

4.1

Since it leads to an important result, let us first do it for the general case where  $X \sim N(\lambda, \zeta)$ , i.e.

$$f_X(x) = \frac{1}{\sqrt{2\pi\zeta}} e^{-\frac{1}{2}\left(\frac{x-\lambda}{\zeta}\right)^2}$$

To obtain PDF  $f_Y(y)$  for  $Y = e^X$ , one may apply Ang & Tang (4.6),

$$f_Y(y) = f_X(g^{-1}) \left| \frac{dg^{-1}}{dy} \right|$$

where

$$y = g(x) = e^x;$$

$$\Rightarrow x = g^{-1}(y) = \ln(y)$$

$$\Rightarrow \left| \frac{dg^{-1}}{dy} \right| = \left| \frac{d}{dy} \ln y \right| = \left| \frac{1}{y} \right|$$

$$= \frac{1}{y} \quad \text{since } y = e^x > 0$$

Hence, expressed as a function of  $y$ , the PDF  $f_Y(y)$  is

$$\begin{aligned} f_X(g^{-1}) \left| \frac{dg^{-1}}{dy} \right| &= f_X(\ln(y)) \frac{1}{y} \\ &= \frac{1}{\sqrt{2\pi y\zeta}} e^{-\frac{1}{2}\left(\frac{\ln y - \lambda}{\zeta}\right)^2} \quad (\text{non-negative } y \text{ only}) \end{aligned}$$

which, by comparison to Ang & Tang (3.29), is exactly what is called a log-normal distribution with parameters  $\lambda$  and  $\zeta$ , i.e. if  $X \sim N(\lambda, \zeta)$ , and  $Y = e^X$ , then  $Y \sim \text{LN}(\lambda, \zeta)$ .

Hence, for the particular case where  $X \sim N(2, 0.4)$ ,  $Y$  is LN with parameters  $\lambda$  being 2 and  $\zeta$  being 0.4.

## 4.2

To have a better physical feel in terms of probability (rather than probability density), let's work with the CDF (which we can later differentiate to get the PDF) of  $Y$ : since  $Y$  cannot be negative, we know that  $P(Y < 0) = 0$ , hence

when  $y < 0$ :

$$\begin{aligned} F_Y(y) &= 0 \\ \Rightarrow f_Y(y) &= [F_Y(y)]' = 0 \end{aligned}$$

But when  $y \geq 0$ ,

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P\left(\frac{1}{2}mX^2 \leq y\right) \\ &= P\left(-\sqrt{\frac{2y}{m}} \leq X \leq \sqrt{\frac{2y}{m}}\right) \\ &= F_X\left(\sqrt{\frac{2y}{m}}\right) - F_X\left(-\sqrt{\frac{2y}{m}}\right) \\ &= F_X\left(\sqrt{\frac{2y}{m}}\right) - 0 \\ &= F_X\left(\sqrt{\frac{2y}{m}}\right) \end{aligned}$$

Hence the PDF,

$$\begin{aligned} f_Y(y) &= \frac{d}{dy} [F_Y(y)] \\ &= f_X\left(\sqrt{\frac{2y}{m}}\right) \frac{d}{dy} \sqrt{\frac{2y}{m}} \\ &= \frac{8y}{ma^3 \sqrt{\pi}} \exp\left(-\frac{2y}{ma^2}\right) \frac{1}{\sqrt{2my}} \\ &= \frac{4}{a^3} \sqrt{\frac{2y}{\pi m^3}} \exp\left(-\frac{2y}{ma^2}\right) \end{aligned}$$

Hence the answer is

$$f_Y(y) = \begin{cases} \frac{4}{a^3} \sqrt{\frac{2y}{\pi m^3}} \exp\left(-\frac{2y}{ma^2}\right) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

