Solution Manual for Solar Engineering of Thermal Processes 4th Edition – John Duffie, William Beckman

P01_01

1.1 From the diameter and effective surface temperature of the sun, estimate the rate at which it emits energy. What fraction of this emitted energy is intercepted by Earth? Estimate the solar constant, given the mean Earth-sun distance.

Equations

Knowns:

$$\sigma = sigma\# \quad (1)$$

$$D_{sun} = 1.39 \times 10^{9} \ [\text{m}] \quad (2)$$

$$D_{earth} = 1.27 \times 10^{7} \ [\text{m}] \quad (3)$$

$$R_{SunEarth} = 1.495 \times 10^{11} \ [\text{m}] \quad (4)$$

$$T_{Sun} = 5777 \ [\text{K}] \quad (5)$$

Calculate the emitted solar energy

$$Area_{sun} = \pi \cdot D_{sun}^2; \quad (6)$$

$$EmitSol = \sigma \cdot Area_{sun} \cdot T_{Sun}^4 \quad (7)$$

Calculate the fraction intercepted by the earth

$$Fraction = Area_{earth,proj} / Area_{1au} \quad (8)$$

$$Area_{earth,proj} = \pi \cdot \frac{D_{earth}^2}{4} \quad (9)$$

$$Area_{1au} = 4 \cdot \pi \cdot R_{SunEarth}^2 \quad (10)$$

Estimate solar constant

$$SolarConstant = Fraction \cdot EmitSol / Area_{earth,proj} \quad (11)$$

Solution

$Area_{1au} = 2.809 \times 10^{23}$ [m^2]
$Area_{earth,proj} = 1.267 \times 10^{14}$ [m^2]
$Area_{sun} = 6.070 \times 10^{18}$ [m^2]
$D_{earth} = 1.270 \times 10^{7}$ [m]
$D_{sun} = 1.390 \times 10^{9}$ [m]
$EmitSol = 3.833 \times 10^{26}$ [W]
$Fraction = 4.510 \times 10^{-10}$ [-]
$R_{SunEarth} = 1.495 \times 10^{11}$ [m]
$\sigma = 5.670 \times 10^{-8}$ [W/m^2-K^4]
$SolarConstant = 1,365$ [W/m^2]
$T_{Sun} = 5,777$ [K]

P01_02

1.2 What would be the "solar constant" for Venus? Mean Venus-sun distance is 0.72 times the mean Earth-sun distance. Assume the sun to be a blackbody emitter at 5777 K.

Equations

Start with Solar Constant = 1367 W/m^2 from Problem 1.1. SolarConst$_{Venus}$ will be inversely proportional to the square of the distance from sun:

$$SolarConst_{Venus} = \frac{SolarConst}{.72^2} \tag{1}$$

$$SolarConst = 1367 \; \left[\mathrm{W/m^2}\right] \tag{2}$$

Solution

$SolarConst = 1,367 \; \left[\mathrm{W/m^2}\right]$ $\quad SolarConst_{Venus} = 2,637 \; \left[\mathrm{W/m^2}\right]$

P01_03

1.3 What fraction of the extraterrestrial radiation is at wavelengths below 0.5 μm? 2 μm? What fraction is included in the wavelength range 0.5 to 2.0 μm?

Equations

From Table 1.3.1.

$$f_{0to0.5} = 0.216 \quad (1)$$

$$f_{0to2.0} = 0.941 \quad (2)$$

Therefore the fraction between 0.5 and 2.0 is:

$$f_{0.5to2.0} = f_{0to2.0} - F_{0to0.5} \quad (3)$$

Solution

$f_{0.5to2.0} = 0.725$ [-] $f_{0to0.5} = 0.216$ [-] $f_{0to2.0} = 0.941$ [-]

P01_04

1.4 Divide the extraterrestrial solar spectrum into 10 equal increments of energy. Specify a characteristic wavelength for each increment.

Equations

This problem is essentially solved in Table 1.3.1b. The wavelength divisions for the increments are the wavelengths given in the table for F_0-λ of 0.1, 0.2, 0.3, etc. The energy midpoints are the wavelengths in the table for 0.05, 0.15, 0.25, etc. The first increment of a tenth is in the wavelength range of 0 to 0.416, and the wavelength at the energy midpoint of the increment is 0.364. Use the 'Insert/Modify an Array' in the Edit menu to simplify data input.

$$\lambda_0 = 0.300; \quad \lambda_1 = .416; \quad \lambda_2 = .489; \quad \lambda_3 = .561; \quad \lambda_4 = .638; \quad \lambda_5 = .731 \tag{1}$$

$$\lambda_6 = .849; \quad \lambda_7 = 1.008; \quad \lambda_8 = 1.244; \quad \lambda_9 = 1.654; \quad \lambda_{10} = 8.000; \tag{2}$$

$$\lambda_{mid,1} = .364; \quad \lambda_{mid,2} = .455; \quad \lambda_{mid,3} = .525; \quad \lambda_{mid,4} = .599; \quad \lambda_{mid,5} = .682 \tag{3}$$

$$\lambda_{mid,6} = .787; \quad \lambda_{mid,7} = .923; \quad \lambda_{mid,8} = 1.113; \quad \lambda_{mid,9} = 1.412; \quad \lambda_{mid,10} = 2.117 \tag{4}$$

So, the range of wavelengths for each of the 10 bands is:

duplicate $i = 1,\ 10$ (5)

$$\lambda_{range,i} = \lambda_i - \lambda_{i-1} \tag{6}$$

end (7)

Solution

Arrays Table: $Main$

Row	λ_i	$\lambda_{mid,i}$	$\lambda_{range,i}$
0	0.300		
1	0.416	0.364	0.116
2	0.489	0.455	0.073
3	0.561	0.525	0.072
4	0.638	0.599	0.077
5	0.731	0.682	0.093
6	0.849	0.787	0.118
7	1.008	0.923	0.159
8	1.244	1.113	0.236
9	1.654	1.412	0.410
10	8.000	2.117	6.346

P01_05

1.5 Calculate the angle of incidence of the beam solar radiation at 1400 (2 PM) solar time on February 10 at latitude 43.3° on surfaces of the following orientations:

a Horizontal
b Sloped to south at 60°
c Slope of 60°, facing 40° west of south
d Vertical, facing south
e Vertical, facing west

