CHAPTER 1

1.1-1. $A = \{0 < \text{integers} < 4\}$, $B = \{6 < \text{even integers} < 16\}$, $C = \{0 < \text{odd integers}\}$. Other definitions also are possible.

1.1-2. Class $= \{A, B, C\}$ or class $= \{\{1,2,3\}, \{8,10,12,14\}, \{1,3,5,7,\cdots\}\}$.

1.1-3. A, B, D, E, and F are countable and finite. C is countable and infinite. G, H, and I are uncountable and infinite.

1.1-4. $A = B$, $A \subset C$, $A \subset G$, $A \subset I$.

$B = A$, $B \subset C$, $B \subset G$, $B \subset I$.

C is not equal to, or a subset of, any of the other sets. The same applies to D.

$E \subset D$.

$F \subset D$, $F \subset E$ (Note that F may be a null set.)

G is not equal to, or a subset of, any of the other sets.

$H \subset G$, if x is in meters and negative length is allowed.

I is not equal to, or a subset of, any of the other sets.

1.1-5. $\{\cdot\}$ (null set); $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$; $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$; $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, $\{b, c, d\}$; $\{a, b, c, d\}$.

1.1-6. (a) $S = \{-40°F \le t \le 130°F\}$, t is temperature. (b) $\{t \le 32°F\}$. (c) $\{32°F < t \le 100°F\}$.

★ 1.1-7. The null set is one subset. There are N subsets with one element each. Taking elements by pairs corresponds to $\binom{N}{2} = N!/2!(N-2)!$ subsets, which is the number of combinations possible of N things taken 2 at a time. For subsets with three elements the number is $\binom{N}{3} = N!/3!(N-3)!$. Continuing the logic gives

$$\text{Subsets} = \sum_{i=0}^{N} \binom{N}{i} = \sum_{i=0}^{N} \frac{N!}{i!(N-i)!} = 2^N$$

where a series has been used from Jolley (1961, p. 36).

1.1-8. (a) $S = \{-10 \le s \le 10\}$. (b) $V = \{0 \le s \le 10\}$. (c) $S = \{-13 \le s \le 7\}$, $V = \{0 \le s \le 7\}$.

1.1-9. $A = \{-7, -6, -5, \cdots, -1, 0, 1, \cdots, 567\}$, $B = \{-1, 1, 3, 5\}$, $A \subset B$ is not true, $B \subset A$ is true.

1.1-10. $A_1 = \{a_1\}$, $A_2 = \{a_2\}$, $A_3 = \{a_3\}$, $A_4 = \{a_1, a_2\}$, $A_5 = \{a_1, a_3\}$, $A_6 = \{a_2, a_3\}$, $A_7 = \{a_1, a_2, a_3\} = A$, $A_8 = \phi$.

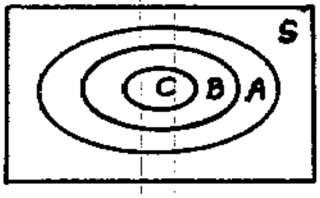
(1.1-11.) (a) $\{2, 3, 4, 5, 6, 7, 8\}$, $\{1 < I = \text{integers} < 9\}$.

(b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $\{1 \le I = \text{integers} \le 9\}$.

(c) $R_{equivalent} = R/n$, $n = 1, 2, \cdots, 5$, where $R = 10\,\Omega$, so $\{10, 5, 10/3, 2.5, 2\}$, $\{R = 10/n, n = 1, 2, \cdots, 5\}$. (d) $R_{equivalent} = nR$, $n = 1, 2, \cdots, 6$, where $R = 2.2\,\Omega$, so $\{2.2, 4.4, 6.6, 8.8, 11.0, 13.2\}$, $\{R = 2.2\,n,\ n = 1, 2, \cdots, 6\}$.

(1.1-12.) (a) 60, (b) 100, (c) 35.

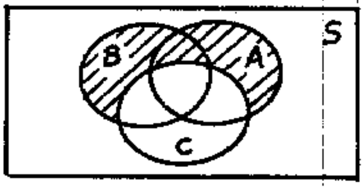
(1.2-1.) A Venn diagram proves $C \subset A$ if $C \subset B$ and $B \subset A$.



(1.2-2.) (a) $A - B = \{-6, -4, 1.6, 8\}$. (b) $B - A = \{1, 2, 4\}$.
(c) $A \cup B = \{-6, -4, -0.5, 0, 1, 1.6, 2, 4, 8\}$.
(d) $A \cap B = \{-0.5, 0\}$.

(1.2-3.) (a) $\bar{A} = S - A = \{6, 8, 12\}$. (b) $A - B = \{2\}$; $B - A = \{6, 8\}$. (c) $A \cup B = \{2, 4, 6, 8, 10\}$.
(d) $A \cap B = \{4, 10\}$. (e) $\bar{A} \cap B = \{6, 8\}$.

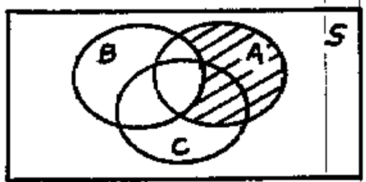
(1.2-4.) (a)



$(A \cup B) - C$ shaded

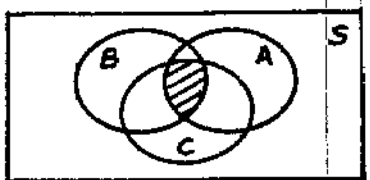
1.2-4. (Continued)

(b)



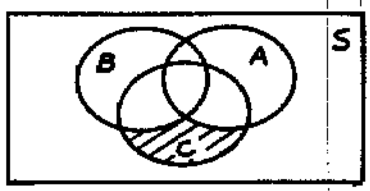
$\bar{B} \cap A$ shaded

(c)



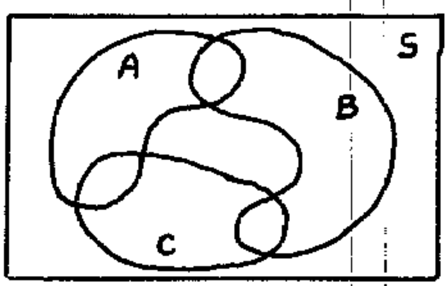
$A \cap B \cap C$ shaded

(d)



$\overline{(A \cup B)} \cap C$ shaded

1.2-5.