4.3

A = volume of air traffic  
C = event of overcrowded

(a) T = total power supply =  $N(\mu_T, \sigma_T)$

Where  $\mu_T = 100 + 200 + 400$

$$\sigma_T = \sqrt{15^2 + 40^2 + 40^2} = 58.5$$

(b) P(Normal weather) = P(W) = 2/3

P(Extreme weather) = P(E) = 1/3

P(Power shortage) = P(S) = P(S | W)P(W) + P(S | E)P(E)

$$= P(T < 400) \times 2/3 + P(T < 600) \times 1/3$$

$$= [\Phi(\frac{400 - 400}{58.5})] \times \frac{2}{3} + [\Phi(\frac{600 - 400}{58.5})] \times \frac{1}{3}$$

$$= [\Phi(0)] \times \frac{2}{3} + [\Phi(3.42)] \times \frac{1}{3}$$

$$= 0.5 \times 0.667 + 0.99968 \times 0.333$$

$$= 0.667$$

(c)  $P(W | S) = \frac{P(S|W)P(W)}{P(S)} = \frac{0.5 \times 0.667}{0.667} = 0.5$

(d) P(all individual power source can meet respective demand)

= P(N > 0.15x400)P(F > 0.3x400)P(H > 0.55x400)

$$= [1 - \Phi(\frac{60 - 100}{15})][1 - \Phi(\frac{120 - 200}{40})][1 - \Phi(\frac{220 - 400}{40})]$$

$$= \Phi(2.67) \times \Phi(2) \times \Phi(4.5)$$

$$= 0.09962 \times 0.977 \times 1$$

$$= 0.973$$

P(at least one source not able to supply respective allocation) = 0.027

4.4

(a) Let  $T_J$  be John's travel time in (hours);  $T_J = T_3 + T_4$  with

$$\mu_{T_J} = 5 + 4 = 9 \text{ (hours), and}$$

$$\sigma_{T_J} = [3^2 + 1^2 + (2)(0.8)(3)(1)]^{1/2} = 3.847 \text{ (hours)}$$

Hence

$$\begin{aligned} P(T_J > 10 \text{ hours}) &= 1 - P\left(\frac{T_J - \mu_{T_J}}{\sigma_{T_J}} \leq \frac{10 - 9}{3.847}\right) \\ &= 1 - \Phi(0.26) = 1 - 0.603 \\ &\cong \mathbf{0.397} \end{aligned}$$

(b) Let  $T_B$  be Bob's travel time in (hours);  $T_B = T_1 + T_2$  with

$$\mu_{T_B} = 6 + 4 = 10 \text{ (hours), and}$$

$$\sigma_{T_B} = [2^2 + 1^2]^{1/2} = \sqrt{5} \text{ (hours)}$$

Hence

$$P(T_J - T_B > 1) = P(T_B - T_J + 1 < 0),$$

now let  $R \equiv T_B - T_J + 1$ ;  $R$  is normal with

$$\mu_R = \mu_{T_B} - \mu_{T_J} + 1 = 10 - 9 + 1 = 2,$$

$$\sigma_R = [\sigma_{T_B}^2 + \sigma_{T_J}^2]^{1/2} = (5 + 14.8)^{1/2} = \sqrt{19.8},$$

hence

$$\begin{aligned} P(R < 0) &= \Phi\left(\frac{0 - 2}{\sqrt{19.8}}\right) \\ &= \Phi(-0.449) \\ &\cong \mathbf{0.327} \end{aligned}$$

(c) Since the lower route (A-C-D) has a smaller expected travel time of  $\mu_{T_J} = 9$  hours as compared to the upper (with expected travel time =  $\mu_{T_B} = 10$  hours), one should take the **lower** route to minimize expected travel time from A to D.

4.5

- (a) To calculate probability, we first need to have the PDF of  $S$ . As a linear combination of three normal variables,  $S$  itself is normal, with parameters

$$\begin{aligned}\mu_S &= 0.3 \times 5 + 0.2 \times 8 + 0.1 \times 7 = 3.8 \text{ (cm)} \\ \sigma_S &= \sqrt{0.3^2 1^2 + 0.2^2 2^2 + 0.1^2 1^2} \cong 0.51 \text{ (cm)}\end{aligned}$$

Hence

$$\begin{aligned}P(S > 4\text{cm}) &= 1 - \Phi\left(\frac{4 - 3.8}{0.51}\right) \\ &= 1 - \Phi(0.3922) = 1 - 0.6526 \\ &\cong \mathbf{0.347}\end{aligned}$$

- (b) Now that we have a constraint  $A + B + C = 20\text{m}$ , these variables are no longer all independent, for example, we have

$$C = 20 - A - B$$

Hence

$$\begin{aligned}S &= 0.3A + 0.2B + 0.1(20 - A - B) \\ \Rightarrow S &= 0.2A + 0.1B + 2, \text{ with } \rho_{AB} = 0.5.\end{aligned}$$