Equations

$$Dec = 23.45\ [\text{deg}] \cdot \sin(.9863 \cdot (284 + n)) \qquad \text{Eqn. 1.6.1} \qquad (1)$$

$$CosIncAng = sin(Dec) \cdot sin(Lat) \cdot \text{Cos}(Slope) - sin(Dec) \cdot \text{Cos}(Lat) \cdot sin(Slope) \cdot \text{Cos}(SurfAzAng) +$$
$$\text{Cos}(Dec) \cdot \text{Cos}(Lat) \cdot \text{Cos}(Slope) \cdot \text{Cos}(HourAngle) +$$
$$\text{Cos}(Dec) \cdot \sin(Lat) \cdot \sin(Slope) \cdot \text{Cos}(SurfAzAng) \cdot \text{Cos}(HourAngle)$$
$$+ \text{Cos}(Dec) \cdot \sin(Slope) \cdot \sin(SurfAzAng) \cdot \sin(HourAngle) \qquad \text{Eqn 1.6.2} \qquad (2)$$

$$\theta = \arccos(CosIncAng) \qquad (3)$$

All data and results are in the Parametric Table

We could have used the SETP function to find θ:

θ = arcCos(CosIncAng(Lat, n, HourAngle, Slope, SurfAzAng))

Solution

Variables in Main program

$CosIncAng = 0.4832$ [-] $Dec = -14.9$ [deg] $HourAngle = 30$ [deg] $Lat = 43.3$ [deg]
$n = 41$ $Slope = 90$ [deg] $SurfAzAng = 90$ [deg] $\theta = 61.1$ [deg]

Parametric Table: $Table\ 1$

Run	n	Lat [deg]	$HourAngle$ [deg]	$SurfAzAng$ [deg]	$Slope$ [deg]	Dec [deg]	θ [deg]
1	41	43.3	30	0	0	-14.9	64.4
2	41	43.3	30	0	60	-14.9	28.9
3	41	43.3	30	40	60	-14.9	8.0
4	41	43.3	30	0	90	-14.9	40.4
5	41	43.3	30	90	90	-14.9	61.1

P01_06

1.6	**a**	When it is 2 PM Mountain Standard Time (MST) on Feb. 3 in North Platte, NE (L = 101°W), what is the solar time?
	b	When it is 2 PM MST in Boise, ID (L = 116°W), on Feb. 3, what is the solar time?
	c	What EDT corresponds to solar noon on July 31 for Portland, ME (L = 70.5°W)?
	d	What CDT corresponds to 10:00 AM on July 31 for Iron Mountain, MI (L = 90°W)?

Equations

Use of Equation of Time (Eqn 1.5.3) in Equation 1.5.2

$$EqTime = 229.2\ [\text{min}]\cdot(.000075 + .001868 \cdot \text{Cos}(B) - .032077 \cdot \sin(B) - .014615 \cdot \text{Cos}(2 \cdot B) - .04089 \cdot \sin(2 \cdot B)) \quad (1)$$

$$B = (n-1) \cdot 360 \frac{[\text{deg}]}{365} \quad \text{Eqn 1.5.3} \quad (2)$$

$$n = nDay\,(month,\ day) \quad \text{Use either the SETP function or Table 1.6.1 to find the day number)} \quad (3)$$

The difference between standard and solar time is:

$$Solar - Standard = (4\ [\text{min/deg}] \cdot (Long_{Std} - Long_{Loc}) + EqTime) \cdot \left| 0.016666667 \frac{\text{hr}}{\text{min}} \right| \quad \text{Eqn 1.5.2} \quad (4)$$

The difference between daylight time and standard time is one hour

$$Daylight - Standard = 1\ [\text{hr}] \quad (5)$$

Note: In part d the time of 10:00 AM is solar time as nothing else is specified.

Solution

$B = 208.1$ [deg]
$EqTime = -6.549$ [min]
$month = 7$ [-]
$Standard = 10:06:33$ [hr]
$day = 31$ [-]
$Long_{Loc} = 90$ [deg]
$n = 212$ [-]
$Daylight = 11:06:33$ [hr]
$Long_{Std} = 90$ [deg]
$Solar = 10:00:00$ [hr]

Parametric Table: $Table\ 1$

Run	$month$ [-]	day [-]	n [-]	B [deg]	$Long_{Loc}$ [deg]	$Long_{Std}$ [deg]	$EqTime$ [min]	$Solar$ [hr]	$Standard$ [hr]	$Daylight$ [hr]
1	2	3	*34*	*32.55*	101	105	*-13.49*	*14:02:31*	14:00:00	*15:00:00*
2	2	3	*34*	*32.55*	116	105	*-13.49*	*13:02:31*	14:00:00	*15:00:00*
3	7	31	*212*	*208.1*	70.5	75	*-6.549*	12:00:00	*11:48:33*	*12:48:33*
4	7	31	*212*	*208.1*	90	90	*-6.549*	10:00:00	*10:06:33*	*11:06:33*

P01_07

1.7 Determine the sunset hour angle and day length for Madison and for Miami for the following dates: **a** January 1, **b** March 22, **c** July 1. **d** the mean day of February.

Equations

Determine the sunset hour angle andday length for various conditions

$$Dec = 23.45\ [\text{deg}] \cdot \sin(.9863 \cdot (284 + n)) \quad \text{Eqn 1.6.1a} \tag{1}$$

$$SunSetHrAngle = \arccos(-\tan(Lat) \cdot \tan(Dec)) \quad \text{Eqn 1.6.10} \tag{2}$$

$$DayLength = 2\ [\text{hr}] \cdot \frac{\arccos(-\tan(Lat) \cdot \tan(Dec))}{15\ [\text{deg}]} \quad \text{Eqn 1.6.11} \tag{3}$$

$$n = nDay(month,\ day) \quad \text{Table 1.6.1 or SETP function} \tag{4}$$

The average day for February is the 22. The average day could be found from Table 1.6.1 or SETP function, AveDay=AveDay$_{(month)}$

The more exact declination equation, 1.6.1b can be used

dec=(0.006918-0.399912*cos(B)+0.070257*sin(B)-0.000758*Cos(2*B) +0.000907*sin(2*B)

Eqn 1.6.1b

B=(n-1)*360[deg]/365

The day lengths using the two equations differ by 2 minutes. For most solar calculations this difference can be ignored.