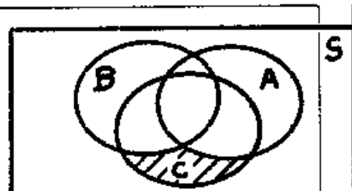
$A \cap B \neq \emptyset$

$B \cap C \neq \emptyset$

$A \cap C \neq \emptyset$

$A \cap B \cap C = \emptyset$.

1.2-6. (a)

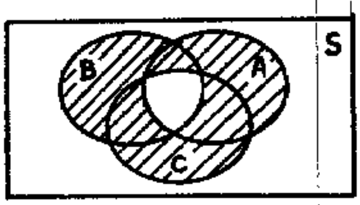


$(\overline{A \cup B}) \cap C$ or

$C - [(A \cap C) \cup (B \cap C)]$

shaded.

(b)

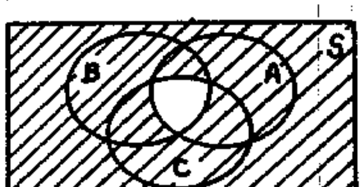


$(A \cup B \cup C) - (A \cap B \cap C)$ or

$(\bar{A} \cap B) \cup (\bar{B} \cap C) \cup (\bar{C} \cap A)$

Shaded.

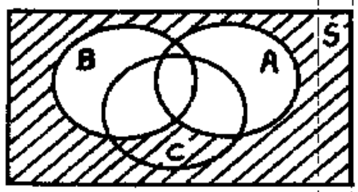
(c)



$\overline{A \cap B \cap C}$ or $\bar{A} \cup \bar{B} \cup \bar{C}$

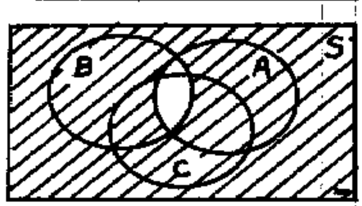
shaded.

1.2-7.



$\overline{A \cup B}$ or

$\bar{A} \cap \bar{B}$ Shaded.

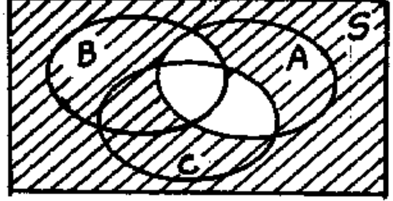


$\overline{A \cap B}$ or

$\bar{A} \cup \bar{B}$ shaded.

1.2-8. (a) $A \cup B = \{-10 \le s < -4\}$. (b) $A \cap B = \{-7 < s \le -5\}$. (c) The set $C = \{-9 \le s \le N\}$ satisfies making $A \cap C$ as large as possible for any $-5 \le N \le -4$. The set $B \cap C$ is largest if $C = \{-9 \le s \le -4\}$. The set satisfying both requirements is therefore $C = \{-9 \le s \le -4\}$. The set $C = \{-9 \le s < -4\}$ is also a valid solution. (d) $A \cap B \cap C = \{-7 < s \le -5\}$.

1.2-9. (a) To prove $\overline{A \cap (B \cup C)} = (\bar{A} \cup \bar{B}) \cap (\bar{A} \cup \bar{C})$ work with the left side. Use (1.2-13) to get $\overline{A \cap (B \cup C)} = \bar{A} \cup \overline{(B \cup C)}$. Next, use (1.2-12) to get $\overline{A \cap (B \cup C)} = \bar{A} \cup (\bar{B} \cap \bar{C})$. Finally, (1.2-9) is used to obtain the desired equation.



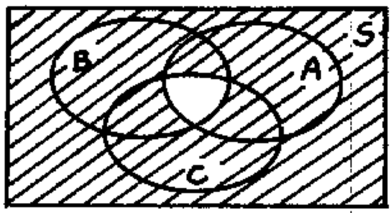
$\overline{A \cap (B \cup C)}$ and

$(\bar{A} \cup \bar{B}) \cap (\bar{A} \cup \bar{C})$ shaded.

(b) By using (1.2-13) on the left side of $\overline{A \cap B \cap C} =$

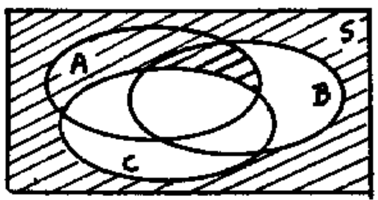
1.2-9. (Continued)

$\bar{A} \cup \bar{B} \cup \bar{C}$ we have $\overline{A \cap B \cap C} = \bar{A} \cup (\overline{B \cap C}) =$
$\bar{A} \cup (\bar{B} \cup \bar{C}) = \bar{A} \cup \bar{B} \cup \bar{C}$.

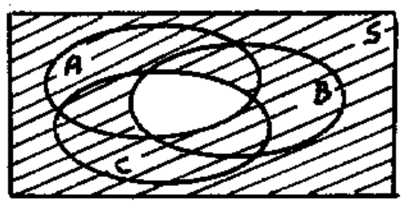


$\overline{A \cap B \cap C}$ and $\bar{A} \cup \bar{B} \cup \bar{C}$ shaded.

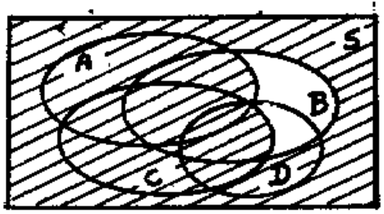
1.2-10.



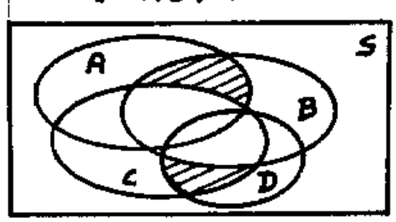
(a) $(A \cup \bar{B}) \cap \bar{C}$



(b) $\overline{(A \cap B)} \cup \bar{C}$

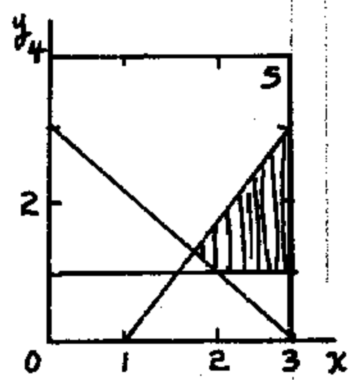


(c) $(A \cup \bar{B}) \cup (C \cap D)$

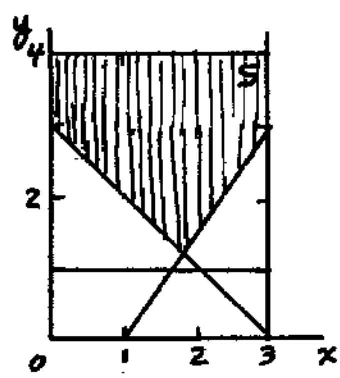


(d) $(A \cap B \cap \bar{C}) \cup (\bar{B} \cap C \cap D)$

1.2-11.