Thus

$$\mu_S = 0.2 \times 5 + 0.1 \times 8 + 2 = 3.8 \text{ (cm) as before, and}$$

$$\begin{aligned}\sigma_S &= \sqrt{0.2^2 1^2 + 0.1^2 2^2 + 2 \times 0.5 \times 0.1 \times 1 \times 2} \\ &= \sqrt{0.12} \cong 0.346 \text{ (cm)}\end{aligned}$$

Hence

$$\begin{aligned}P(S > 4\text{cm}) &= 1 - \Phi\left(\frac{4 - 3.8}{\sqrt{0.12}}\right) \\ &= 1 - \Phi(0.577) \cong \mathbf{0.282}\end{aligned}$$

4.6

$$\begin{aligned}Q &= 4A + B + 2C \\ &= 4A + B + 2(30 - A - B) \\ &= 2A - B + 60, \text{ hence}\end{aligned}$$

$$\mu_Q = 2 \times 5 - 8 + 60 = 62,$$

$$\begin{aligned}\sigma_Q &= [4 \times 3^2 + 1 \times 2^2 + 2(2)(-1)(-0.5)(3)(2)]^{1/2} \\ &= (36 + 4 + 12)^{1/2} = \sqrt{52}\end{aligned}$$

$$\text{Hence } P(Q < 40) = P\left(\frac{Q - \mu_Q}{\sigma_Q} < \frac{40 - 62}{\sqrt{52}}\right)$$

$$= \Phi(-3.0508)$$

$$\cong \mathbf{0.00114}$$

4.7

(a) Let  $Q_1$  and  $Q_2$  be the annual maximum flood peak in rivers 1 and 2, respectively. We have

$$Q_1 \sim N(35, 10), Q_2 \sim N(25, 10)$$

The annual max. peak discharge passing through the city,  $Q$ , is the sum of them,

$$\begin{aligned} Q &= Q_1 + Q_2, \text{ hence} \\ \mu_Q &= \mu_{Q_1} + \mu_{Q_2} = 35 + 25 = \mathbf{60} \text{ (m}^3\text{/sec), and} \\ \sigma_Q^2 &= \sigma_{Q_1}^2 + \sigma_{Q_2}^2 + 2\rho_{Q_1, Q_2}\sigma_{Q_1}\sigma_{Q_2} \\ &= 10^2 + 10^2 + 2 \times 0.5 \times 10 \times 10 = 300 \text{ (m}^3\text{/sec),} \\ \Rightarrow \sigma_Q &= \sqrt{300} \cong \mathbf{17.32} \text{ (m}^3\text{/sec)} \end{aligned}$$

(b) The annual risk of flooding,  $p = P(Q > 100)$

$$\begin{aligned} &= 1 - \Phi\left(\frac{100 - 60}{\sqrt{300}}\right) \\ &= 1 - \Phi(2.309) = 1 - 0.9895 \\ &= \mathbf{0.0105} \text{ (probability each year)} \end{aligned}$$

Hence the return period is  $\tau = \frac{1}{p}$

$$\begin{aligned} &= \frac{1}{0.0105} = 95.59643882 \\ &\cong \mathbf{95} \text{ years.} \end{aligned}$$

(c) Since the yearly risk of flooding is  $p = 0.0105$ , and we have a course of  $n = 10$  years, we adopt a binomial model for  $X$ , the total number of flood years over a 10-year period.

$$\begin{aligned} P(\text{city experiences (any) flooding}) &= 1 - P(\text{city experiences no flooding at all}) \\ &= 1 - P(X = 0) \\ &= 1 - (1 - p)^{10} = 1 - (1 - 0.0105)^{10} \\ &= 1 - 0.9895^{10} \\ &\cong \mathbf{10\%} \end{aligned}$$

(d) The requirement on  $p$  is, using the flooding probability expression from part (c):

$$\begin{aligned} 1 - (1 - p)^{10} &= 0.1 \div 2 = 0.05 \\ \Rightarrow p &= 1 - (1 - 0.05)^{1/10} = 0.0051, \end{aligned}$$

which translates into a condition on the design channel capacity  $Q_0$ , following what's done in (b),

$$\begin{aligned} 1 - \Phi\left(\frac{Q_0 - 60}{\sqrt{300}}\right) &= 0.0051 \\ \Rightarrow \Phi\left(\frac{Q_0 - 60}{\sqrt{300}}\right) &= 0.9949 \\ \Rightarrow Q_0 &= 60 + \Phi^{-1}(0.9949) \sqrt{300} \\ &= 60 + 2.57 \sqrt{300} = 104.5 \end{aligned}$$

∴ Extending the channel capacity to about **104.5 m<sup>3</sup>/sec** will cut the risk by half.

