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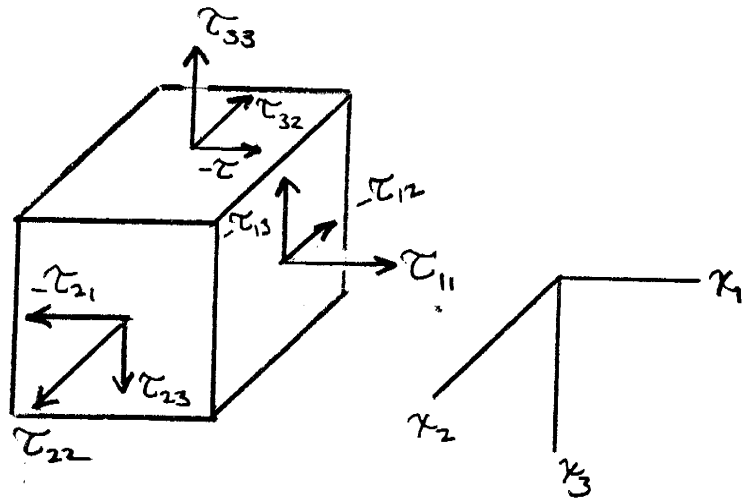
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CHAPTER 1

1.1



1.2

$$2. (a) T_i^{(v)} = \tau_{ij} v_j$$

$$T_1^{(v)} = \tau_{11}(0.11) + \tau_{12}(0.35) + \tau_{13}(0.93) = 180 \text{ psi}$$

$$T_2^{(v)} = -2450 \text{ psi}$$

$$T_3^{(v)} = -140 \text{ psi}$$

(b) To find component in direction of unit vector \hat{e} , compute

$$\begin{aligned} \vec{T}(\hat{e}) = \vec{T}^{(v)} \cdot \hat{e} &= T_i^{(v)} \cdot \epsilon_i = 180(.33) - 2450(.90) - 140(.284) \\ &= -2185 \text{ psi} \end{aligned}$$

1.3

If there was a body-couple vector \vec{M} , with components M_i , it would follow that

$$M_i = \epsilon_{ijk} \tau_{jk}; \quad \therefore M_1 = \epsilon_{123} \tau_{23} + \epsilon_{132} \tau_{32}$$

$$\therefore \tau_{32} = \tau_{23} - M_1 = 100 - 200 = -100 \text{ psi}$$

1.4 From Eq. (1.11), $\tau'_{pq} = a_{pj} a_{qi} \tau_{ji}$, corresponding to the transformation $x'_i = a_{ij} x_j$. For the rotation given, the transformation matrix is

$$a_{ij} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is a straightforward exercise to obtain the stress matrix τ'_{pq} :

$$\tau'_{pq} = \begin{pmatrix} 137 & -137 & 0 \\ -137 & 63 & 0 \\ 0 & 0 & 500 \end{pmatrix}$$

Note that in calculating the above, the only non-zero terms in the

τ_{ij} matrix are τ_{11} , $\tau_{12} = \tau_{21}$, τ_{33} . Thus

$$\tau'_{pq} = (a_{p1} a_{q1}) \tau_{11} + (a_{p2} a_{q1} + a_{p1} a_{q2}) \tau_{12} + (a_{p3} a_{q3}) \tau_{33}.$$

1.5

(a) The principal stresses are found from the equation

$$\tau^3 - I_{\tau} \tau^2 + II_{\tau} \tau - III_{\tau} = 0.$$

In the present example the invariants of the given stress state are easily computed as

$$I_{\tau} = \tau_{kk} = 0; \quad III_{\tau} = |\tau_{ij}| \equiv 0.$$

$$II_{\tau} = \begin{vmatrix} -1000 & 1000 \\ 1000 & 1000 \end{vmatrix} + |0| + |0| = -2 \times 10^6 \text{ (psi)}^2$$

$$\tau^3 - 2 \times 10^6 \tau = 0 \rightarrow \begin{cases} \tau = 0 \\ \tau^2 = 2 \times 10^6 \end{cases}$$

$$\therefore \tau_1 = \sqrt{2} \times 10^3 \text{ psi}, \quad \tau_2 = 0, \quad \tau_3 = -\sqrt{2} \times 10^3 \text{ psi}$$