Solution

Variables in Main program

$day = 16$	$DayLength = 11:41$ [hr]	$Dec = -2.42$ [deg]	$Lat = 43.3$ [deg]
$month = 3$	$n = 75$	$SunSetHrAngle = 87.7$ [deg]	

Parametric Table: $Dec\ with\ Eqn\ 1.6.1a$

Run	$month$	day	n	Lat [deg]	Dec [deg]	$DayLength$ [hr]	$SunSetHrAngle$ [deg]
1	1	1	*1*	22.5	*-23.01*	*10:38*	*79.9*
2	3	22	*81*	22.5	*-0.00*	*12:00*	*90.0*
3	7	1	*182*	22.5	*23.12*	*13:21*	*100.2*
4	3	16	*75*	22.5	*-2.42*	*11:51*	*89.0*
5	1	1	*1*	43.3	*-23.01*	*8:51*	*66.4*
6	3	22	*81*	43.3	*-0.00*	*12:00*	*90.0*
7	7	1	*182*	43.3	*23.12*	*15:09*	*113.7*
8	3	16	*75*	43.3	*-2.42*	*11:41*	*87.7*

Parametric Table: $Dec\ with\ Eqn\ 1.6.1b$

Run	$month$	day	n	Lat [deg]	Dec [deg]	$DayLength$ [hr]	$SunSetHrAngle$ [deg]
1	1	1	*1*	22.5	*-23.01*	*10:38*	*79.9*
2	3	22	*81*	22.5	*-0.00*	*12:00*	*90.0*
3	7	1	*182*	22.5	*23.12*	*13:21*	*100.2*
4	3	16	*75*	22.5	*-2.42*	*11:51*	*89.0*
5	1	1	*1*	43.3	*-23.01*	*8:51*	*66.4*
6	3	22	*81*	43.3	*-0.00*	*12:00*	*90.0*
7	7	1	*182*	43.3	*23.12*	*15:09*	*113.7*
8	3	16	*75*	43.3	*-2.42*	*11:41*	*87.7*

P01_08

1.8 A concentrating collector is located at $\phi = 27°$ and is rotated about a single axis so as to always minimize the angle of incidence of beam radiation on it. On April 5, at solar times of 9 AM and noon, calculate the angle of incidence:

a If the axis is horizontal and east-west
b If the axis is horizontal and north-south
c If the axis is parallel to the earth's axis

Equations

Calculation of θ_{beam} for tracking surfaces. Use parametric table for the two times.

$$Lat = 27; \quad month = 3; \quad Day = 5; \quad n = nDay\,(month,\ day) \quad \text{could use Table 1.6.1 to find n} \tag{1}$$

$$HourAngle = (Time - 12\ [\text{hr}]) \cdot 15\ [\text{deg/hr}] \tag{2}$$

$$Dec = 23.45\ [\text{deg}] \cdot \sin\left(.9863 \cdot (284 + n)\right) \quad 1.6.1 \tag{3}$$

$$CosZenithAngle = \text{Cos}(Lat) \cdot \text{Cos}(Dec) \cdot \text{Cos}(HourAngle) + \sin(Lat) \cdot \sin(Dec) \quad 1.6.5 \tag{4}$$

$$ZenithAngle = \arccos(CosZenithAngle) \tag{5}$$

For horizontal E-W axis, use Equation 1.7.2a:

$$\theta_{EW} = \arccos\left(1 - (cos\,(Dec))^2 \cdot (\sin(HourAngle))^2\right) \tag{6}$$

For horizontal N-S axis, use Equation 1.7.3a:

$$\theta_{NS} = \arccos\left(\left((cos\,(ZenithAngle))^2 + (cos\,(Dec))^2 \cdot (\sin(HourAngle))^2\right)^{.5}\right) \tag{7}$$

For polar axis, use Equation 1.7.5a:

$$\theta_{Polar} = \arccos\left(cos\,(Dec)\right) \tag{8}$$

Solution

$CosZenithAngle = 0.8313$ [-] $\quad Day = 5 \quad Dec = -6.765$ [deg]
$HourAngle = 0$ [deg] $\quad Lat = 27$ [deg] $\quad month = 3$
$n = 64 \quad \theta_{EW} = 0$ [deg] $\quad \theta_{NS} = 33.77$ [deg]
$\theta_{Polar} = 6.765$ [deg] $\quad Time = 12$ [hr] $\quad ZenithAngle = 33.77$ [deg]

Parametric Table: $Table\ 1$

Run	$Time$ [hr]	Dec [deg]	$HourAngle$ [deg]	$ZenithAngle$ [deg]	θ_{EW} [deg]	θ_{NS} [deg]	θ_{Polar} [deg]
1	9	*-6.765*	*-45*	*55.1*	*59.54*	*25.07*	*6.765*
2	12	*-6.765*	*0*	*33.77*	*0*	*33.77*	*6.765*

P01_09

1.9 Estimate R_b for a collector at Madison ($\phi = 43º$):
a Sloped 60º from horizontal, with $\gamma = 0º$, at 2:30 on March 5
b Sloped 45º, with surface azimuth angle of 15º, at 10:30 solar time on March 5

Equations

Calculate R_b from Equation 1.8.2. Use parametric table for the two conditions.

$$Dec = 23.45 \text{ [deg]} \cdot \sin(.9863 \cdot (284 + n)) \quad 1.6.1 \tag{1}$$

$$n = nDay(month,\ day) \quad \text{Table 1.6.1} \tag{2}$$

$$HrAng = (Time - 12 \text{ [hr]}) \cdot 15 \text{ [deg/hr]} \tag{3}$$

$$CosZen = \text{Cos}(Lat) \cdot \text{Cos}(Dec) \cdot \text{Cos}(HrAng) + \sin(Lat) \cdot \sin(Dec) \quad 1.6.5 \tag{4}$$

$$CosTheta = \sin(Dec) \cdot \sin(Lat) \cdot \text{Cos}(Slope) - \sin(Dec) \cdot \text{Cos}(Lat) \cdot \sin(Slope) \cdot \text{Cos}(SurfAzAng) + \text{Cos}(Dec) \cdot \text{Cos}(Lat) \cdot \text{Cos}(Slope) \cdot \text{Cos}(HrAng) + \text{Cos}(Dec) \cdot \sin(Lat) \cdot \sin(Slope) \cdot \text{Cos}(SurfAzAng) \cdot \text{Cos}(HrAng) + \text{Cos}(Dec) \cdot \sin(Slope) \cdot \sin(SurfAzAng) \cdot \sin(HrAng) \tag{5}$$

$$R_b = CosTheta / CosZen \quad 1.8.2 \tag{6}$$

We could use the SETP function to find CosTheta
CosTheta = CosTheta(Lat, Dec, HrAng, Slope, SurfAzAngle)