4.8

$$\begin{aligned} P(T < 30) &= P(T < 30 | N=0)P(N=0) + P(T < 30 | N=1)P(N=1) + P(T < 30 | N=2)P(N=2) \\ &= \Phi\left(\frac{30-30}{5}\right) \times 0.2 + \Phi\left(\frac{30-35}{\sqrt{25+3^2}}\right) \times 0.5 + \Phi\left(\frac{30-40}{\sqrt{25+2 \times 3^2}}\right) \times 0.3 \\ &= 0.5 \times 0.2 + \Phi(-0.857) \times 0.5 + \Phi(-1.525) \times 0.3 \\ &= 0.217 \end{aligned}$$

4.9

$$\begin{aligned} N &= \text{total time for one round trip in normal traffic} \\ &= N(30+20+40, \sqrt{9^2 + 4^2 + 12^2}) \\ &= N(90, 15.52) \end{aligned}$$

$$\begin{aligned} R &= \text{total round trip time in rush hour traffic} \\ &= N(30+30+40, \sqrt{9^2 + 6^2 + 12^2}) \\ &= N(100, 16.16) \end{aligned}$$

$$\begin{aligned} \text{(a)} \quad &P(\text{on schedule in normal traffic}) \\ &= P(N < 20) \\ &= \Phi\left(\frac{120 - 90}{15.52}\right) = \Phi(1.933) = 0.974 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad &T_A = \text{Normal time taken for passenger starting from A} = T_2 + T_3 \\ &= N(20+40, \sqrt{4^2 + 12^2}) = N(60, 12.65) \\ P(T_A < 60) &= \Phi\left(\frac{60 - 60}{12.65}\right) = \Phi(0) = 0.5 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad &\text{Under rush hour traffic} \\ T_A &= N(30+40, \sqrt{6^2 + 12^2}) = N(70, 13.4) \\ T_B &= N(40, 12) \\ P(T_A < 60) &= \Phi\left(\frac{60 - 70}{13.4}\right) = \Phi(-0.746) = 0.227 \\ P(T_B < 60) &= \Phi\left(\frac{60 - 40}{12}\right) = \Phi(1.667) = 0.952 \end{aligned}$$

$$\begin{aligned} &\text{Percentage of passengers arriving in less than an hour} \\ &= 0.227 \times 1/3 + 0.952 \times 2/3 \\ &= 0.7 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad &P(T_B < 60 \mid T_B > 45) = \frac{P(45 < T_B < 60)}{P(T_B > 45)} \\ &= \frac{\Phi\left(\frac{60 - 40}{12}\right) - \Phi\left(\frac{45 - 40}{12}\right)}{1 - \Phi\left(\frac{45 - 40}{12}\right)} = \frac{\Phi(1.667) - \Phi(0.417)}{1 - \Phi(0.417)} = \frac{0.952 - 0.661}{0.339} = 0.858 \end{aligned}$$

4.10

(a)  $D = S_1 - S_2$

$$\mu_D = \mu_{S_1} - \mu_{S_2} = 2 - 2 = 0$$

$$\text{Var}(D) = \text{Var}(S_1) + \text{Var}(S_2) - 2\rho\sqrt{\text{Var}(S_1)\text{Var}(S_2)}$$

$$= (0.3 \times 2)^2 + (0.3 \times 2)^2 - 2 \times 0.7(0.3 \times 2)^2$$

$$= .216$$

(b)  $P(-.5 < D < .5) = \Phi\left(\frac{0.5 - 0}{\sqrt{0.216}}\right) - \Phi\left(\frac{-0.5 - 0}{\sqrt{0.216}}\right) = \Phi(1.076) - \Phi(-1.076) = 0.718$

