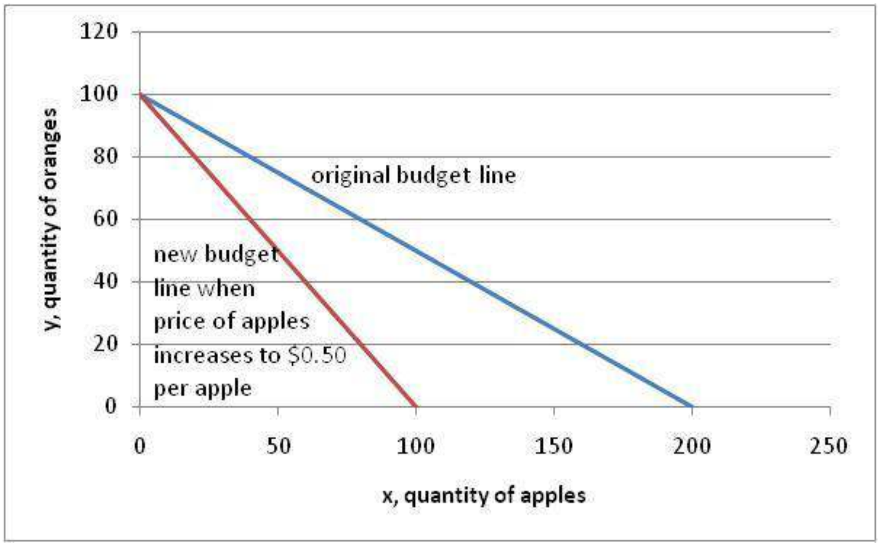
e. Suppose Sarah decides to cut her monthly budget for fruit in half. Coincidentally, the next time she goes to the grocery store, she learns that oranges and apples are on sale for half price, will remain so for the next month, i.e., the price of apples falls from $0.25 per apple to $0.125 per apple and the price of oranges falls from $0.50 per orange to $0.25 per orange. What happens to the graph of Sarah's budget line?

a) $0.25x + 0.50y \leq 50$.

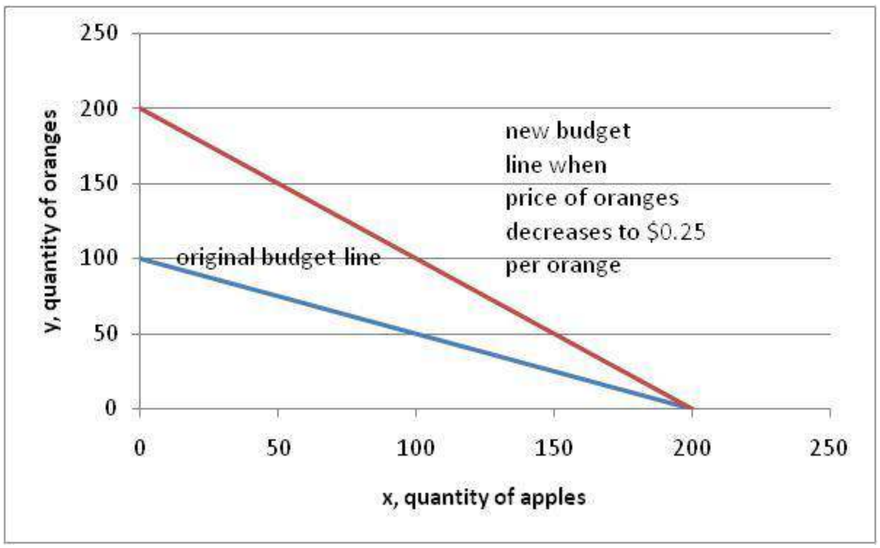
b)



c)



d)



e) Sarah's budget line would not change.

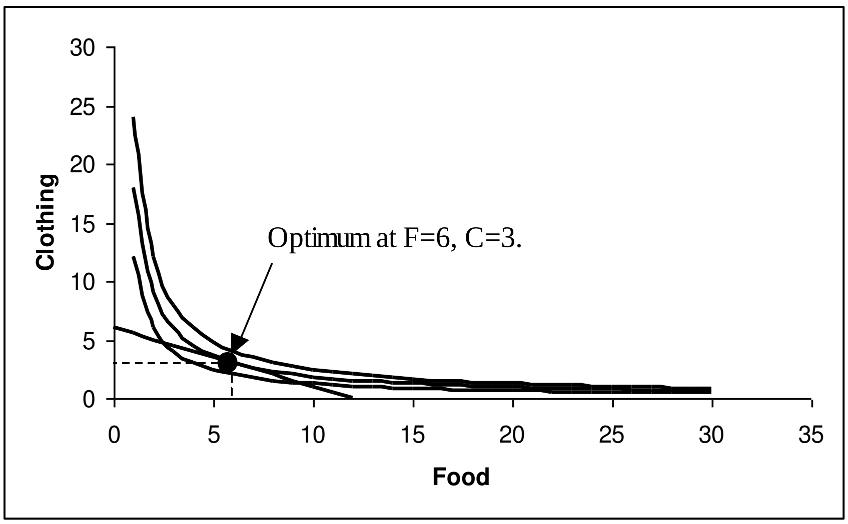
- Horizontal intercept of the budget line: (0.5)$50/((0.5)(0.25) = 200
- Vertical intercept of the budget line: (0.5)$50/((0.5)(0.50) = 100
- Slope of the budget line = -(0.5)(0.25)/((0.5)(0.50)) = 0.50

These are the same as before, and thus the budget line does not change.