Solution

$CosTheta = 0.8343$ [-]	$CosZen = 0.5906$ [-]	$day = 5$
$Dec = -6.765$ [deg]	$HrAng = -22.5$ [deg]	$Lat = 43$ [deg]
$month = 3$	$n = 64$	$R_b = 1.413$ [-]
$Slope = 45$ [deg]	$SurfAzAng = 15$ [deg]	$Time = 10:30$ [hr]

Parametric Table: $Table\ 1$

Run	Lat [deg]	$month$	day	n	$Slope$ [deg]	$SurfAzAng$ [deg]	$Time$ [hr]	$HrAng$ [deg]	Dec [deg]	$CosTheta$ [-]	$CosZen$ [-]	R_b [-]
1	43	3	5	64	60	0	14:30	37.5	-6.765	0.7878	0.4958	1.589
2	43	3	5	64	45	15	10:30	-22.5	-6.765	0.8343	0.5906	1.413

P01_10

1.10 For Minneapolis ($\phi = 45^o$) on Feb. 8:
a What is H_o?
b For the month, what is $\bar{H}_o$?
c What is I_o for the hour ending at 11:00 AM on the 8th?

Equations

Calculation of extraterrestrial radiation on horizontal surface

$$month = 2; \quad day = 8; \quad lat = 45; \quad G_{sc} = 1367 \ [\mathrm{W/m^2}]; \quad n = 39 \quad \text{Table 1.6.1} \tag{1}$$

$$B = (n-1) \cdot 360 \ \frac{[\mathrm{deg}]}{365} \quad \text{note that we could have used Eqn 1.6.1a but 1.6.1b is more accurate.} \tag{2}$$

$$Dec = \left(180 \ \frac{[\mathrm{deg}]}{\pi}\right) \cdot (0.006918 - 0.399912 \cdot \mathrm{Cos}(B) + 0.070257 \cdot \sin(B) - 0.0067518 \cdot \mathrm{Cos}(2 \cdot B)$$

$$+0.000907 \cdot \sin(2 \cdot B) - 0.002697 \cdot \mathrm{Cos}(3 \cdot B) + 0.00148 \cdot \sin(3 \cdot B)) \quad \text{Eqn 1.6.1b} \tag{3}$$

$$SunSetHrAng = \arccos(-\tan(Lat) \cdot \tan(Dec)) \quad \text{Eqn 1.6.10} \tag{4}$$

From Equation 1.10.3 the day's extraterrestrial radiations is:

$$H_o = \frac{24 \cdot 3600 \ [\mathrm{s}] \cdot G_{sc}}{\pi} \cdot (1 + 0.033 \cdot \mathrm{Cos}(360 \ [\mathrm{deg}] \cdot n/365)) \cdot (\mathrm{Cos}(lat) \cdot \mathrm{Cos}(dec) \cdot \sin(SunSetHrAng)$$

$$+ \left(\pi \cdot \frac{SunSetHrAng}{180 \ [\mathrm{deg}]} \cdot \sin(lat) \cdot \sin(dec)\right)\right) \cdot \left|1 \times 10^{-6} \ \frac{\mathrm{MJ}}{\mathrm{J}}\right| \tag{5}$$

To find the monthly average daily radiation we will use the SETP function with the average day in February.

$$n_{ave} = 47 \quad \text{Table 1.6.1} \tag{6}$$

$$B1 = (n_{ave} - 1) \cdot 360 \ \frac{[\mathrm{deg}]}{365} \tag{7}$$

$$Dec_{ave} = \left(180 \ \frac{[\mathrm{deg}]}{\pi}\right) \cdot (0.006918 - 0.399912 \cdot \mathrm{Cos}(B1) + 0.070257 \cdot \sin(B1) - 0.0067518 \cdot \mathrm{Cos}(2 \cdot B1)$$

$$+0.000907 \cdot \sin(2 \cdot B1) - 0.002697 \cdot \mathrm{Cos}(3 \cdot B1) + 0.00148 \cdot \sin(3 \cdot B1)) \quad \text{Eqn 1.6.1b} \tag{8}$$

$$SunSetHrAng_{ave} = \arccos(-\tan(Lat) \cdot \tan(Dec_{ave})) \quad \text{Eqn 1.6.10} \tag{9}$$

$$\bar{H}_o = \frac{24 \cdot 3600 \ [\mathrm{s}] \cdot G_{sc}}{\pi} \cdot (1 + 0.033 \cdot \mathrm{Cos}(360 \ [\mathrm{deg}] \cdot n_{ave}/365)) \cdot (\mathrm{Cos}(lat) \cdot \mathrm{Cos}(dec_{ave})$$

$$\cdot \sin(SunSetHrAng_{ave}) + \left(\pi \cdot \frac{SunSetHrAng_{ave}}{180 \ [\mathrm{deg}]} \cdot \sin(lat) \cdot \sin(dec_{ave})\right)\right) \cdot \left|1 \times 10^{-6} \ \frac{\mathrm{MJ}}{\mathrm{J}}\right| \tag{10}$$

To find the energy in the hour 10 to 11 use Equation 1.10.4

$$Time = 10:30 \quad \text{note that this is the midpoint of the hour} \tag{11}$$

$$HrAng = (Time - 12\ [\text{hr}]) \cdot 15\ [\text{deg/hr}]; \quad HrAng2 = HrAng + 7.5\ [\text{deg}]; \quad HrAng1 = HrAng - 7.5\ [\text{deg}] \tag{12}$$

$$I_o = \frac{12 \cdot 3600\ [\text{s}] \cdot G_{sc}}{\pi} \cdot (1 + .033 \cdot \text{Cos}(360\ [\text{deg}] \cdot n/365)) \cdot (\text{Cos}(Lat) \cdot \text{Cos}(Dec) \cdot (\sin(HrAng2) - \sin(HrAng1))$$

$$+ \left| 0.017453293\ \frac{\text{rad}}{\text{deg}} \right| \cdot (HrAng2 - HrAng1) \cdot \sin(Lat) \cdot \sin(Dec) \Big) \cdot \left| 1 \times 10^{-6}\ \frac{\text{MJ}}{\text{J}} \right| \tag{13}$$