(a) $A \cap B \cap C$



(b) $C \cap B \cap \bar{A}$

(1.2-12.) (a) $A = \{80\,m \le d \le 1050\,m\}$, $B = \{950\,m \le d \le 1750\,m\}$, $C = \{1750\,m \le d \le 2000\,m\}$. (b) $A \cap B = \{950\,m \le d \le 1050\,m\}$ is the set of distances where both propeller and jet aircraft take off. (c) $\overline{A \cup B} = \{0 \le d \le 80\,m$ and $1750\,m < d \le 2000\,m\}$ is the portion of the runway used by both types of aircraft and the portion used by neither (safety margin). (d) $\overline{A \cup B \cup C} = \{0 \le d < 80\,m\}$ is portion of runway used by all aircraft in takeoff. $A \cup B = \{80\,m \le d \le 1750\,m\}$ is the runway distances used by all aircraft in taking off.

(1.2-13.) $\overline{A_1 \cap A_2} = \bar{A}_1 \cup \bar{A}_2 \triangleq C$ so $A_1 \cap A_2 = \bar{C}$

$\overline{A_1 \cap A_2 \cap A_3} = \overline{\bar{C} \cap A_3} = C \cup \bar{A}_3 = \bar{A}_1 \cup \bar{A}_2 \cup \bar{A}_3$

Continuation of the iteration proves the desired result.

(1.2-14.) Apply (1.2-12) and define C as

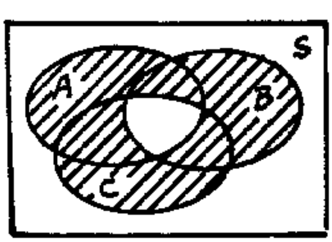
$\overline{A_1 \cup A_2} = \bar{A}_1 \cap \bar{A}_2 \triangleq C$ so $\bar{C} = A_1 \cup A_2$

$\overline{A_1 \cup A_2 \cup A_3} = \overline{\bar{C} \cup A_3} = C \cap \bar{A}_3 = \bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3$

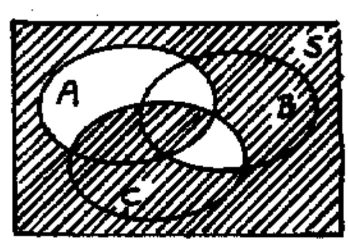
Continued iteration proves the desired result.

(1.2-15.) (a) False, (b) true, (c) false, (d) false, (e) true, (f) true, and (g) false.

(1.2-16.)

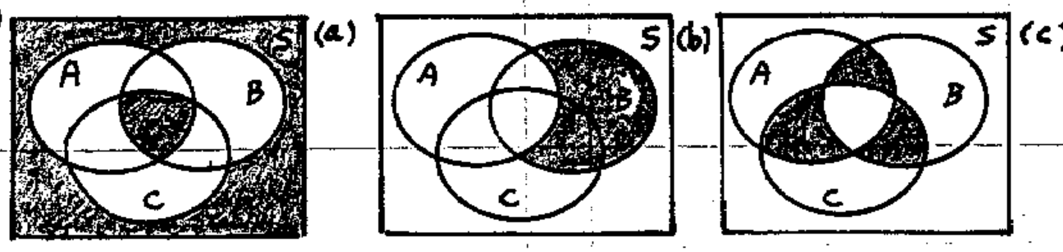


(a)



(b)

1.2-17.



1.3-1. The events are $A = \{1, 3, 5\}$, $B = \{4, 5, 6\}$, $A \cup B = \{1, 3, 4, 5, 6\}$, $A \cap B = \{5\}$. By assuming a fair die the probability of each of the six mutually exclusive outcomes is $1/6$. Thus, from axiom 3: $P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1/2$, $P(B) = 1/2$, $P(A \cup B) = 5/6$ and $P(A \cap B) = 1/6$.

1.3-2. There are 36 possible mutually exclusive possible outcomes, each with probability $1/36$. Only six produce a seven: $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$ and $(6,1)$. Two produce an eleven: $(5,6)$ and $(6,5)$. Thus, eight outcomes satisfy the event "7 or 11." Therefore $P(\text{sum is } 7 \text{ or } 11) = 8/36 = 2/9$.

1.3-3. (a) $S = \{0 < s \le 100\}$. (b) $P\{20 < s \le 35\} = (35-20)/100 = 15/100$. (c) $P\{s = 58\} = 0$ since the number 58 is only one of an infinite number of numbers in S.

1.3-4. (a) $P(A \cup C) = P\{a_1, a_5, a_6, a_9\} = 4/10$. (b) $P(B \cup \bar{C}) = P\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\} = P(S) = 1$. (c) $P[A \cap (B \cup C)] = P\{a_1, a_9\} = 2/10$.

1.3-4. (Continued)

(d) $P(\overline{A \cup B}) = P\{a_3, a_4, a_7, a_8, a_{10}\} = 5/10 = 1/2$.

(e) $P[A \cup (B \cap C)] = P\{a_6, a_9\} = 2/10 = 1/5$.

1.3-5. A and $\bar{A}$ are mutually exclusive and span the entire sample space. Therefore, from axiom 3: $P(A \cup \bar{A}) = P(S) = 1 = P(A) + P(\bar{A})$ or $P(\bar{A}) = 1 - P(A)$.

1.3-6. (a) $P(A) = P\{6\} = 1/6$; $P(B) = P\{2,5\} = 2/6 = 1/3$.
(b) $P(A) + P(B) + P(C) = 1$ is guaranteed if C is exclusive of A and B and comprises the balance of the sample space. Thus, $C = \overline{A \cup B} = \{1, 3, 4\}$.

1.3-7. $P(\text{black}) = 75/500$, $P(\text{green}) = 150/500$, $P(\text{red}) = 175/500$, $P(\text{white}) = 70/500$ and $P(\text{blue}) = 30/500$.

1.3-8. (a) $P(\text{jack}) = 4 \text{ jacks}/52 \text{ cards} = 4/52 = 1/13$.
(b) $P(5 \text{ or smaller}) = [4 \text{ fives} + 4 \text{ fours} + 4 \text{ threes} + 4 \text{ twos}]/52 \text{ cards} = 16/52 = 4/13$. (C) $P(\text{red ten}) = [1 \text{ ten of hearts} + 1 \text{ ten of diamonds}]/52 \text{ cards} = 2/52$.