4.3 In Problem 3.7 of Chapter 3, we considered Julie's preferences for food *F* and clothing *C*. Her utility function was *U*(*F*, *C*) = *FC*. Her marginal utilities were *MUF* = *C* and *MUC* = *F*. You were asked to draw the indifference curves *U* = 12, *U* = 18, and *U* = 24, and to show that she had a diminishing marginal rate of substitution of food for clothing.

Suppose that food costs $1 a unit and that clothing costs $2 a unit. Julie has $12 to spend on food and clothing.
a) Using a graph (and no algebra), find the optimal (utility-maximizing) choice of food and clothing. Let the amount of food be on the horizontal axis and the amount of clothing be on the vertical axis.
b) Using algebra (the tangency condition and the budget line), find the optimal choice of food and clothing.
c) What is the marginal rate of substitution of food for clothing at her optimal basket? Show this graphically and algebraically.
d) Suppose Julie decides to buy 4 units of food and 4 units of clothing with her $12 budget (instead of the optimal basket). Would her marginal utility per dollar spent on food be greater than or less than her marginal utility per dollar spent on clothing? What does this tell you about how she should substitute food for clothing if she wanted to increase her utility without spending any more money?

a)



b) The tangency condition implies that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information results in

$$\frac{C}{F} = \frac{1}{2}$$
$$2C = F$$

Substituting this result into the budget line, $F + 2C = 12$, yields

$$2C + 2C = 12$$
$$4C = 12$$
$$C = 3$$

Finally, plugging this result back into the tangency condition implies $F = 6$. At the optimum the consumer choose 6 units of food and 3 units of clothing.

c) At the optimum, $MRS_{F,C} = C/F = 3/6 = 1/2$. Note that this is equal to the ratio of the price of food to the price of clothing. The equality of the price ration and $MRS_{F,C}$ is seen in the graph above as the tangency between the budget line and the indifference curve for $U = 18$.

d) If the consumer purchases 4 units of food and 4 units of clothing, then

$$\frac{MU_F}{P_F} = \frac{4}{1} = 4 > \frac{MU_C}{P_C} = \frac{4}{2} = 2.$$

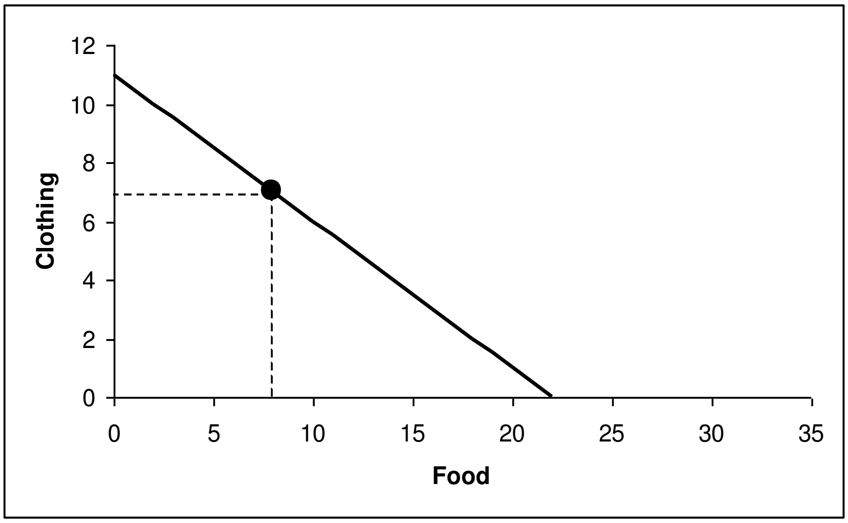
This implies that the consumer could reallocate spending by purchasing more food and less clothing to increase total utility. In fact, at the basket (4, 4) total utility is 16 and the consumer spent $12. By giving up one unit of clothing the consumer saves $2 which can than be used to purchase two units of food (they each cost $1). This will result in a new basket (6,3), total utility of 18, and spending of $12. By reallocating spending toward the good with the higher "bang for the buck" the consumer increased total utility while remaining within the budget constraint.

4.4 The utility that Ann receives by consuming food *F* and clothing *C* is given by *U*(*F*, *C*) = *FC* + *F*. The marginal utilities of food and clothing are *MUF* = *C* + 1 and *MUC* = *F*. Food costs $1 a unit, and clothing costs $2 a unit. Ann's income is $22.
a) Ann is currently spending all of her income. She is buying 8 units of food. How many units of clothing is she consuming?
b) Graph her budget line. Place the number of units of clothing on the vertical axis and the number of units of food on the horizontal axis. Plot her current consumption basket.
c) Draw the indifference curve associated with a utility level of 36 and the indifference curve associated with a utility level of 72. Are the indifference curves bowed in toward the origin?
d) Using a graph (and no algebra), find the utility maximizing choice of food and clothing.
e) Using algebra, find the utility-maximizing choice of food and clothing.
f) What is the marginal rate of substitution of food for clothing when utility is maximized? Show this graphically and algebraically.
g) Does Ann have a diminishing marginal rate of substitution of food for clothing? Show this graphically and algebraically.