The following solution uses the EES SETP functions

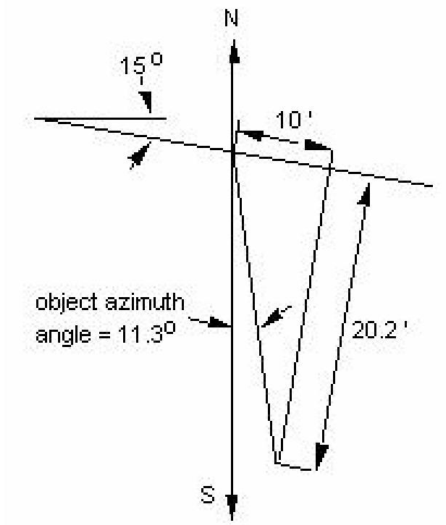
month=2; day=8; lat=45; G_{sc}=G_{sc}#; n=nDay$_{(month}$, day)
Dec = Dec(n)
SunSetHrAng = SunSetHrAng(Lat,n)
H_{Zero} = H_{Zero}(n, Lat)
n_{ave}=AveDay(month)
$\bar{H}_o$=H_{Zero}(n_{ave}, Lat)
Time=10:30
HrAng=(Time-12[hr])*15[deg/hr]
HrAng2=HrAng+7.5[deg]; HrAng1=HrAng-7.5[deg]
I_o=I_{Zero}(n, Lat, HrAng1, HrAng2)

Solution

$B = 37.48$ [deg]	$B1 = 45.37$ [deg]	$day = 8$	$Dec = -15.25$ [deg]
$Dec_{ave} = -12.61$ [deg]	$G_{sc} = 1,367$ [W/m^2]	$HrAng = -22.5$ [deg]	$HrAng1 = -30$ [deg]
$HrAng2 = -15$ [deg]	$\bar{H}_o = 17.88$ [MJ/m^2]	$H_o = 16.03$ [MJ/m^2]	$I_o = 2.234$ [MJ/m^2]
$lat = 45$ [deg]	$month = 2$	$n = 39$ [-]	$n_{ave} = 47$ [-]
$SunSetHrAng = 74.18$ [deg]	$SunSetHrAng_{ave} = 77.07$ [deg]	$Time = 10:30:00$ [hr]	

P01_11

1.11 A window on a building in Kansas City faces 15° west of south. An "ell" of the building is 10 ft away from the east edge of the window, and projects out 20.2 ft from the plane of the window. If the vertical dimension of the edge of the ell is large compared to that of the window, plot the times of day at which the east edge of the window will start to receive beam radiation as a function of the time of year.



Equations

1.11 Calculation of shading by a building ELL

As the ell is high compared to the window, the Object Altitude Angle varies from 0 to 90. An equation for the Object Azimuth Angle(which has a single value) is:

$$ObjAzAng = -\left(90\ [\text{deg}] - \left(\arctan\left(20.2/10\right) + 15\ [\text{deg}]\right)\right) \quad (1)$$

When the Solar Azimuth Angle (γ_s) is less than ObjAzAng the corner of the window is shaded. So, for the mean day of each month we need to find the time (i.e., the hour angle, ω) at which the ObjAzAng is equal to $\gamma_{s.}$ We will use the EES Parametric Table to work through the 12 months.

$$lat = 40 \quad (2)$$

$$n_{ave} = AveDay\left(month\right) \quad (3)$$

$$dec = 23.45\ [\text{deg}] \cdot \sin\left(360 \cdot \frac{284 + n_{ave}}{365}\right) \quad (4)$$

$$\theta_z = \arccos\left(cos\left(lat\right) \cdot cos\left(dec\right) \cdot Cos\left(\omega\right) + \sin\left(lat\right) \cdot \sin\left(dec\right)\right) \quad (5)$$

$$\gamma_s = -\arccos\left(\frac{cos\left(\theta_z\right) \cdot \sin\left(lat\right) - \sin\left(Dec\right)}{\sin\left(\theta_z\right) \cdot Cos\left(lat\right)}\right) \quad (6)$$

$$Time = -\frac{\omega}{15\ [\text{deg/hr}]} \cdot 60\ [\text{min/hr}] \quad (7)$$

Here Time is the number of minutes before solar noon; this makes an easier to understand plot.

These equations are diffucult to solve for the hour angle ω unless very good guesses are provided. An alternative method of solution is to define an error as the absolute difference between γ_s and the ObjAzAng and minimize this error.

$$error = abs\left(\gamma_s - ObjAzAng\right) \tag{8}$$

The Object Azimuth Angle could have been plotted on a solar position plot for Latitude 40. The times when shading ceases is determined for the time of year from this plot. .

The plot shows the number of minutes before solar noon at which the sun is incident on the east edge of the window as a function of month.

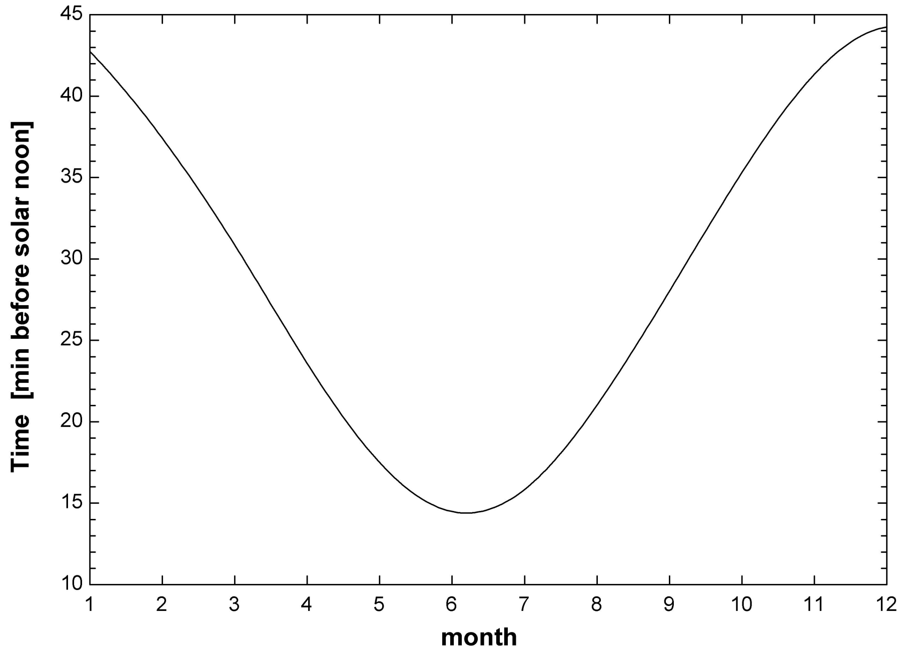
Solution

$dec = -23.05$ [deg] $error = 2.53 \times 10^{-4}$ [deg] $\gamma_s = -11.34$ [deg] $lat = 40$ [deg]
$month = 12$ $n_{ave} = 344$ $ObjAzAng = -11.34$ [deg] $\omega = -11.06$ [deg]
$\theta_z = 63.9$ [deg] $Time = 44$ [min]