1.3-9. (a) $P(A \text{ wins}) = P(2,4) + P(1,4) + P(4,1) + P(4,2) = 4/36$, (b) $P(B \text{ wins}) = P(4,1) + P(4,2) + P(4,3) + P(4,4) + P(4,5) + P(4,6) + P(1,4) + P(2,4) + P(3,4) + P(5,4) + P(6,4) = 11/36$. (c) $P(A \text{ and } B \text{ win}) = P(A \text{ wins}) = 4/36$ because $A \subset B$.

(1.3-10.) (a) P(stay in after one toss) $= P(H,H,T) + P(H,T,H) + P(T,T,H) + P(T,H,T) = 4/8$. (b) P(out after first toss) $= P(H,T,T) + P(T,H,H) = 2/8$. (c) P(no "odd man") $= P(H,H,H) + P(T,T,T) = 2/8$.

(1.3-11.)

Ω \ W	0.25	0.50	1.0
10	0.08	0.10	0.01
22	0.20	0.26	0.05
48	0.12	0.15	0.03
	0.40	0.51	0.09

$S = \{$all resistors$\}$

All resistors span S.

From table: (a) $P(48\,\Omega$ and $0.25\,W) = 0.12$, (b) $P(48\,\Omega$ and $0.5\,W) = 0.15$, (c) $P(48\,\Omega$ and $1.0\,W) = 0.03$.

(1.3-12.)

(a) P(one is a two and other is 3 or more) $= P(2,3) + P(2,4) + P(2,5) + P(2,6) + P(3,2) + P(4,2) + P(5,2) + P(6,2) = 8(1/36) = 2/9$. (b) $P(10 \le \text{sum}$ and $\text{sum} \le 4) = P(4,6) + P(5,5) + P(5,6) + P(6,4) + P(6,5) + P(6,6) + P(1,1) + P(1,2) + P(1,3) + P(2,1) + P(2,2) + P(3,1) = 12/36 = 1/3$.

1.3-13.

Probabilities of sums showing up are:

	1	2	3	4	5	6
1			2/49			
2			2/49			
3			2/49			
4	2/49	2/49	4/49	2/49	2/49	2/49
5			2/49			
6			2/49			

all other probabilities not shown are 1/49

$P\{7\} = P(6,1) + P(5,2) + P(4,3) + P(3,4) + P(2,5) + P(1,6)$

$= 9/49 \approx 0.1837$

For fair dice each outcome has probability $= 1/36$ so

$P\{7\} = 6/36 = 1/6 \approx 0.1667$. Improvement $= (9/49)/(6/36) \approx 1.102$, (10.2%).

1.4-1. (a) $P(\text{second is queen} \mid \text{first is queen}) = 3/51$.

(b) $P(\text{second is seven} \mid \text{first is queen}) = 4/51$.

(c) $P(\text{queen} \cap \text{queen}) = P(\text{queen} \mid \text{queen})\, P(\text{queen})$

$= \frac{3}{51} \cdot \frac{4}{52} = 1/221$.

1.4-2. $P(\text{four sevens}) = P(\text{fourth is seven} \mid \text{first three sevens})$

$\cdot P(\text{first three sevens}) = \frac{1}{49} P(\text{first three sevens}) =$

$\frac{1}{49} P(\text{third is seven} \mid \text{first two sevens})\, P(\text{first two sevens})$

$= \frac{1}{49} \cdot \frac{2}{50} \cdot P(\text{first two sevens}) = \frac{1}{49} \cdot \frac{2}{50} P(\text{second is seven} \mid \text{first is seven})\, P(\text{first is seven}) = \frac{1}{49} \cdot \frac{2}{50} \cdot \frac{3}{51} \cdot \frac{4}{52} = \frac{1}{270725}$

$= 3.694(10^{-6})$.

1.4-3. $P(D) = 24/100$, $P(E) = 38/100$,

$P(D \cap E) = 14/100$, $P(D \mid E) = 14/38$,

$P(E \mid D) = 14/24$.

1.4-4. Define B_1 = "draw a 5% resistor" and B_2 = "draw a 10% resistor." (a) From (1.4-10):

$$P(D) = P(D|B_1)P(B_1) + P(D|B_2)P(B_2)$$
$$= \frac{10}{62}\left(\frac{62}{100}\right) + \frac{14}{38}\left(\frac{38}{100}\right) = 24/100.$$

(b) $P(5\% \,|\, 22\,\Omega) = P(B_1|D) = P(D|B_1)P(B_1)/P(D)$

$$= \frac{10}{62}\left(\frac{62}{100}\right)\Big/\frac{24}{100} = 10/24.$$

1.4-5. (a) $P(0.01\mu F \,|\, \text{box } 2) = 95/210.$

(b) $P(0.01\mu F \,|\, \text{box } 3)\,P(\text{box } 3) = P(\text{box } 3 \,|\, 0.01\mu F)\,P(0.01\mu F)$

Thus,
$$P(\text{box } 3 \,|\, 0.01\mu F) = \frac{P(0.01\mu F \,|\, \text{box } 3)\,P(\text{box } 3)}{P(0.01\mu F)}$$

From the total probability theorem:

$$P(0.01\mu F) = P(0.01\mu F \,|\, \text{box } 1)\,P(\text{box } 1)$$
$$+ P(0.01\mu F \,|\, \text{box } 2)\,P(\text{box } 2) + P(0.01\mu F \,|\, \text{box } 3)\,P(\text{box } 3)$$
$$= \frac{20}{145}\left(\frac{1}{3}\right) + \frac{95}{210}\left(\frac{1}{3}\right) + \frac{25}{245}\left(\frac{1}{3}\right) = \frac{5903}{3(29)42(7)}.$$

Thus,
$$P(\text{box } 3 \,|\, 0.01\mu F) = \frac{\frac{25}{245}\left(\frac{1}{3}\right)}{\frac{5903}{3(29)42(7)}} = \frac{870}{5903} \approx 0.1474.$$

1.4-6. $P(0.01\mu F \,|\, \text{box } 1) = \frac{20}{145}$, $P(0.01\mu F \,|\, \text{box } 2) = \frac{95}{210}$, $P(0.01\mu F \,|\, \text{box } 3) = \frac{25}{245}$, $P(0.1\mu F \,|\, \text{box } 1) = \frac{55}{145}$, $P(0.1\mu F \,|\, \text{box } 2) = \frac{35}{210}$, $P(0.1\mu F \,|\, \text{box } 3) = \frac{75}{245}$, $P(1.0\mu F \,|\, \text{box } 1) = \frac{70}{145}$, $P(1.0\mu F \,|\, \text{box } 2) = \frac{80}{210}$, $P(1.0\mu F \,|\, \text{box } 3) = \frac{145}{245}$.