4.11

(a)  $X_5 = X_1 + X_2 + X_3$

$$\mu_5 = \mu_1 + \mu_2 + \mu_3 = 10 + 15 + 20 = 45$$

$$\sigma_5^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2 = 3^2 + 3^2 + 2 \times 0.6 \times 3 \times 3 = 28.8$$

$$\therefore \sigma_5 = 5.37$$

(b)  $V_1 = 60X_1$

$$V_2 = 60X_2$$

$$\therefore P(V_2 - V_1 > 400) = P(Z > 400)$$

$$\therefore \mu_Z = 60\mu_2 - 60\mu_1 = 60 \times 15 - 60 \times 10 = 300$$

$$\sigma_Z^2 = 60^2\sigma_2^2 + 60^2\sigma_1^2 - 2\rho_{1,2} \times 60^2\sigma_1\sigma_2 = 25920$$

$$\therefore \sigma_Z = 161$$

$$\therefore P(V_2 - V_1 > 400) = 1 - \Phi\left(\frac{400 - 300}{161}\right) = 1 - \Phi(.621) = .267$$

(c)  $X_5 = X_1 + X_2 + X_3$

$$\mu_5 = \mu_1 + \mu_2 + \mu_3 = 10 + 15 + 20 + 3n = 45 + 3n$$

$$\sigma_5^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2 = 3^2 + 3^2 + 2 \times 0.6 \times 3 \times 3 = 28.8$$

$$\therefore \sigma_5 = 5.37$$

$$\therefore P(X_5 > 70) = .05$$

$$\therefore P(X_5 < 70) = .95$$

$$\Phi\left(\frac{70 - 45 - 3n}{5.37}\right) = .95$$

$$\Rightarrow 25 - 3n = 5.37\Phi^{-1}(.95) = 5.37 * 1.645 = 8.839$$

$$\Rightarrow n = 5.4$$

4.12

(a)  $P(X > 20 \cup Y > 20) = P(X > 20) + P(Y > 20) - P(X > 20)P(Y > 20)$

$$\begin{aligned} &= (1 - \Phi(\frac{20-20}{4})) + (1 - \Phi(\frac{20-15}{3})) - (1 - \Phi(\frac{20-20}{4})) \times (1 - \Phi(\frac{20-15}{3})) \\ &= 0.5 + 0.0478 - 0.5 \times 0.0478 \\ &= 0.524 \end{aligned}$$

(b) Z in normal distribution with

$$\mu_z = 0.6 \times 20 + 0.4 \times 15 = 18$$

$$\sigma_z = \sqrt{0.6^2 4^2 + 0.4^2 3^2} = 2.683$$

$$\therefore P(Z > 20) = 1 - \Phi(\frac{20-18}{2.683}) = 1 - 0.7718 = 0.2282$$

(c)  $\mu_z = 0.6 \times 20 + 0.4 \times 15 = 18$

$$\sigma_z = \sqrt{0.6^2 4^2 + 0.4^2 3^2 + 2 \times 0.8 \times 0.6 \times 0.4 \times 4 \times 3} = 3.436$$

$$\therefore P(Z > 20) = 1 - \Phi(\frac{20-18}{3.436}) = 1 - 0.72 = 0.28$$

4.13

(a)  $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$$= 1 - \binom{5}{0} 0.6^0 0.4^5 - \binom{5}{1} 0.6^1 0.4^4$$

$$= 1 - 0.4^5 - 5 \times 0.6 \times 0.4^4$$

$$= 0.046$$

(b)  $N_H, N_B$  are the number of highway and building jobs won

$$P(N_H = 1 \cap N_B = 0) = P(N_H = 1)P(N_B = 0)$$

$$= \binom{3}{1} 0.6^1 0.4^2 \binom{2}{0} 0.6^0 0.4^2$$

$$= 0.046$$

(c)  $T = H_1 + H_2 + B$  where  $H_1, H_2$  and  $B$  are the profits from the respective jobs.  
 $T$  in Normal distribution with

$$\mu_T = 100 + 100 + 80 = 280$$

$$\sigma_T = \sqrt{40^2 + 40^2 + 20^2} = 60$$

$$\therefore P(T > 300) = 1 - \Phi\left(\frac{300 - 280}{60}\right) = 1 - \Phi(0.333) = 0.2695$$

(d)  $\mu_T = 100 + 100 + 80 = 280$

$$\sigma_T = \sqrt{40^2 + 40^2 + 20^2 + 2 \times 0.8 \times 40 \times 40} = 78.5$$

$$\therefore P(T > 300) = 1 - \Phi\left(\frac{300 - 280}{78.5}\right) = 1 - \Phi(0.255) = 0.4$$