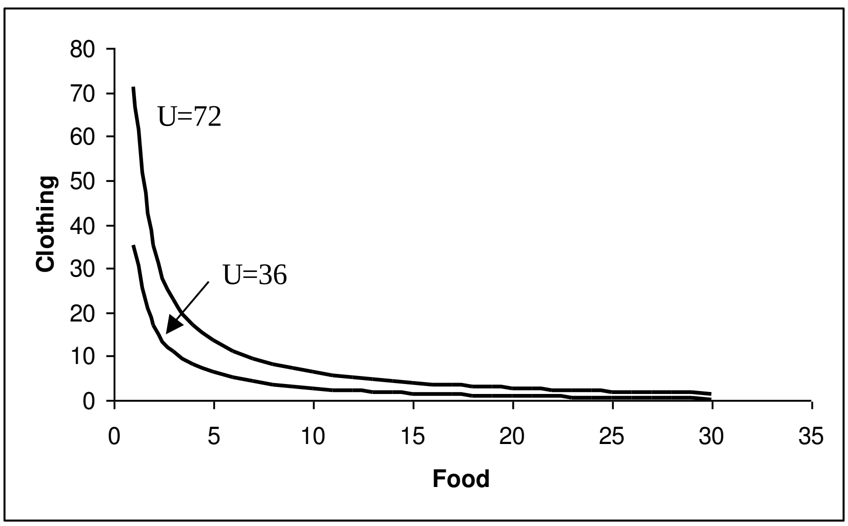
a) If Ann is spending all of her income then

$$F + 2C = 22$$
$$8 + 2C = 22$$
$$2C = 14$$
$$C = 7$$

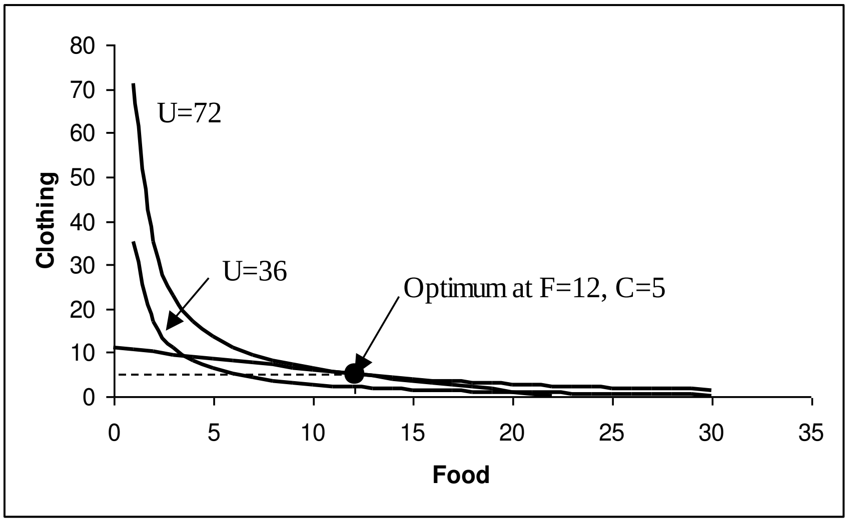
b)



c) Yes, the indifference curves are convex, i.e., bowed in toward the origin. Also, note that they intersect the *F*-axis.



d)



e) The tangency condition requires that

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$$

Plugging in the known information yields

$$\frac{C+1}{F} = \frac{1}{2}$$
$$2C + 2 = F$$

Substituting this result into the budget line, $F + 2C = 22$ results in

$$(2C+2) + 2C = 22$$
$$4C = 20$$
$$C = 5$$

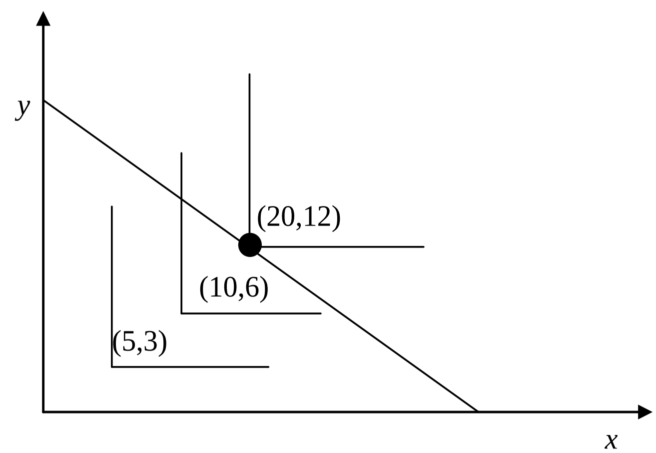
Finally, plugging this result back into the tangency condition implies that $F = 2(5) + 2 = 12$. At the optimum the consumer chooses 5 units of clothing and 12 units of food.

f) $MRS_{F,C} = \frac{C+1}{F} = \frac{5+1}{12} = \frac{1}{2}$ The marginal rate of substitution is equal to the price ratio.

g) Yes, the indifference curves do exhibit diminishing $MRS_{F,C}$. We can see this in the graph in part c) because the indifference curves are bowed in toward the origin. Algebraically, $MRS_{F,C} = {}^{C+1}\!/_{F}$. As F increases and C decreases along an isoquant, $MRS_{F,C}$ diminishes.