1.4-7. From equations in Example 1.4-2: $P(A_1) = 0.95(0.6) + 0.05(0.4) = 0.59$, $P(A_2) = 0.05(0.6) + 0.95(0.4) = 0.41$, $P(B_1|A_1) = 0.95(0.6)/0.59 = 0.966$, $P(B_2|A_2) = 0.95(0.4)/0.41 = 0.927$, $P(B_1|A_2) = 0.05(0.6)/0.41 = 0.073$, $P(B_2|A_1) = 0.05(0.4)/0.59 = 0.034$.

1.4-8. From equations in Example 1.4-2: $P(A_1) = 1.0(0.7) + 0.0(0.3) = 0.7$, $P(A_2) = 0.0(0.7) + 1.0(0.3) = 0.3$, $P(B_1|A_1) = 1.0(0.7)/0.7 = 1.0$, $P(B_2|A_2) = 1.0(0.3)/0.3 = 1.0$, $P(B_1|A_2) = 0$, $P(B_2|A_1) = 0$. The system is ideal (or noise-free) since the probabilities of an error in symbol reception are zero.

1.4-9.

Event	10 W	25 W	50 W	Totals
D (Defective)	15	7	3	25
G (Good)	85	63	27	175
Totals	100	70	30	200

(a) $P(D|10W) = 15/100$. (b) $P(D \cap 50W) = P(D|50W) \cdot P(50W) = \frac{3}{30}\left(\frac{1}{3}\right) = 1/30$. (c) $P(D) =$

$$P(D|10W)P(10W) + P(D|25W)P(25W) + P(D|50W)P(50W)$$

$$= \frac{15}{100}\left(\frac{1}{3}\right) + \frac{7}{70}\left(\frac{1}{3}\right) + \frac{3}{30}\left(\frac{1}{3}\right) = 0.35/3 \approx 0.1167.$$

(1.4-10.) (a) $P(\text{launch}) = P(A \text{ fails} \cap B \text{ fails})$

$= P(B \text{ fails} \mid A \text{ fails}) P(A \text{ fails}) = 0.06(0.01)$

$= 0.0006.$

(b) $P(A \text{ fails} \mid B \text{ fails}) = P(A \text{ fails} \cap B \text{ fails}) / P(B \text{ fails}) =$

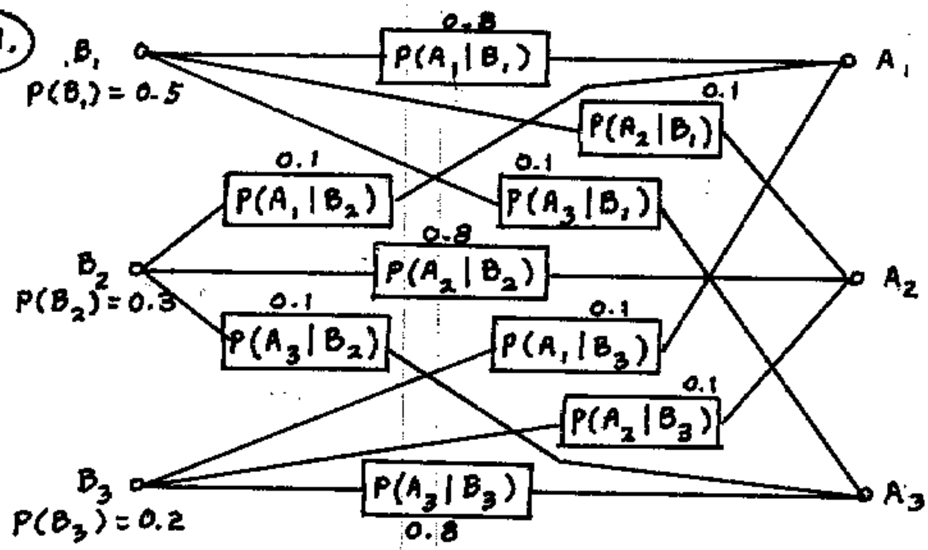
$6(10^{-4}) / 3(10^{-2}) = 0.02.$

(c) $P(A \text{ fails}) P(B \text{ fails}) = 0.01(0.03) = 3(10^{-4}) \neq 6(10^{-6}) =$

$P(A \text{ fails} \cap B \text{ fails})$ so events "A fails" and "B fails" are not independent.

★(1.4-11.)

(a)



(b) $P(A_1) = P(A_1|B_1)P(B_1) + P(A_1|B_2)P(B_2) + P(A_1|B_3)P(B_3)$

$= 0.8(0.5) + 0.1(0.3) + 0.1(0.2) = 0.45$, $P(A_2) =$

$0.1(0.5) + 0.8(0.3) + 0.1(0.2) = 0.31$, $P(A_3) = 0.1(0.5)$

$+ 0.1(0.3) + 0.8(0.2) = 0.24.$ (c)

(c) $P(B_1|A_1) = 0.8(0.5)/0.45 = 0.8889$, Bayes' rule.

$P(B_1|A_2) = 0.1(0.5)/0.31 = 0.1613$

$P(B_1|A_3) = 0.1(0.5)/0.24 = 0.2083$

★ 1.4-11. (Continued)

$P(B_2 \mid A_1) = 0.1(0.3)/0.45 = 0.0667$

$P(B_2 \mid A_2) = 0.8(0.3)/0.31 = 0.7742$

$P(B_2 \mid A_3) = 0.1(0.3)/0.24 = 0.1250$

$P(B_3 \mid A_1) = 0.1(0.2)/0.45 = 0.0444$

$P(B_2 \mid A_2) = 0.1(0.2)/0.31 = 0.0645$

$P(B_3 \mid A_3) = 0.8(0.2)/0.24 = 0.6667$

(d) When $P(B_i) = 1/3$, $i = 1, 2, 3$, then $P(A_1) = \frac{1}{3}[P(A_1|B_1) + P(A_1|B_2) + P(A_1|B_3)] = \frac{1}{3}[0.8 + 0.1 + 0.1] = 1/3$. Similarly $P(A_2) = P(A_3) = P(A_1) = 1/3$ and also $P(B_i \mid A_k) = 0.1$, $k \neq i$, and $P(B_i \mid A_i) = 0.8$, $i = 1, 2, 3$.