Parametric Table: *Results*

Run	$month$	dec [deg]	n_{ave}	γ_s [deg]	θ_z [deg]	ω [deg]	$error$ [deg]	$Time$ [min]
1	1	-20.92	17	-11.34	61.7	-10.68	9.08E-4	43
2	2	-12.95	47	-11.34	53.7	-9.352	1.42E-4	37
3	3	-2.418	75	-11.34	43.0	-7.711	2.02E-3	31
4	4	9.415	105	-11.34	31.0	-5.896	6.01E-5	24
5	5	18.79	135	-11.34	21.5	-4.373	1.88E-3	17
6	6	23.09	162	-11.34	17.2	-3.621	6.34E-5	14
7	7	21.18	198	-11.34	19.1	-3.959	4.31E-5	16
8	8	13.45	228	-11.34	26.9	-5.255	3.20E-4	21
9	9	2.217	258	-11.34	38.3	-7.005	8.44E-4	28
10	10	-9.599	288	-11.34	50.3	-8.825	6.14E-3	35
11	11	-18.91	318	-11.34	59.7	-10.34	2.62E-4	41
12	12	-23.05	344	-11.34	63.9	-11.06	2.53E-4	44

Plot Window 1: $minutes\ before\ solar\ noon$

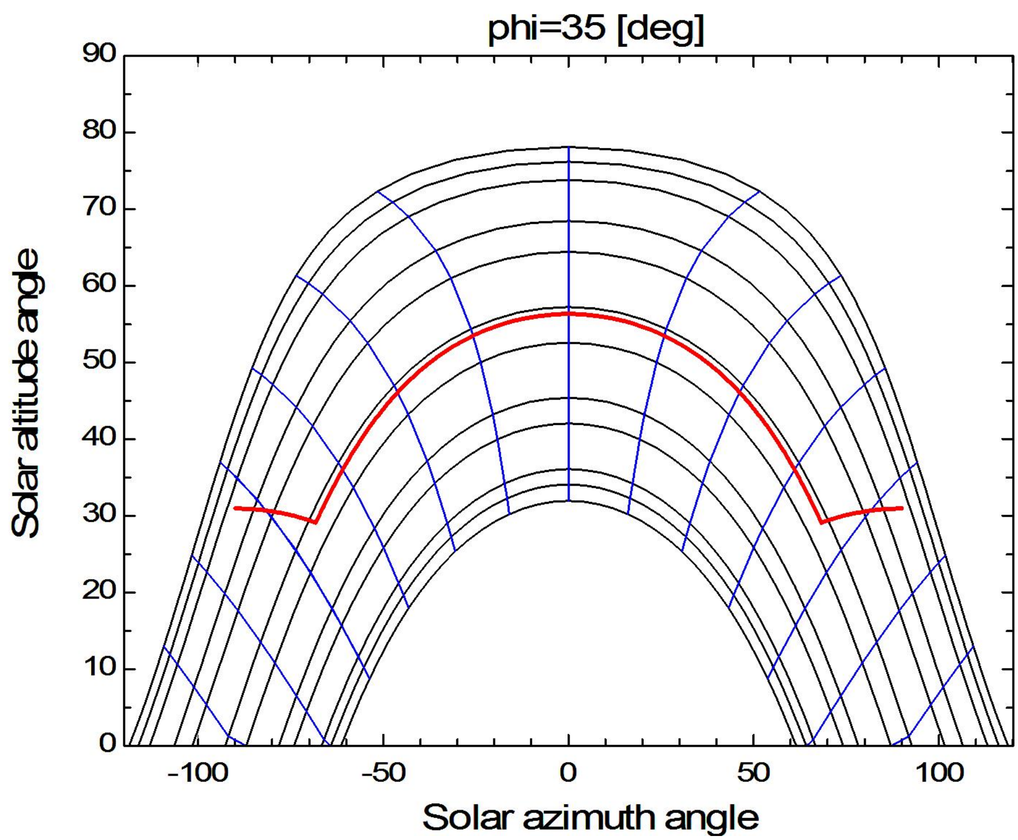


P01_12

1.12 An overhang over a south-facing window has dimensions as follows (see Figure 14.4.1): $G = 0.25$ m, $H = 1.75$ m, $W = 3.25$ m, $P = 0.75$ m, $E_L = E_R = 0.50$ m.

a For a location at $\phi = 35°$ for a point at the middle of the window, plot a shading diagram.

b Will this point receive beam radiation at 1 PM solar time on February 16? At 3 PM on July 17? At 5 PM on August 16?



Equations

Calculation of shading by an overhang

Since the system is symmetrical about the N-S line, it is necessary to calculate only points on one side (e.g., west). Use a coordinate system with the origin at the center of the window. The Z axis is vertical, the X axis is towards the west (i.e., left in Fig 14.4.1) and the Y axis is towards the south. A general point on the overhang is x, y, z. For example, the corner of the overhang is at x=(W+E)/2, y=P, and z=H/2+G. The azimuth angle is arcTan(x/y) and the altitude angle is arcTan(z/sqrt(x^2+y^2))

Expressions for the relationships of distances and angles for any point on the overhang are as follows:

$$W = 3.25; \quad E = 0.5; \quad P = 0.75; \quad H = 1.75; \quad G = 0.25 \tag{1}$$

$$\gamma = \arctan\left(x/y\right); \qquad \gamma_{east} = -\gamma \tag{2}$$

$$\alpha = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right) \tag{3}$$

Set up a parametric table for γ and α with various values of x, y, and z around the boarder of the overhang. Both positive and negative values of γ were placed in the parametric table for plotting purposes. The parametric table is saved as an EES LKT table. The program, SolarPositionDiagram.EES reads this lookup table and overlays the values on a solar postion diagram for a latitude of 34 degrees. The figure shows the shading boundary plotted on a solar position diagram. Area above the red line are shaded. The shading diagram ends at plus or minus 90 degrees since the sun is then behind the window.

```
$SaveTable 'Parametric'  'shading.lkt'  /N
```

Part b: From the diagram we see that all of February (the 4th month from the bottom) is unshaded and all of July (the second month from the top) is shaded. In August (the 4th month from the top) the center is shaded after 8am and before 4pm.

