

2.1 $v = \sqrt{2} \times 120 \cos(\omega t + 30^\circ) \Rightarrow V = 120 \angle 30^\circ \text{ V}$
 $i = \sqrt{2} \times 10 \cos(\omega t - 30^\circ) \Rightarrow I = 10 \angle -30^\circ \text{ A}$

(a) $p(t) = |V||I| [\cos \phi + \cos(2\omega t + \angle V + \angle I)]$
 $= 600 + 1200 \cos 2\omega t \text{ W}$

$S = VI^* = 1200 \angle 60^\circ = P + jQ \Rightarrow P = 600 \text{ W}, Q = 1039 \text{ VAR}$

(b) $Z = V/I = 12 \angle 60^\circ = 6 + j10.39 = R + jX \Rightarrow R = 6, X = 10.39$

2.2 (a) Using (2.3) we find $P_{\max} = 1707 = |V||I|(\cos \phi + 1)$
 and $P_{\min} = -293 = |V||I|(\cos \phi - 1)$. Then, since $|V| = 100$, we
 get $|I| = 10$ and $\cos \phi = \pm 45^\circ$. Pick $\phi = 45^\circ \Rightarrow Z = 10 \angle 45^\circ =$
 $7.07 + j7.07 = R + jX \Rightarrow R = 7.07, X = 7.07$

(b) $S = VI^* = Z|I|^2 = (7.07 + j7.07) 10^2 \Rightarrow P = 707, Q = 707$

(c) For simplicity assume $i(t) = \sqrt{2}|I| \cos \omega t$. Then
 $p_L(t) = v_L(t) i(t) = L \frac{di}{dt} i = -2\omega L|I|^2 \cos \omega t \sin \omega t = -\omega L|I|^2 \sin 2\omega t$
 $P_{L\max} = \omega L|I|^2 = 707 = Q$. Thus $P_{\max} = Q$. The same!

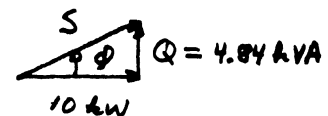
2.3 0.7 PF lagging $\Rightarrow \phi = 45.57^\circ, Q = 5.10 \text{ MVAR}$
 0.9 PF lagging $\Rightarrow \phi = 25.84^\circ, Q = 2.42 \text{ MVAR}$. Capacitor must
 supply $5.10 - 2.42 = 2.68 \text{ MVAR}$.

2.4 0.707 PF lagging $\Rightarrow S_{3\phi} = 200 + j200 \text{ kVA}$. Cap supplies
 50 kVAR . Resultant $S_{3\phi} = 200 + j150 \text{ kVA} \Rightarrow PF = 0.80$.

$|S| = \frac{|S_{3\phi}|}{3} = \frac{250 \times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}} |I| \Rightarrow |I| = 328 \text{ A}$

2.5 0.9 PF lagging $\Rightarrow \phi = 25.84^\circ$

(a) $S = 10 + j4.84 \text{ kVA}$



(b) $10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71 \text{ A}$

(c) Using (2.3), (or first principles) we get

$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84^\circ)$

Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208 \angle 120^\circ$, $V_{bc} = 208 \angle 0^\circ$.
 Using (2.17) or Fig 2.11, $V_{an} = 120 \angle 90^\circ \Rightarrow V_{bn} = 120 \angle -30^\circ$,
 $V_{cn} = 120 \angle -150^\circ$. Using per phase analysis, $I_a = 12 \angle 105^\circ \Rightarrow$
 $I_b = 12 \angle -15^\circ$, $I_c = 12 \angle -135^\circ$.

2.7 $S = VI^* = V(YV)^* = Y^* |V|^2 = Y_C^* + Y_L^* + Y_R^*$
 $= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,
 $I_a = 0.9123 \angle -90.351^\circ$, $I_b = 0.9123 \angle -209.65^\circ$, $I_c = 0.9929 \angle 30^\circ$.

(b) Using per phase analysis $I_a = 1 \angle 0^\circ / j1.1 = 0.9091 \angle -90^\circ$,
 then, $I_b = 0.9091 \angle -210^\circ$, $I_c = 0.9091 \angle 30^\circ$.