1.4-12.

	No. Pills	Line L	
		L_1	L_2
A_1	102	0.02	0.03
A_2	101	0.06	0.08
A_3	100	0.88	0.83
A_4	99	0.03	0.04
A_5	98	0.01	0.02

$P(L_1) = 0.45$, $P(L_2) = 0.55$

$P(102|L_1) = 0.02$, $P(102|L_2) = 0.03$

$P(101|L_1) = 0.06$, $P(101|L_2) = 0.08$

$P(100|L_1) = 0.88$, $P(100|L_2) = 0.83$

$P(99|L_1) = 0.03$, $P(99|L_2) = 0.04$

$P(98|L_1) = 0.01$, $P(98|L_2) = 0.02$

(a) $P(102) = P(102|L_1)P(L_1) + P(102|L_2)P(L_2) = 0.02(0.45) + 0.03(0.55) = 0.0255$. Similarly, $P(101) = 0.0710$, $P(100) = 0.8525$, $P(99) = 0.0355$, and $P(98) = 0.0155$. (b) $P(L_1|100) = \dfrac{P(100|L_1)\,P(L_1)}{P(100)} =$

(1.4-12.) (Continued) $\dfrac{0.88(0.45)}{0.8525} \approx 0.4645$. (c) $P(\text{pills} < 100) = P(98) + P(99) = 0.0355 + 0.0155 = 0.0510$.

(1.4-13.)

Define:

$A = \{\text{IC from source } A\}$

$B = \{\text{IC from source } B\}$

$C = \{\text{IC from source } C\}$

$D = \{\text{defective IC}\}$

Then:

$P(D|A) = 0.001, \quad P(A) = 1/3$

$P(D|B) = 0.003, \quad P(B) = 1/3$

$P(D|C) = 0.002, \quad P(C) = 1/3$

(a) $P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) = (0.001/3) + (0.003/3) + (0.002/3) = 0.006/3 = 0.002$. (b) $P(A|D) = P(D|A)P(A)/P(D) = 0.001(1/3)/0.002 = 1/6$. Similarly, $P(B|D) = 1/2$, $P(C|D) = 1/3$.

(1.4-14.)

(a) $P(\text{ace}) = P(A|D_1)P(D_1) + P(A|D_2)P(D_2) + P(A|D_3)P(D_3) = \frac{4}{52}\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right) + 0\left(\frac{1}{6}\right) = \frac{19}{156} \approx 0.1218$. (b) $P(3) = P(3|D_1)P(D_1) + P(3|D_2)P(D_2) + P(3|D_3)P(D_3) = \frac{4}{52}\left(\frac{1}{2}\right) + 0\left(\frac{1}{3}\right) + \frac{4}{36}\left(\frac{1}{6}\right) = \frac{20}{351} \approx 0.0570$. (c) Similarly $P(\text{red card}) = \frac{26}{52}\left(\frac{1}{2}\right) + \frac{8}{16}\left(\frac{1}{3}\right) + \frac{18}{36}\left(\frac{1}{6}\right) = \frac{1}{2} = 0.50$.

(1.5-1.) $P(A \cap B) = 28/100 = 0.28 \neq P(A)P(B) = (44/100) \cdot (62/100) = 0.273$ so A and B are not independent. $P(A \cap C) = 0.0$ while $P(A)P(C) \neq 0$ so A and C are not independent. $P(B \cap C) = 0.24 \neq P(B)P(C) = (62/100)(32/100) = 0.198$ so B and C are not

(1.5-1.) (Continued)

independent. Finally, $P(A \cap B \cap C) = 0.0 \neq P(A) \cdot P(B) P(C)$ so A, B, and C are not statistically independent, even by pairs.

(1.5-2.) $P(A_1 \cap A_2) = P(A_1)P(A_2)$, $\quad P(A_1 \cap A_3) = P(A_1)P(A_3)$,

$P(A_1 \cap A_4) = P(A_1)P(A_4)$, $\quad P(A_2 \cap A_3) = P(A_2)P(A_3)$,

$P(A_2 \cap A_4) = P(A_2)P(A_4)$, $\quad P(A_3 \cap A_4) = P(A_3)P(A_4)$,

$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$, $P(A_1 \cap A_2 \cap A_4) = P(A_1)P(A_2)P(A_4)$,

$P(A_1 \cap A_3 \cap A_4) = P(A_1)P(A_3)P(A_4)$, $P(A_2 \cap A_3 \cap A_4) = P(A_2)P(A_3)P(A_4)$,

and $P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2)P(A_3)P(A_4)$.

★ (1.5-3.) (a) First form the product $P(A_1)P(\bar{A}_2) = P(A_1)[1 - P(A_2)] = P(A_1) - P(A_1)P(A_2)$. Since A_1 and A_2 are independent this becomes

$$P(A_1)P(\bar{A}_2) = P(A_1) - P(A_1 \cap A_2). \qquad (1)$$

a Venn diagram shows that $A_1 = (A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)$ so $P(A_1) = P[(A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)]$. From (1.4-2): $P[(A_1 \cap \bar{A}_2) \cup (A_1 \cap A_2)] = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2) - P[(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2)]$. However, a Venn diagram will show that $(A_1 \cap \bar{A}_2) \cap (A_1 \cap A_2) = \emptyset$ so these last two results give

$$P(A_1) = P(A_1 \cap \bar{A}_2) + P(A_1 \cap A_2). \qquad (2)$$

By combining (1) and (2) we have $P(A_1)P(\bar{A}_2) = P(A_1 \cap \bar{A}_2)$ so A_1 and $\bar{A}_2$ are statistically indep-

*(1.5-3.) (Continued)

endent from (1.5-3).

(b) The procedure is the same as in (a): $P(\bar{A}_1)P(A_2) = P(A_2)[1-P(A_1)] = P(A_2) - P(A_1 \cap A_2)$. But $A_2 = (A_2 \cap \bar{A}_1) \cup (A_2 \cap A_1)$ and $(A_2 \cap \bar{A}_1) \cap (A_2 \cap A_1) = \emptyset$ so $P(A_2) = P[(A_2 \cap \bar{A}_1) \cup (A_2 \cap A_1)] = P(A_2 \cap \bar{A}_1) + P(A_2 \cap A_1) - P[(A_2 \cap \bar{A}_1) \cap (A_2 \cap A_1)] = P(A_2 \cap \bar{A}_1) + P(A_2 \cap A_1)$. Thus, from the first and last equations:

$$P(\bar{A}_1)P(A_2) = P(\bar{A}_1 \cap A_2)$$

so $\bar{A}_1$ and A_2 are independent.