4.5 Consider a consumer with the utility function $U(x, y) = \min(3x, 5y)$, that is, the two goods are perfect complements in the ratio 3:5. The prices of the two goods are $P_x = \$5$ and $P_y = \$10$, and the consumer's income is \$220. Determine the optimum consumption basket.

This question cannot be solved using the usual tangency condition. However, you can see from the graph below that the optimum basket will necessarily lie on the "elbow" of some indifference curve, such as (5, 3), (10, 6) etc. If the consumer were at some other point, he could always move to such a point, keeping utility constant and decreasing his expenditure. The equation of all these "elbow" points is $3x = 5y$, or $y = 0.6x$. Therefore the optimum point must be such that $3x = 5y$. The usual budget constraint must hold of course. That is, $5x + 10y = 220$. Combining these two conditions, we get $(x, y) = (20, 12)$.



4.6 Jane likes hamburgers (*H*) and milkshakes (*M*). Her indifference curves are bowed in toward the origin and do not intersect the axes. The price of a milkshake is \$1 and the price of a hamburger is \$3. She is spending all her income at the basket she is currently consuming, and her marginal rate of substitution of hamburgers for milkshakes is 2. Is she at an optimum? If so, show why. If not, should she buy fewer hamburgers and more milkshakes, or the reverse?

From the given information we know that $P_H = 3$, $P_M = 1$, and $MRS_{H,M} = 2$. Comparing the $MRS_{H,M}$ to the price ratio,

$$MRS_{H,M} = 2 < \frac{P_H}{P_M} = \frac{3}{1}$$

Since these are not equal Jane is not currently at an optimum. In addition, we can say that

$$\frac{P_H}{P_M} > MRS_{H,M} = \frac{MU_H}{MU_M}$$

which is equivalent to

$$\frac{MU_M}{P_M} > \frac{MU_H}{P_H}$$

That is, the "bang for the buck" from milkshakes is greater than the "bang for the buck" from hamburgers. So Jane can increase her total utility by reallocating her spending to purchase fewer hamburgers and more milkshakes.

4.7 Ray buys only hamburgers and bottles of root beer out of a weekly income of $100. He currently consumes 20 bottles of root beer per week, and his marginal utility of root beer is 6. The price of root beer is $2 per bottle. Currently, he also consumes 15 hamburgers per week, and his marginal utility of a hamburger is 8. Is Ray maximizing utility at his current consumption basket? If not, should he buy more hamburgers each week, or fewer?

Compare MU_H/P_H with MU_R/P_R, where the subscripts "H" and "R" refer respectively to hamburgers and root beer. We have all the information to make this comparison except for the price of a hamburger. But we can determine the price of a hamburger from Sam's budget constraint:

$P_H H + P_R R$ = Income, or $P_H(15) + 2(20) = 100$.

So P_H = $4 per hamburger.
Now we can see that $MU_H/P_H = 8/4 = 2$ and $MU_R/P_R = 6/2 = 3$.
Since the "bang for the buck" is higher for root beer than for hamburgers, he should buy fewer hamburgers (and more root beer).

4.8 Dave currently consumes 10 hot dogs and 6 sodas each week. At his current consumption basket, his marginal utility for hot dogs is 5 and his marginal utility for sodas is 3. If the price of one hot dog is $1 and the price of one soda is $0.50, is Dave currently maximizing his utility? If not, how should he reallocate his spending in order to increase his utility?

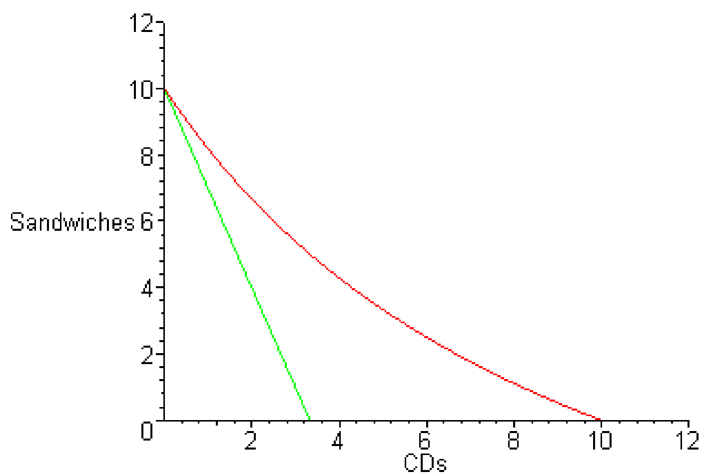
To determine if this situation is optimal, determine if the tangency condition holds.
Is $\frac{MU_H}{P_H} = \frac{MU_S}{P_S}$? That is, is $\frac{5}{1} = \frac{3}{0.50}$? No ($5 \neq 6$). So $\frac{MU_H}{P_H} < \frac{MU_S}{P_S}$.
Since the tangency condition does not hold, Dave is not currently maximizing his utility. To increase his utility he should purchase more soda and fewer hot dogs (since the 'bang for the buck' for sodas is higher).