2.9 Proceeding by analogy with 3ϕ , we note

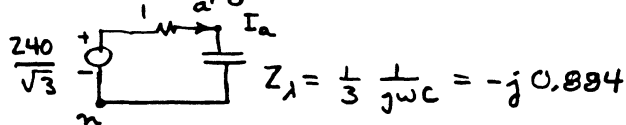
$$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2} E_{an} e^{j\pi/4}$$

Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab} e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form
 a pos. seq. set of 4ϕ voltages. Doing per phase (phase a)
 analysis we have

$$\frac{1}{\sqrt{2}} \angle 45^\circ \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} -j0.5 \Rightarrow I_a = \frac{\frac{1}{\sqrt{2}} \angle -45^\circ}{j0.5} = \sqrt{2} \angle -135^\circ$$

Then $I_b = \sqrt{2} \angle -225^\circ$, $I_c = \sqrt{2} \angle -315^\circ$, $I_d = \sqrt{2} \angle -405^\circ$

2.10 Using per phase circuit we find $I_a = 103.8 \angle 41.5^\circ$.



Then $|I_b| = 103.8 \text{ A}$

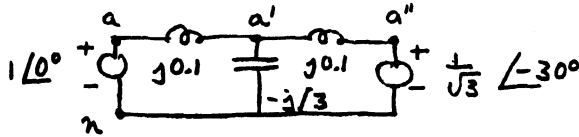
$$V_{a'n} = Z_1 I_a = 91.76 \angle -48.5^\circ \Rightarrow |V_{a'b'}| = \sqrt{3} \cdot 91.76 = 158.9 \text{ V}$$

$$S_{\text{load}} = V_{a'n} I_a^* = 9524 \angle -90^\circ$$

$$S_{\text{load}}^{3\phi} = 3 S_{\text{load}} = 28574 \angle -90^\circ \text{ W}$$

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3} V_{a''n} e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''} e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle -30^\circ$

Per Phase Ckt

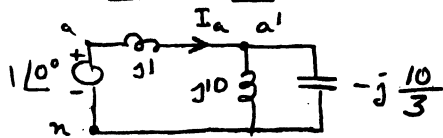


Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle -10.89^\circ \Rightarrow V_{b'n} = 0.899 \angle -130.89^\circ$ and
 $V_{c'n} = 0.899 \angle -250.89^\circ$. Then $V_{a'b'c'} = 1.557 \angle 19.11^\circ$

2.12 Assume pos. seq..

Per Phase Ckt



Combining parallel elements we have $Z_{||} = -j5$. $I_a = 0.25 \angle 90^\circ$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle 0^\circ$

$V_{a'b'c'} = 2.165 \angle 30^\circ \Rightarrow I_{cap} = 2.165 \angle 120^\circ$

$I_{load} = 3 V_{a'n} I_a^* = 0.3125 \angle -90^\circ$

2.13

(a) $V_{bc} = 208 \angle -120^\circ$, $V_{ca} = 208 \angle 120^\circ$

$V_{an} = \frac{208}{\sqrt{3}} \angle -30^\circ \Rightarrow I_a = 1.20 \angle -90^\circ$

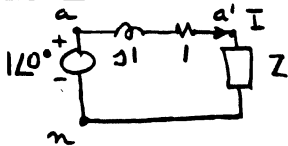
Then $I_b = 1.20 \angle -210^\circ$, $I_c = 1.20 \angle -330^\circ$

(b) $V_{bc} = 208 \angle 120^\circ$, $V_{ca} = 208 \angle -120^\circ$

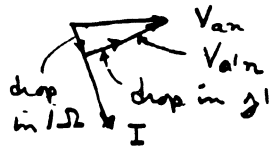
$V_{an} = \frac{208}{\sqrt{3}} \angle 30^\circ \Rightarrow I_a = 1.20 \angle -30^\circ$

$I_b = 1.20 \angle 90^\circ$, $I_c = 1.20 \angle -150^\circ$

2.14 Per Phase Ckt. Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.



I. $Z = j\omega L$



Clearly $|V_{a'n}| < |V_{an}|$

II $Z = R$



Clearly $|V_{a'n}| < |V_{an}|$

III $Z = -j \frac{1}{\omega C}$

For example $Z = -j2$

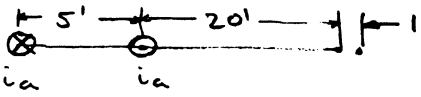
$I = \frac{1}{\sqrt{2}} \angle 45^\circ$

$V_{a'n} = \sqrt{2} \angle -45^\circ$

$|V_{a'n}| > |V_{an}|$

This is O.K.

2.15 Since $E_a + E_b + E_c = 0$, neutrals are at same potential.
 $Z = E_a / I_a = \sqrt{2} \angle 55^\circ$. For each Z , $S = VI^* = |V|^2 / Z^*$. Thus
 $S^{3\phi} = \frac{(\sqrt{2})^2 + 1^2 + 1^2}{\sqrt{2} \angle -55^\circ} = 2\sqrt{2} \angle 55^\circ$.