(c) Again repeat the procedures of (a) and (b):

$$P(\bar{A}_1)P(\bar{A}_2) = [1-P(A_1)][1-P(A_2)] = 1 - P(A_1) - P(A_2) + P(A_1)P(A_2) = 1 - P(A_1) - P(A_2) + P(A_1 \cap A_2)$$

$= 1 - [P(A_1) + P(A_2) - P(A_1 \cap A_2)] = 1 - P(A_1 \cup A_2)$ from (1.4-2). But since $1 - P(A_1 \cup A_2) = P(\overline{A_1 \cup A_2})$, then $P(\bar{A}_1)P(\bar{A}_2) = P(\overline{A_1 \cup A_2})$. However, a Venn diagram shows that $\overline{A_1 \cup A_2} = \bar{A}_1 \cap \bar{A}_2$ so

$$P(\bar{A}_1)P(\bar{A}_2) = P(\bar{A}_1 \cap \bar{A}_2)$$

which, from (1.5-3) shows $\bar{A}_1$ and $\bar{A}_2$ to be independent.

(1.5-4.) Comb. N things 2 at time $= \binom{N}{2}$ (pairs)

Comb. N things 3 at time $= \binom{N}{3}$ (triple)

$\vdots$

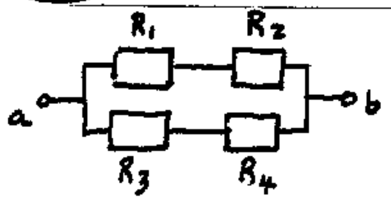
Comb. N things N at time $= \binom{N}{N}$ (N-tuple)

(1.5-4.) (Continued)

$$\text{Total} = \text{sum} = \sum_{i=2}^{N}\binom{N}{i} = -\binom{N}{0} - \binom{N}{1} + \sum_{i=0}^{N}\binom{N}{i}$$

Use (C-61). $\text{Total} = -1 - N + \sum_{i=0}^{N}\binom{N}{i} = 2^N - N - 1.$

(1.5-5.)



$R_i = \{\text{relay } R_i \text{ fails}, \ i = 1,2,3,4\} = \{R_i \text{ opens}\}$

$$P(R_i) = \begin{cases} p_1 = 0.005, & i = 1,2 \\ p_2 = 0.008, & i = 3,4 \end{cases}$$

$P(\text{signal does not arrive}) = P\{(R_1 \text{ or } R_2 \text{ opens}) \text{ and } (R_3 \text{ or } R_4 \text{ opens})\}$
$= P\{(R_1 \cup R_2) \cap (R_3 \cup R_4)\} = P(R_1 \cup R_2)P(R_3 \cup R_4)$ (independent failures) $= [P(R_1) + P(R_2) - P(R_1 \cap R_2)][P(R_3) + P(R_4) - P(R_3 \cap R_4)]$
$= (2p_1 - p_1^2)(2p_2 - p_2^2) = [0.01 - 25(10^{-6})][0.016 - 64(10^{-6})] \approx 0.00016.$

(1.5-6.)

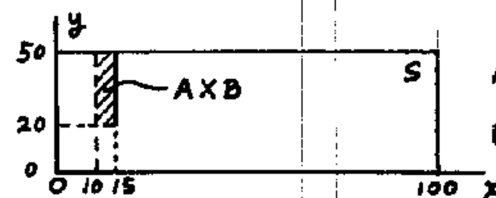
~~P{signal does not arrive} = P{(upper path fails) and (lower~~
path fails)} $= P\{(\text{upper path fails}) \cap (\text{lower path fails})\}$
$= P(\text{upper path fails})P(\text{lower path fails})$. But $P(\text{upper path fails})$
$= P(R_1) + P(R_2 \cap R_3) - P(R_1 \cap R_2 \cap R_3) = p_1 + p_2^2 - p_1 p_2^2 = 5.0995(10^{-3})$, and
$P(\text{lower path fails}) = p_2 + p_3^2 - p_2 p_3^2 = 12.475(10^{-3})$. Thus,
$P\{\text{signal does not arrive}\} = 5.0995(10^{-3})\,12.475(10^{-3}) = 63.6163(10^{-6})$.

(1.6-1.) Let c_1, c_2, c_3, c_4 and c_5 represent the five cities and let m_1, m_2, m_3 and m_4 represent the motels. Then $S_1 = \{c_1, c_2, c_3, c_4, c_5\}$ and $S_2 = \{m_1, m_2, m_3, m_4\}$. The combined sample space becomes

1.6-1. (Continued)

$$S = S_1 \times S_2 = \{(c_1,m_1),\ (c_1,m_2),\ (c_1,m_3),\ (c_1,m_4),$$
$$(c_2,m_1),\ (c_2,m_2),\ (c_2,m_3),\ (c_2,m_4),$$
$$(c_3,m_1),\ (c_3,m_2),\ (c_3,m_3),\ (c_3,m_4),$$
$$(c_4,m_1),\ (c_4,m_2),\ (c_4,m_3),\ (c_4,m_4),$$
$$(c_5,m_1),\ (c_5,m_2),\ (c_5,m_3),\ (c_5,m_4)\}.$$

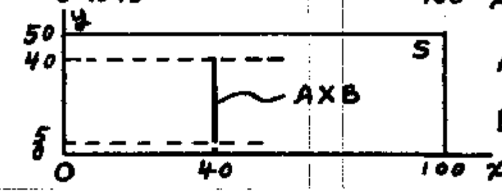
1.6-2. (a)



$A = \{10 < x \le 15\}$

$B = \{20 < y \le 50\}$

(b)



$A = \{x = 40\}$

$B = \{5 < y \le 40\}$

1.6-3. Sequences $= 6(6)6(6) = 6^4 = 1296$.

1.6-4. Number of poker hands $= \binom{52}{5} = \dfrac{52(51)50(49)48}{5(4)3(2)}$

$$= 2,598,960.$$

1.7-1. This is a Bernoulli trials experiment with $N=4$,

$p = P(\text{a can is out of tolerance}) = 0.03$.

(a) $P(4 \text{ out of tolerance}) = \binom{4}{4}(0.03)^4(1-0.03)^0 = 8.1(10^{-7})$.