4.9 Helen's preferences over CDs (*C*) and sandwiches (*S*) are given by *U*(*S*, *C*) = *SC* + 10(*S* + *C*), with *MUC* = *S* + 10 and *MUS* = *C* + 10. If the price of a CD is $9 and the price of a sandwich is $3, and Helen can spend a combined total of $30 each day on these goods, find Helen's optimal consumption basket.

See the graph below. The fact that Helen's indifference curves touch the axes should immediately make you want to check for a corner point solution.

To see the corner point optimum algebraically, notice if there was an interior solution, the tangency condition implies $(S + 10)/(C + 10) = 3$, or $S = 3C + 20$. Combining this with the budget constraint, $9C + 3S = 30$, we find that the optimal number of CDs would be given by $18C = -30$ which implies a negative number of CDs. Since it's impossible to purchase a negative amount of something, our assumption that there was an interior solution must be false. Instead, the optimum will consist of $C = 0$ and Helen spending all her income on sandwiches: $S = 10$.

Graphically, the corner optimum is reflected in the fact that the slope of the budget line is steeper than that of the indifference curve, even when $C = 0$. Specifically, note that at $(C, S) = (0, 10)$ we have $P_C / P_S = 3 > MRS_{C,S} = 2$. Thus, even at the corner point, the marginal utility per dollar spent on CDs is lower than on sandwiches. However, since she is already at a corner point with $C = 0$, she cannot give up any more CDs. Therefore the best Helen can do is to spend all her income on sandwiches: $(C, S) = (0, 10)$. [Note: At the other corner with $S = 0$ and $C = 3.3$, $P_C / P_S = 3 > MRS_{C,S} = 0.75$. Thus, Helen would prefer to buy more sandwiches and less CDs, which is of course entirely feasible at this corner point. Thus the $S = 0$ corner cannot be an optimum.]



4.10 The utility that Corey obtains by consuming hamburgers (H) and hot dogs (S) is given by $U(H,S) = \sqrt{H} + \sqrt{S+4}$. The marginal utility of hamburgers is $\frac{0.5}{\sqrt{H}}$ and the marginal utility of steaks is equal to $\frac{0.5}{\sqrt{S+4}}$.

a) Sketch the indifference curve corresponding to the utility level $U = 12$.

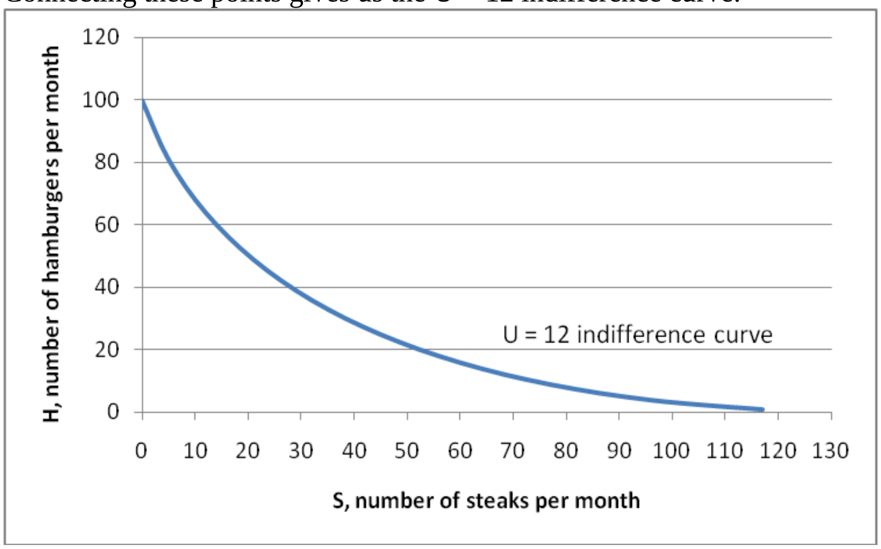
b) Suppose that the price of hamburgers is \$1 per hamburger, and the price of steak is \$8 per steak. Moreover, suppose that Corey can spend \$100 per month on these two foods. Sketch Corey's budget line for hamburgers and steak given this budget.

c) Based on your answer to parts (a) and (b), what is Corey's optimal consumption basket given his budget?

a) Some points on the *U* = 12 indifference curve include

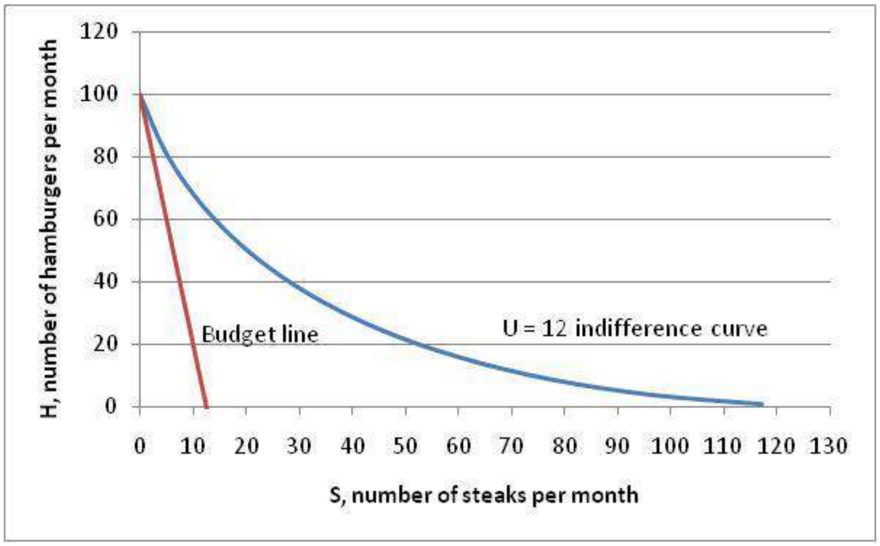
S	*H*	*U*
0	100	12
5	81`	12
12	64	12
21	49	12
32	36	12
45	25	12
60	16	12