4.14

S: the amount of water available. S follows LogNormal Distribution with

$$E(S)=1, \text{ c.o.v}=0.4$$

$$\zeta_S^2 = \ln(1 + 0.4^2) = 0.14842$$

$$\lambda_S = \ln \mu_S - \frac{1}{2} \zeta_S^2 = \ln 1 - 0.5 \times 0.14842 = -0.07421$$

D: the total demand of water.

$$E(D)=1.5, \text{ c.o.v}=0.1$$

$$\zeta_D^2 = 0.1^2 = 0.01$$

$$\lambda_D = \ln \mu_D - \frac{1}{2} \zeta_D^2 = \ln 1.5 - 0.5 \times 0.01 = 0.4005$$

P(water shortage)=P(D>S)

$$=P(D/S > 1)$$

$$=P((Z=\ln(D/S)) > 0)$$

Z follows Normal Distribution with:

$$\text{Var}(Z) = \zeta_D^2 + \zeta_S^2 = 0.1^2 + 0.148 = 0.158$$

$$\mu_Z = \lambda_D - \lambda_S = 0.4005 - (-0.07421) = 0.4747$$

$$\Rightarrow P(Z > 0) = 1 - \Phi\left(\frac{0 - 0.4747}{\sqrt{0.158}}\right) = 0.883$$

4.15

- (a) P is lognormally distributed with

$$\begin{aligned}\lambda_P &= \lambda_C + \lambda_R + 2\lambda_V - \ln 2 \\ &= (\ln 1.8 - 0.5 \times 0.2^2) + (\ln 2.3 \times 10^{-3} - 0.5 \times 0.1^2) + 2[\ln 120 - 0.5 \times \ln(1 + 0.45^2)] - \ln 2 \\ &= 0.568 + (-6.08) + 2(4.6953) - 0.693 \\ &= 3.186\end{aligned}$$

$$\begin{aligned}\xi_P \\ \sqrt{\xi_C^2 + \xi_R^2 + 4\xi_V^2} &= \sqrt{0.2^2 + 0.1^2 + 4 \times \ln(1 + 0.45^2)} = \sqrt{0.2^2 + 0.1^2 + 4 \times 0.1844} \\ &= 0.887\end{aligned}$$

- (b)  $P(P > 30) = 1 - \Phi\left(\frac{\ln 30 - 3.186}{0.887}\right) = 1 - \Phi(0.243) = 0.596$

- (c) C is lognormally distributed with

$$\begin{aligned}\lambda_C &= \ln 90 - 0.5 \times 0.15^2 = 4.4886 \\ \xi_C &= 0.15\end{aligned}$$

$$P(\text{Failure of antenna}) = P(C < P) = P(C/P < 1)$$

Define  $B = C/D$

B also follows a lognormal distribution with

$$\begin{aligned}\lambda_B &= \lambda_C - \lambda_P = 4.4886 - 3.186 = 1.7 \\ \xi_B &= \sqrt{\xi_C^2 + \xi_P^2} = \sqrt{0.15^2 + 0.887^2} = 0.9\end{aligned}$$

Hence,  $P(\text{Failure of antenna}) = P(B < 1)$

$$= \Phi\left(\frac{\ln 1 - \lambda_B}{\xi_B}\right) = \Phi\left(\frac{-1.7}{0.9}\right) = \Phi(-1.89) = 0.029$$

- (d) Mean rate of wind storm causing failure

$$= 1/5 \times 0.029 = 0.0058$$

$P(\text{antenna failure in 25 years}) = 1 - P(\text{no damaging storm in 25 years})$

$$= 1 - e^{-0.0058 \times 25} = 0.135$$

- (e)  $P(\text{at least two out of 5 antenna failures})$

$$\begin{aligned}&= 1 - P(X=0) - P(X=1) \\ &= 1 - 0.865^5 - 5(0.135)(0.865)^4 \\ &= 0.138\end{aligned}$$