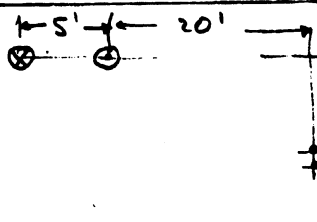
3.1  $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$, $|I_a| = 100$

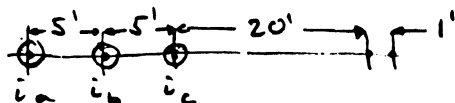
(Upward) flux linkages of telephone wires

$$\lambda = 2 \times 10^{-7} \left[i_a \ln \frac{26}{25} - i_b \ln \frac{21}{20} \right] = -0.00957 \times 2 \times 10^{-7} i_a$$

$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = j\omega \times (-0.00957 \times 2 \times 10^{-7}) \times 100 \text{ V/meter}$$

Since 1 mile = 1.609 km, $|V_{tel}| = 0.116 \text{ V/mile}$

3.2  $\lambda = 2 \times 10^{-7} i_a \left[\ln \frac{\sqrt{25^2 + 11^2}}{\sqrt{25^2 + 10^2}} - \ln \frac{\sqrt{20^2 + 11^2}}{\sqrt{20^2 + 10^2}} \right]$
 $= -0.066294 \times 2 \times 10^{-7} i_a$
 $\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$

3.3 

$$\lambda = 2 \times 10^{-7} \left[i_a \ln \frac{31}{30} + i_b \ln \frac{26}{25} + i_c \ln \frac{21}{20} \right]$$

$$= 2 \times 10^{-7} \left[0.0328 i_a + 0.0392 i_b + 0.0488 i_c \right]$$

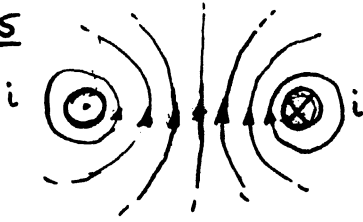
$$V_{tel} = j\omega \times 2 \times 10^{-7} \left[0.0328 I_a + 0.0392 I_b + 0.0488 I_c \right]$$

$$|V_{tel}| = 377 \times 2 \times 10^{-7} \times 100 \times \left| 0.0328 + 0.0392 \angle -120^\circ + 0.0488 \angle 120^\circ \right|$$

$$= 1048 \times 10^{-7} \text{ V/meter} = 0.1692 \text{ V/mile}$$

3.4 The telephone wires are transposed every 1000 ft. Cancellation occurs in all but 720' of line $\Rightarrow 0.0158 \text{ V}$.

3.5



The flux linkages due to the current in the left conductor is \approx

$$\lambda_1 = \frac{\mu_0}{2\pi} i \left[\frac{\mu_r}{4} + \ln \frac{D}{r} \right] = i \frac{\mu_0}{2\pi} \ln \frac{D}{r'}$$

Here we are neglecting partial flux linkages of the far (right) conductor. The current in the right conductor contributes an equal number of flux linkages. Thus (at least approximately),

$$L = 2\lambda_1 / i = \frac{\mu_0}{\pi} \ln \frac{D}{r'} = 4 \times 10^{-7} \ln \frac{D}{r'} \text{ H/meter}$$

3.6 If each conductor is hollow then there are no partial flux linkages and in (3.13) the term involving μ_r is absent. Then $r' = r$ and

$$L = \frac{\mu_0}{2\pi} \ln \frac{D}{r} \quad \text{H/m}$$

3.7 The hint is misleading. A better hint for combining the 7 (unequal) inductances L_1, L_2, \dots, L_7 would be to use the average inductance i.e. let

$$L_a = \frac{L_{av}}{7} = \frac{L_1 + L_2 + \dots + L_7}{7}$$

Returning to the problem, paralleling the steps which lead to (3.27) we have

$$\begin{aligned} \lambda_1 &= \frac{\mu_0}{2\pi} \left[\frac{i_a}{7} \left(\ln \frac{1}{d_{11}} + \ln \frac{1}{d_{12}} + \dots + \ln \frac{1}{d_{17}} \right) \right. \\ &\quad + \frac{i_b}{7} \left(\ln \frac{1}{d_{18}} + \ln \frac{1}{d_{19}} + \dots + \ln \frac{1}{d_{1,21}} \right) \\ &\quad \left. + \frac{i_c}{7} \left(\ln \frac{1}{d_{1,15}} + \ln \frac{1}{d_{1,16}} + \dots + \ln \frac{1}{d_{1,21}} \right) \right] \\ &\approx \frac{\mu_0}{2\pi} i_a \ln \frac{D}{(d_{11} \dots d_{17})^{1/7}} \Rightarrow L_1 = 7 \times \frac{\mu_0}{2\pi} \ln \frac{D}{(d_{11} \dots d_{17})^{1/7}} \end{aligned}$$