(b) $P(2 \text{ out of tolerance}) = \binom{4}{2}(0.03)^2(1-0.03)^2$

$$= \frac{4!}{2!2!}(9)10^{-4}(0.97)^2 \approx 5.081(10^{-3}).$$

(c) $P(\text{all in tolerance}) = P(\text{none is out of tolerance})$

$$= \binom{4}{0}(0.03)^0(1-0.03)^4 = (0.97)^4 \approx 0.8853.$$

1.7-2. This is a Bernoulli trials experiment with $N = 6$, $p = P(\text{land in recovery zone}) = 0.8$.

(a) $P(\text{none in zone}) = \binom{6}{0}(0.8)^0(1-0.8)^6 = (0.2)^6 = 6.4(10^{-5})$.

(b) $P(\text{at least one in zone}) = 1 - P(\text{none in zone}) =$

$$1 - 6.4(10^{-5}) = 0.999936.$$

(c) $P(\text{success}) = P(3 \text{ in zone}) + P(4 \text{ in zone}) + P(5 \text{ in zone})$

$$+ P(6 \text{ in zone}) = \binom{6}{3}(0.8)^3(0.2)^3 + \binom{6}{4}(0.8)^4(0.2)^2$$

$$+ \binom{6}{5}(0.8)^5(0.2)^1 + \binom{6}{6}(0.8)^6(0.2)^0 \approx 0.983.$$

Yes, the program is successful.

1.7-3. For $N = 2$: $P\{\text{carrier sunk}\} = P\{2 \text{ hits}\} = \binom{2}{2}(0.4)^2 \cdot (1-0.4)^0 = 0.16$. For $N = 4$: $P\{\text{carrier sunk}\} = P\{2 \text{ hits}\} + P\{3 \text{ hits}\} + P\{4 \text{ hits}\} = \binom{4}{2}(0.4)^2(0.6)^2 + \binom{4}{3}(0.4)^3(0.6)^1 + \binom{4}{4}(0.4)^4(0.6)^0 = 0.5248$.

1.7-4. $P(\text{late } k \text{ of } N \text{ times}) = \binom{N}{k} 0.4^k (0.6)^{N-k}$, $N = 5$.

(a) $P(\text{late 3 or more times in one week}) = \binom{5}{3} 0.4^3 (0.6)^2 + \binom{5}{4} 0.4^4 (0.6)^1 + \binom{5}{5} 0.4^5 = 0.2304 + 0.0768 + 0.01024 = 0.31744$. (b) $P(\text{not late at all}) = \binom{5}{0} 0.4^0 (0.6)^5 = 0.07776$.

1.7-5. (a) $P(0 \text{ late}) = \binom{5}{0}(0.3)^0(0.7)^5 = 0.16807$. (b) $P(\text{all late}) = \binom{5}{5} 0.3^5 (0.7)^0 = 0.00243$. (c) $P(3 \text{ or more on time}) = P(0 \text{ late}) + P(1 \text{ late}) + P(2 \text{ late}) = 0.16807 + \binom{5}{1} 0.3 (0.7)^4 + \binom{5}{2} 0.3^2 (0.7)^3 = 0.83692$.

1.7-6. $P(\text{all on time}) = P(0 \text{ late}) = \binom{5}{0} x^0 (1-x)^5 = (1-x)^5 = 0.9$. Thus, $x = P(\text{a flight is late}) = 1 - 0.9^{1/5} = 0.020852$. Yes — to reduce 0.3 to 0.020852!

1.7-7. (a) $P(\text{win}) = P(2 \text{ heads}) = \binom{5}{2} 0.7^2 (0.3)^3 = 0.13230$. No!

(b) $P(\text{win}) = P(4 \text{ heads}) + P(5 \text{ heads}) = \binom{5}{4} 0.7^4 (0.3)^1 + \binom{5}{5} 0.7^5 (0.3)^0 = 0.36015 + 0.16807 = 0.52822$. Yes!

★**1.7-8.** (a) $P(\text{win set}) = P(5 \text{ hits in } 6) + P(6 \text{ hits in } 6) = \binom{6}{5}(0.8)^5(0.2)^1 + \binom{6}{6}(0.8)^6(0.2)^0 = 0.393216 + 0.262144 = 0.65536$. (b) Second Bernoulli trial problem. $P(3 \text{ sets of } 3) = \binom{3}{3}(0.65536)^3(1-0.65536)^0 = 0.28147$.

1.7-9.

$P(\text{successful arrival}) = P\{\text{engine survives and navigation system survives}\} = P\{\text{engine survives}\}P\{\text{navigation system survives}\} = [1 - P(\text{engine fails})][1 - P(\text{navigation system fails})] = 0.95(0.999) = 0.94905$.

1.7-10.

(a) $P(\text{all operate}) = P(0 \text{ fail}) = \dfrac{6!}{0!(6-0)!}(0.06)^0(0.94)^{6-0} = 0.6899$.

(b) $P(\text{all fail}) = P(6 \text{ fail}) = \dfrac{6!}{6!(6-6)!}(0.06)^6(0.94)^{6-6} = 4.666(10^{-8})$.

(c) $P(1 \text{ fails}) = \dfrac{6!}{1!(6-1)!}(0.06)^1(0.94)^{6-1} = 0.2642$.

1.7-11.

(a) $P(2 \text{ or more not repaired}) = 1 - P(0 \text{ not repaired}) - P(1 \text{ not repaired}) = 1 - \frac{8!}{0!(8-0)!}(0.1)^0(0.9)^{8-0} - \frac{8!}{1!(8-1)!}(0.1)^1(0.9)^{8-1}$ $\approx 1 - 0.4305 - 0.3826 = 0.1869$. (b) $P(8 \text{ properly repaired}) = P(0 \text{ not repaired}) \approx 0.4305$.

1.7-12.

$P(0 \text{ fail to return key}) = \frac{(Np)^{k=0} e^{-Np}}{(k=0)!} = \frac{2.5^0 e^{-2.5}}{0!} = 0.0821$.

$P(1 \text{ fails to return key}) = \frac{2.5^1 e^{-2.5}}{1!} = 0.2052$.

$P(2 \text{ fail to return keys}) = \frac{2.5^2 e^{-2.5}}{2!} = 0.2565$.

$P(3 \text{ fail to return keys}) = \frac{2.5^3 e^{-2.5}}{3!} = 0.2138$.

$P(\text{no more than 3 fail to return keys}) = 0.0821 + 0.2052 + 0.2565 + 0.2138 = 0.7576$.