(b) The array of principal values can be written

$$\tau_i = \begin{pmatrix} \sqrt{2} \times 10^3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} \times 10^3 \end{pmatrix}$$

From this array we can again compute the invariants, to compare with the results in (a). Trivially, $I_\tau = III_\tau = 0$. And

$$II_\tau = \begin{vmatrix} \sqrt{2} \times 10^3 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} \sqrt{2} \times 10^3 & 0 \\ 0 & \sqrt{2} \times 10^3 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & -\sqrt{2} \times 10^3 \end{vmatrix}$$

$$= -2 \times 10^6. \text{ Thus, agreement!}$$

(c) To obtain the direction cosines, need to solve $(\tau_{ij} - \sigma \delta_{ij}) v_j = 0$. See note below.**

For $\tau_1 = \sqrt{2} \times 10^3$, after some algebra: $v_3^1 = 0$
 $(1 + \sqrt{2}) v_1^1 = v_2^1$

But $v_i^1 v_i^1 = 1 \rightarrow v_1^1 = \frac{\sqrt{2} - 1}{\sqrt{4 - 2\sqrt{2}}}$ $v_2^1 = \frac{1}{\sqrt{4 - 2\sqrt{2}}}$

$$v_3^1 = 0$$

For $\tau_2 = 0$, get $v_1^2 = v_2^2 = 0$, $v_3^2 = 1$.

For $\tau_3 = -\sqrt{2} \times 10^3$, get $v_1^3 = \frac{-1 - \sqrt{2}}{\sqrt{4 + 2\sqrt{2}}}$

$$v_2^3 = \frac{1}{\sqrt{4 + 2\sqrt{2}}}, \quad v_3^3 = 0.$$

** Since $\tau_{13} = \tau_{23} = \tau_{33} = 0$, these equations reduce simply to the set:

$$(\tau_{11} - \tau) v_1 + \tau_{12} v_2 = 0$$

$$\tau_{12} v_1 + (\tau_{22} - \tau) v_2 = 0$$

$$\tau v_3 = 0.$$

The orthogonality relations, for $\tau_1 \neq \tau_2 \neq \tau_3$ that must be satisfied are

$$0 = \sum_j^1 \sum_j^2 = 0 + 0 + 0 = 0$$

$$0 = \sum_j^1 \sum_j^3 = \frac{\sqrt{2}-1-(1+\sqrt{2})}{\sqrt{4-2\sqrt{2}}\sqrt{4+2\sqrt{2}}} + \frac{1}{\sqrt{4-2\sqrt{2}}}\frac{1}{\sqrt{4+2\sqrt{2}}} = \frac{1-1}{\sqrt{4-2\sqrt{2}}\sqrt{4+2\sqrt{2}}} = 0$$

$$0 = \sum_j^2 \sum_j^3 = 0 + 0 + 0 = 0$$

∴ Have obtained correct principal stresses and directions.

1.6

The stress quadric is $\sum_i \sum_j \tau_{ij} = \pm d^2$.

(a) Uniaxial tension: $\tau_{11} = c^2 = \text{constant}$; other $\tau_{ij} = 0$.

$$\therefore \sum_1^2 \tau_{11} = \sum_1^2 c^2 = \pm d^2 = +d^2$$

$\sum_1 = \pm \frac{d}{c}$ } represents a pair of plane surfaces.