Connecting these points gives us the U = 12 indifference curve:

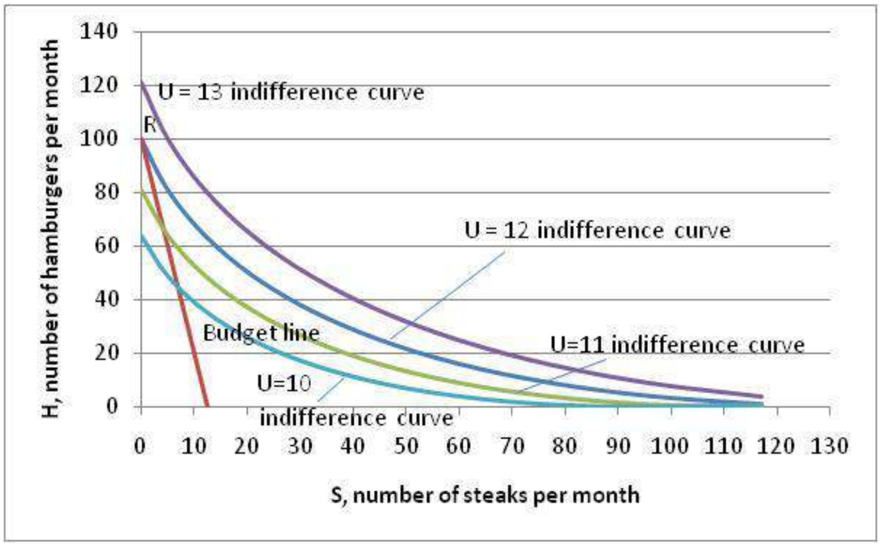


b) The equation of the budget line is $H + 8S = 100$

Graphing this on the same axes as the U = 12 indifference curve gives us:



c) The optimal consumption basket is $S = 0$, $H = 100$, i.e., point *R* in the figure below. There are several ways to see this. One way is to sketch a few more indifference curves (each corresponding to a different level of utility). This picture strongly suggests that the point of maximum utility occurs at point *R*.



Another way is to compare the marginal utility per dollar of spent on hamburger and the marginal utility per dollar spent on steak at point *R*. From the information given in the statement of the problem, $MU_H = \frac{0.5}{\sqrt{H}}$ and $MU_S = \frac{0.5}{\sqrt{S+4}}$, and so at point R

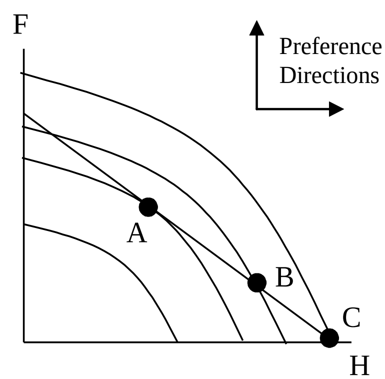
$$\frac{MU_S}{P_s} = \frac{\frac{0.5}{\sqrt{0+4}}}{8} = \frac{1}{32}$$

$$\frac{MU_H}{P_H} = \frac{\frac{0.5}{\sqrt{100}}}{1} = \frac{1}{20}$$

Thus, at point *R*, the marginal utility per dollar spent on hamburger is greater than the marginal utility per dollar spent on steak, and so the consumer would like to purchase more hamburger and less steak. However, at point *R*, no further reduction in the quantity of steak is possible, and thus *R* is the optimal consumption basket.

4.11 This problem will help you understand what happens if the marginal rate of substitution is not diminishing. Dr. Strangetaste buys only french fries (*F*) and hot dogs (*H*) out of his income. He has positive marginal utilities for both goods, and his $MRS_{H,F}$ is *increasing*. The price of hot dogs is P_H, and the price of french fries is P_F.
a) Draw several of Dr. Strangetaste's indifference curves, including one that is tangent to his budget line.
b) Show that the point of tangency does *not* represent a basket at which utility is maximized, given the budget constraint. Using the indifference curves you have drawn, indicate on your graph where the optimal basket is located.

a)



b) At point A, Dr. Strangetaste's indifference curve, which is bowed out from the origin, is tangent to his budget line. This point is not an optimum because, for example, Dr. Strangetaste could move to point B on his budget line and achieve a higher level of total utility. Point B, though, is not an optimum either because Dr. Strangetaste could move to point C, a corner point, to achieve an even higher level of total utility. When the MRS is increasing, a corner point optimum will occur (with *F* = 0 in this picture, though it could equivalently be with *H* = 0 for another set of indifference curves).