4.16

(a)  $P(\text{failure})=P(L>C)=P(C/L<1)=P(Z<1)$

Z in LN with  $\lambda_z = \lambda_c - \lambda_L$

$$\zeta_z = \sqrt{\zeta_c^2 + \zeta_L^2}$$

in which  $\zeta_c = 0.2$

$$\lambda_c = \ln 20 - \frac{1}{2}(0.2)^2 = 2.259$$

$$\zeta_L = \sqrt{\ln(1 + \delta^2)} = .294$$

$$\lambda_L = \ln 10 - \frac{1}{2}(0.294)^2 = 2.259$$

$$\therefore \lambda_z = 2.976 - 2.259 = 0.717$$

$$\zeta_z = \sqrt{0.2^2 + 0.294^2} = 0.356$$

$$\therefore P_F = \Phi\left(\frac{\ln 1 - 0.717}{0.356}\right) = \Phi(-2.014) = 1 - 0.978 = 0.022$$

(b)  $T=C_1+C_2$

$$E(T)=E(C_1)+E(C_2)=20+20=40$$

$$Var(T) = Var(C_1) + Var(C_2) + 2\rho\sigma_{C_1}\sigma_{C_2} = 57.6$$

$$\delta_T = \frac{\sqrt{57.6}}{40} = 0.19$$

(c) some other distribution

4.17

(a)  $C=F+B$

$$\mu_C = \mu_F + \mu_B = 20 + 30 = 50$$

$$\sigma_C = \sqrt{(0.2 \times 20)^2 + (0.3 \times 30)^2} = 9.85$$

(b)  $T=C_1+C_2=2 \mu_C=0.197$

$$\sigma_T = \sqrt{9.85^2 + 9.85^2 + 2 \times 0.8 \times 9.85^2} = 18.69$$

$$\therefore \delta_T = 18.69 / 100 = 0.187$$

(c)  $P(\text{failure})=P(T<L)=P(T-L<0)$

$$= \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right)$$

$$= \Phi\left(\frac{-(100 - 50)}{\sqrt{18.69^2 + (.3 \times 50)^2}}\right)$$

$$= \Phi\left(\frac{-50}{23.95}\right)$$

$$= 0.0184$$

4.18

$$(a) \quad F = 18 + \sum_{i=1}^{16} A_i$$

$$\mu_F = 18 + 16 \times 0.1 = 19.6$$

$$\sigma_F = \sqrt{(0.3 \times 0.1)^2 \times 16} = 0.12$$

$$P(F > 20) = 1 - \Phi\left(\frac{20 - 19.6}{0.12}\right) = 0.00043$$

(b)

$$i. \quad P(\text{no collapse}) = P(C') = P(F < M) = P(F - M < 0) = P(Z < 0)$$

$$\mu_Z = \mu_F - \mu_M = 19.6 - 20 = -0.4$$

$$\sigma_Z = \sqrt{(0.12)^2 + (0.01 \times 20)^2} = 0.233$$

$$P(C') = \Phi\left(\frac{0 - (-0.4)}{0.233}\right) = 0.957$$

$$ii. \quad F = 18 + 16A$$

$$\mu_F = 18 + 16 \times 0.1 = 19.6$$

$$\sigma_F = \sqrt{(0.3 \times 0.1)^2 \times 16^2} = 0.48$$

$$\sigma_Z = \sqrt{(0.48)^2 + (0.01 \times 20)^2} = 0.52$$

$$P(C') = \Phi\left(\frac{0 - (-0.4)}{0.52}\right) = 0.779$$

4.19

- (a)  $a$  is lognormal with mean  $0.3g$  and c.o.v. of 25%

$$W = 200 \text{ kips}$$

$F = wa/g$  is also lognormal with

$$\lambda_F = \ln 200 + \lambda_a = 5.298 - 1.204 = 4.094$$

$$\xi_F = \xi_a = 0.25$$

$R = \text{frictional resistance} = WC$

Where  $C = \text{coefficient of friction}$  is lognormal with median  $0.4$  and a c.o.v. of  $0.2$

Hence  $R$  is lognormal with

$$\lambda_R = \ln 200 + \lambda_C = \ln 200 + \ln 0.4 = 4.382$$

$$\xi_R = \xi_C = 0.2$$

$$P(\text{failure}) = P(R < F)$$

$$= \Phi\left(\frac{-\lambda_R + \lambda_F}{\sqrt{\xi_R^2 + \xi_F^2}}\right) = \Phi\left(\frac{-4.382 + 4.094}{0.32}\right) = \Phi(-0.9) = 0.184$$

- (b)  $P(\text{none out of five tanks will fail})$

$$= (0.184)^5 = 0.00021$$