Solution

$\alpha = 31.0$ [deg]	$E = 0.5$ [m]	$G = 0.25$ [m]
$\gamma = 90.0$ [deg]	$\gamma_{east} = -90.0$ [deg]	$H = 1.75$ [m]
$P = 0.75$ [m]	$W = 3.25$ [m]	$x = 1.875$ [m]
$y = 0.00001$ [m]	$z = 1.125$ [m]	

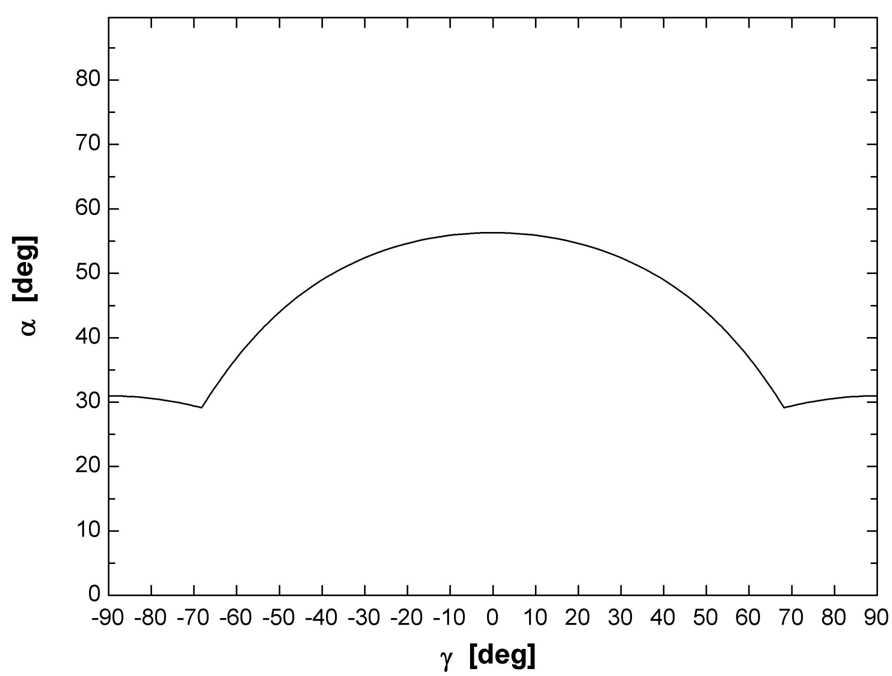
Parametric Table: *Overhang*

Run	x [m]	y [m]	z [m]	α [deg]	γ [deg]	γ_{east} [deg]
1	0	0.75	1.125	56.3	0.0	0.0
2	0.02534	0.75	1.125	56.3	1.9	-1.9
3	0.05068	0.75	1.125	56.2	3.9	-3.9
4	0.07601	0.75	1.125	56.2	5.8	-5.8
5	0.1014	0.75	1.125	56.1	7.7	-7.7
6	0.1267	0.75	1.125	55.9	9.6	-9.6
7	0.152	0.75	1.125	55.8	11.5	-11.5
8	0.1774	0.75	1.125	55.6	13.3	-13.3
9	0.2027	0.75	1.125	55.4	15.1	-15.1
10	0.228	0.75	1.125	55.1	16.9	-16.9
11	0.2534	0.75	1.125	54.9	18.7	-18.7
12	0.2787	0.75	1.125	54.6	20.4	-20.4
13	0.3041	0.75	1.125	54.3	22.1	-22.1
14	0.3294	0.75	1.125	53.9	23.7	-23.7
15	0.3547	0.75	1.125	53.6	25.3	-25.3
16	0.3801	0.75	1.125	53.2	26.9	-26.9
17	0.4054	0.75	1.125	52.8	28.4	-28.4
18	0.4307	0.75	1.125	52.4	29.9	-29.9
19	0.4561	0.75	1.125	52.0	31.3	-31.3
20	0.4814	0.75	1.125	51.6	32.7	-32.7
21	0.5068	0.75	1.125	51.2	34.0	-34.0
22	0.5321	0.75	1.125	50.7	35.4	-35.4
23	0.5574	0.75	1.125	50.3	36.6	-36.6
24	0.5828	0.75	1.125	49.8	37.8	-37.8
25	0.6081	0.75	1.125	49.4	39.0	-39.0