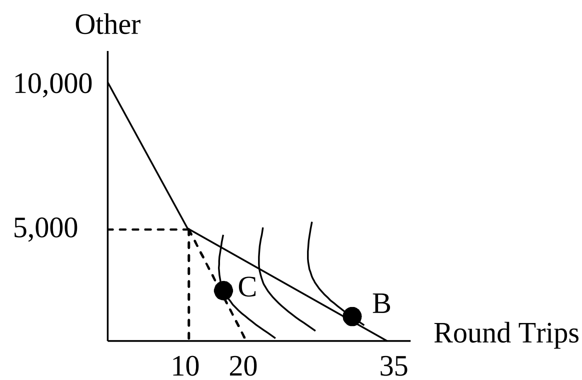
4.12 Julie consumes two goods, food and clothing, and always has a positive marginal utility for each good. Her income is 24. Initially, the price of food is 2 and the price of clothing is 2. After new government policies are implemented, the price of food falls to 1

and the price of clothing rises to 4. Suppose, under the initial budget constraint, her optimal choice is 10 units of food and 2 units of clothing.
a) After the prices change, can you predict whether her utility will be higher, lower, or the same as under the initial prices?
b) Does your answer require that there be a diminishing marginal rate of substitution of food for clothing? Explain.

As given, Julie consumes F = 10 and C = 2 with an income of 24.
Initially (with $P_F = P_C = 2$) she spends all her income: $P_FF + P_CC = 2(10) + 2(2) = 24$.
To buy her initial basket at the new prices, she would only need to spend
$P_FF + P_CC = 1(10) + 4(2) = 18$.
Thus, her initial basket lies inside her new budget constraint (assuming her income stays at 24). With her new budget line she would be able to choose a new basket to the "northeast" of (i.e., a basket involving more food and clothing than) her initial basket, making her better off.

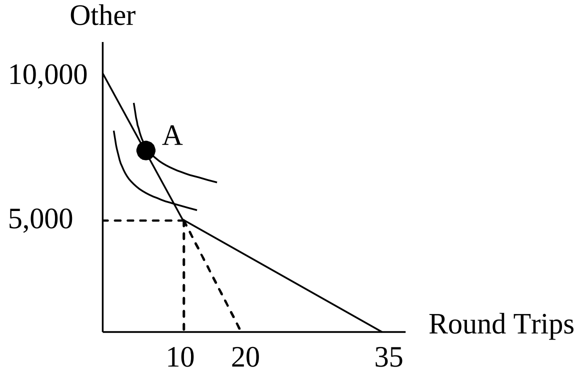
4.13 Toni likes to purchase round trips between the cities of Pulmonia and Castoria and other goods out of her income of $10,000. Fortunately, Pulmonian Airways provides air service and has a frequent-flyer program. Around trip between the two cities normally costs $500, but any customer who makes more than 10 trips a year gets to make additional trips during the year for only $200 per round trip.
a) On a graph with round trips on the horizontal axis and "other goods" on the vertical axis, draw Toni's budget line. (*Hint:* This problem demonstrates that a budget line need not always be a straight line.)
b) On the graph you drew in part (a), draw a set of indifference curves that illustrates why Toni may be better off with the frequent-flyer program.
c) On a new graph draw the same budget line you found in part (a). Now draw a set of indifference curves that illustrates why Toni might *not* be better off with the frequent-flyer program.

a) The budget line will have a kink where round trips = 10 and other goods = 5,000. Northwest of the kink, the budget line's slope will be –500 . Southeast of the kink, the slope will be –200.



b) With the indifference curves drawn on the above graph, Toni is better off with the frequent flyer program (at point B) than she would be without it (at point C). Without the frequent flyer program the best she could achieve is point C, which lies on the hypothetical budget line where the price of round trips is always $500.

c) With the indifference curves drawn on graph below, Toni is no better off with the frequent flyer program than she would be without it (at point A). At this point, her indifference curve is tangent to a portion of the budget line where the frequent flyer program does not apply (less than 10 round trips).

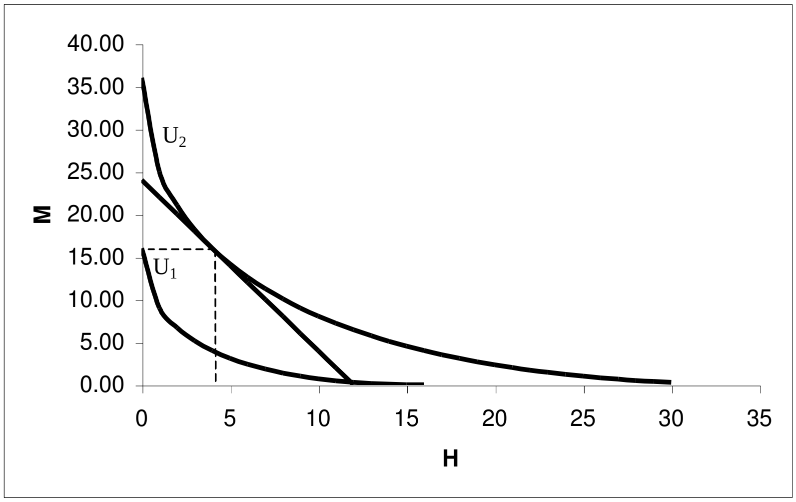


4.14 A consumer has preferences between two goods, hamburgers (measured by *H*) and milkshakes (measured by *M*). His preferences over the two goods are represented by the utility function *U* = √*H* + √*M*. For this utility function *MUH* = 1/(2√*H*) and *MUM* = 1/(2√*M*).
a) Determine if there is a diminishing *MRSH,M* for this utility function.
b) Draw a graph to illustrate the shape of a typical indifference curve. Label the curve *U*1. Does the indifference curve intersect either axis? On the same graph, draw a second indifference curve *U*2, with *U*2 > *U*1.
c) The consumer has an income of $24 per week. The price of a hamburger is $2 and the price of a milkshake is $1. How many milkshakes and hamburgers will he buy each week if he maximizes utility? Illustrate your answer on a graph.

a) $$MRS_{H,M} = \frac{MU_H}{MU_M} = \frac{1/(2\sqrt{H})}{1/(2\sqrt{M})} = \frac{\sqrt{M}}{\sqrt{H}}$$

This utility function has a diminishing marginal rate of substitution since $MRS_{H,M}$ declines as H increases and M decreases.

b)



Since it is possible to have $U > 0$ if either $H = 0$ (and $M > 0$) or $M = 0$ (and $H > 0$), the indifference curves will intersect both axes.

c) We know from the tangency condition that

$$\frac{\sqrt{M}}{\sqrt{H}} = \frac{2}{1}$$
$$M = 4H$$

Substituting this into the budget line, $2H + M = 24$, yields

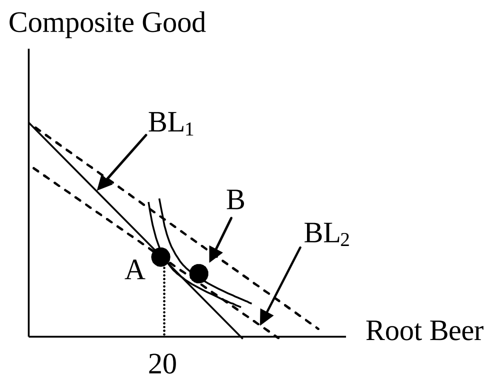
$$2H + 4H = 24$$
$$H = 4$$

Finally, plugging this back into the tangency condition implies $M = 4(4) = 16$. At the optimum the consumer will choose 4 hamburgers and 16 milkshakes. This can be seen in the graph above.

4.15 Justin has the utility function *U = xy*, with the marginal utilities *MUx = y* and *MUy = x*. The price of *x* is 2, the price of *y* is *py*, and his income is 40. When he maximizes utility subject to his budget constraint, he purchases 5 units of *y*. What must be the price of *y* and the amount of *x* consumed?

When Justin maximizes utility, his optimal consumption basket will be on the budget constraint and satisfy the tangency condition.
Any basket on the budget line will satisfy $p_x x + p_y y = I$, or $2x + 5p_y = 40$.
The tangency condition requires that $MU_x / p_x = MU_y / p_y$, or that $5 / 2 = x / p_y$. This implies that $5p_y = 2x$.
Putting these two equations together reveals that $5p_y + 5p_y = 40$; thus $p_y = 4$.

4.16 A student consumes root beer and a composite good whose price is $1. Currently, the government imposes an excise tax of $0.50 per six-pack of root beer. The student now purchases 20 six-packs of root beer per month. (Think of the excise tax as increasing the price of root beer by $0.50 per six-pack over what the price would be without the tax.) The government is considering eliminating the excise tax on root beer and, instead, requiring consumers to pay $10.00 per month as a lump sum tax (i.e., the student pays a tax of $10.00 per month, regardless of how much root beer is consumed). If the new proposal is adopted, how will the student's consumption pattern (in particular, the amount of root beer consumed) and welfare be affected? (Assume that the student's marginal rate of substitution of root beer for other goods is diminishing.)



Assume the student is initially at an interior optimum, point *A*. Denote the initial price of root beer by *P* and the student's income as *M*. Point *A* then consists of $R_A = 20$ units of root beer and $Y_A = M - 20P$ units of the composite good. The effect of the proposal is to rotate the budget line outward (the price change) and then shift it inward (the lump sum tax), for a total movement from BL_1 to BL_2. Notice that BL_2 intersects BL_1 exactly at point *A*: under the proposal, (R_A, Y_A) costs the student $20(P - 0.5) + M - 20P = M - 10$, which is equal to her income under the proposal.

Because *A* was initially optimal, $MRS_{R,Y} = P$ at point *A*. Yet the price ratio along BL_2 is $(P - 0.5)$. Hence $MRS_{R,Y} > P_R / P_Y$, so the student can increase her utility by purchasing more root beer and less of the composite good, at a point such as *B* depicted in the graph above. Thus, the proposal will make the student better off.

4.17 When the price of gasoline is $2.00 per gallon, Joe consumes 1,000 gallons per year. The price increases to $2.50, and to offset the harm to Joe, the government gives him a cash transfer of $500 per year. Will Joe be better off or worse off after the price increase and cash transfer than he was before? What will happen to his gasoline consumption? (Assume that Joe's marginal rate of substitution of gasoline for other goods is diminishing.)