26	0.6334	0.75	1.125	*48.9*	*40.2*	*-40.2*
27	0.6588	0.75	1.125	*48.4*	*41.3*	*-41.3*
28	0.6841	0.75	1.125	*47.9*	*42.4*	*-42.4*
29	0.7095	0.75	1.125	*47.5*	*43.4*	*-43.4*
30	0.7348	0.75	1.125	*47.0*	*44.4*	*-44.4*
31	0.7601	0.75	1.125	*46.5*	*45.4*	*-45.4*
32	0.7855	0.75	1.125	*46.0*	*46.3*	*-46.3*
33	0.8108	0.75	1.125	*45.5*	*47.2*	*-47.2*
34	0.8361	0.75	1.125	*45.0*	*48.1*	*-48.1*
35	0.8615	0.75	1.125	*44.6*	*49.0*	*-49.0*
36	0.8868	0.75	1.125	*44.1*	*49.8*	*-49.8*
37	0.9122	0.75	1.125	*43.6*	*50.6*	*-50.6*
38	0.9375	0.75	1.125	*43.1*	*51.3*	*-51.3*
39	0.9628	0.75	1.125	*42.7*	*52.1*	*-52.1*
40	0.9882	0.75	1.125	*42.2*	*52.8*	*-52.8*
41	1.014	0.75	1.125	*41.7*	*53.5*	*-53.5*
42	1.039	0.75	1.125	*41.3*	*54.2*	*-54.2*
43	1.064	0.75	1.125	*40.8*	*54.8*	*-54.8*
44	1.09	0.75	1.125	*40.4*	*55.5*	*-55.5*
45	1.115	0.75	1.125	*39.9*	*56.1*	*-56.1*
46	1.14	0.75	1.125	*39.5*	*56.7*	*-56.7*
47	1.166	0.75	1.125	*39.1*	*57.2*	*-57.2*
48	1.191	0.75	1.125	*38.6*	*57.8*	*-57.8*
49	1.216	0.75	1.125	*38.2*	*58.3*	*-58.3*
50	1.242	0.75	1.125	*37.8*	*58.9*	*-58.9*
51	1.267	0.75	1.125	*37.4*	*59.4*	*-59.4*
52	1.292	0.75	1.125	*37.0*	*59.9*	*-59.9*
53	1.318	0.75	1.125	*36.6*	*60.4*	*-60.4*
54	1.343	0.75	1.125	*36.2*	*60.8*	*-60.8*
55	1.368	0.75	1.125	*35.8*	*61.3*	*-61.3*
56	1.394	0.75	1.125	*35.4*	*61.7*	*-61.7*
57	1.419	0.75	1.125	*35.0*	*62.1*	*-62.1*
58	1.444	0.75	1.125	*34.7*	*62.6*	*-62.6*
59	1.47	0.75	1.125	*34.3*	*63.0*	*-63.0*
60	1.495	0.75	1.125	*33.9*	*63.4*	*-63.4*
61	1.52	0.75	1.125	*33.6*	*63.7*	*-63.7*
62	1.546	0.75	1.125	*33.2*	*64.1*	*-64.1*
63	1.571	0.75	1.125	*32.9*	*64.5*	*-64.5*
64	1.596	0.75	1.125	*32.5*	*64.8*	*-64.8*
65	1.622	0.75	1.125	*32.2*	*65.2*	*-65.2*
66	1.647	0.75	1.125	*31.9*	*65.5*	*-65.5*
67	1.672	0.75	1.125	*31.5*	*65.8*	*-65.8*
68	1.698	0.75	1.125	*31.2*	*66.2*	*-66.2*
69	1.723	0.75	1.125	*30.9*	*66.5*	*-66.5*
70	1.748	0.75	1.125	*30.6*	*66.8*	*-66.8*
71	1.774	0.75	1.125	*30.3*	*67.1*	*-67.1*
72	1.799	0.75	1.125	*30.0*	*67.4*	*-67.4*
73	1.824	0.75	1.125	*29.7*	*67.7*	*-67.7*
74	1.85	0.75	1.125	*29.4*	*67.9*	*-67.9*
75	1.875	0.75	1.125	*29.1*	*68.2*	*-68.2*
76	1.875	0.75	1.125	*29.1*	*68.2*	*-68.2*
77	1.875	0.7188	1.125	*29.3*	*69.0*	*-69.0*
78	1.875	0.6875	1.125	*29.4*	*69.9*	*-69.9*
79	1.875	0.6563	1.125	*29.5*	*70.7*	*-70.7*

80	1.875	0.625	1.125	*29.6*	*71.6*	*-71.6*
81	1.875	0.5938	1.125	*29.8*	*72.4*	*-72.4*
82	1.875	0.5625	1.125	*29.9*	*73.3*	*-73.3*
83	1.875	0.5313	1.125	*30.0*	*74.2*	*-74.2*
84	1.875	0.5	1.125	*30.1*	*75.1*	*-75.1*
85	1.875	0.4688	1.125	*30.2*	*76.0*	*-76.0*
86	1.875	0.4375	1.125	*30.3*	*76.9*	*-76.9*
87	1.875	0.4063	1.125	*30.4*	*77.8*	*-77.8*
88	1.875	0.375	1.125	*30.5*	*78.7*	*-78.7*
89	1.875	0.3438	1.125	*30.5*	*79.6*	*-79.6*
90	1.875	0.3125	1.125	*30.6*	*80.5*	*-80.5*
91	1.875	0.2813	1.125	*30.7*	*81.5*	*-81.5*
92	1.875	0.25	1.125	*30.7*	*82.4*	*-82.4*
93	1.875	0.2188	1.125	*30.8*	*83.3*	*-83.3*
94	1.875	0.1875	1.125	*30.8*	*84.3*	*-84.3*
95	1.875	0.1563	1.125	*30.9*	*85.2*	*-85.2*
96	1.875	0.125	1.125	*30.9*	*86.2*	*-86.2*
97	1.875	0.09375	1.125	*30.9*	*87.1*	*-87.1*
98	1.875	0.0625	1.125	*30.9*	*88.1*	*-88.1*
99	1.875	0.03125	1.125	*31.0*	*89.0*	*-89.0*
100	1.875	0.00001	1.125	*31.0*	*90.0*	*-90.0*

Plot Window 1: *Plot* 1



P01_13

1.13 For February 16 (the average day for February from table 1.6.1) in Pueblo, CO:
- **a** What is the day length and the sunrise hour angle?
- **b** For this day, what is the declination? Is it significantly different at sunrise and sunset?
- **c** How does theta, the angle of incidence of beam radiation, vary through the day for a surface sloped to the south as the slope varies? Show plots of theta vs. slope.
- **d** For a collector slope of 60º, show a plot of R_b vs. time of day.
- **e** What is the extraterrestrial radiation on a horizontal surface for this day?
- **f** How does I_o vary through this day?
- **g** For the hour 10 to 11 AM (solar time), how much of the extraterrestrial solar radiation is in a wavelength range of 0.640 µm to 1.100 µm? Express the result as a fraction of the total and as energy in W/m^2.
- **h** At 10 AM solar time, what is the local clock time?

Equations

Calculation of solar information

We will use the EES SETP functions for the calculations.
See Key variables in Solutions window.

Knowns

$$month = 2; \quad day = 16; \quad \gamma = 0 \ \text{[deg]} \tag{1}$$

$$Lat = 38.3 \ \text{[deg]}; \quad Long = 104.6 \ \text{[deg]}; \quad n = AveDay(month) \quad \text{Or from Table 1.6.1} \tag{2}$$

a) Day Length

$$\omega_s = SunsetHrAng(Lat, \ n) \quad \text{Equation 1.6.10} \tag{3}$$

$$Hr_{day} = \omega_s \cdot \frac{2}{15 \ \text{[deg/hr]}} \quad \text{Equation 1.6.11} \tag{4}$$

b) Declination - assumed to be at noon.

$$\delta = dec(n) \quad \text{Equation 1.6.1a} \tag{5}$$

$$\delta_{ha} = dec(-n) \quad \text{Equation 1.6.1b - high accuracy declination} \tag{6}$$

At sunrise and sunset the equivalent n is

$$n_{sunrise} = n - \frac{\omega_s}{15 \ \text{[deg/hr]} \cdot 24 \ \text{[hr]}} \tag{7}$$

$$n_{sunset} = n + \frac{\omega_s}{15 \ \text{[deg/hr]} \cdot 24 \ \text{[hr]}} \tag{8}$$

$$\delta_{change} = dec(n_{sunrise}) - dec(n_{sunset}) \tag{9}$$