(b) Plane stress: Let $\tau_{13} = \tau_{23} = \tau_{33} = 0$.

$$\therefore \sum_i \sum_j \tau_{ij} = \sum_1^2 \tau_{11} + 2\sum_1 \sum_2 \tau_{12} + \sum_2^2 \tau_{22} = \pm d^2$$

$$\therefore \sum_1^2 + 2\frac{\tau_{12}}{\tau_{11}} \sum_1 \sum_2 + \frac{\tau_{22}}{\tau_{11}} \sum_2^2 = \pm \frac{d^2}{\tau_{11}}$$

This is clearly the equation of a closed curve in the \sum_1, \sum_2 plane, or a cylinder in \sum_1, \sum_2, \sum_3 space.

(c) For simple shear: only $\tau_{12} = \tau_{21} = c \neq 0$. Then

$$\sum_i \sum_j \tau_{ij} = 2c \sum_1 \sum_2 = \pm d^2$$

∴ $\sum_1 \sum_2 = \pm \frac{d^2}{2c}$, which is the eq. of a rectangular hyperbola in \sum_1, \sum_2 space.

1.7

To find $(\tau_{nn})_{\text{oct}}$ we can use Cauchy's formula. We assume that the x_i are coincident with the axes of principal stress τ_i . As the plane under consideration is at equal orientation to the principal axes, the components v_i of the plane's normal \vec{v} must be equal. Therefore, $v_i = 1/\sqrt{3}$, $i = 1, 2, 3$. Then

$$\begin{aligned} (\tau_{nn})_{\text{oct}} &= \vec{T}^{(v)} \cdot \vec{v} = T_i^{(v)} v_i = \tau_{ij} v_i v_j \\ &= \tau_i v_i^2 = \frac{1}{3}(\tau_1 + \tau_2 + \tau_3). \end{aligned}$$

The total stress vector on the inclined plane can be resolved into two components, the normal component $T^{(n)} \equiv (\tau_{nn})_{\text{oct}}$, and the shear component $T^{(s)} \equiv \tau_{\text{oct}}$. If $\vec{T}^{(v)}$ is the total stress vector on the inclined plane

$$(T^{(s)})^2 = \vec{T}^{(v)} \cdot \vec{T}^{(v)} - (\tau_{nn})_{\text{oct}}^2.$$

But $\vec{T}^{(v)} = \tau_{ij} v_j$ which is Cauchy's law, derived from Newton's law, thus

$$\vec{T}^{(v)} \cdot \vec{T}^{(v)} = \tau_{ij} v_j \tau_{ik} v_k = \tau_1^2 v_1^2 + \tau_2^2 v_2^2 + \tau_3^2 v_3^2.$$

$$\text{Thus } \tau_{\text{oct}}^2 = \frac{1}{3}(\tau_1^2 + \tau_2^2 + \tau_3^2) - \frac{1}{9}(\tau_1 + \tau_2 + \tau_3)^2$$

$$\begin{aligned} \therefore \tau_{\text{oct}}^2 &= \tau_1^2 \left(\frac{1}{3} - \frac{1}{9}\right) + \tau_2^2 \left(\frac{1}{3} - \frac{1}{9}\right) + \tau_3^2 \left(\frac{1}{3} - \frac{1}{9}\right) - \frac{2}{9}(\tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3) \\ &= \frac{1}{9} [2\tau_1^2 + 2\tau_2^2 + 2\tau_3^2 - 2\tau_1 \tau_2 - 2\tau_1 \tau_3 - 2\tau_2 \tau_3] \end{aligned}$$

$$\therefore \tau_{\text{oct}}^2 = \frac{1}{9} [(\tau_1 - \tau_2)^2 + (\tau_2 - \tau_3)^2 + (\tau_1 - \tau_3)^2]$$

For the uniaxial tension test, let $\tau_1 = Y$, $\tau_2 = \tau_3 = 0$.

Then $\tau_{\text{oct}}^2 = \frac{2}{9} Y^2$. Then the criterion of Mises-Hencky follows as

$$(\tau_1 - \tau_2)^2 + (\tau_2 - \tau_3)^2 + (\tau_1 - \tau_3)^2 = 2 Y^2$$

or, more simply, $\tau_{\text{oct}} = \frac{\sqrt{2}}{3} Y$ for yielding.

1.8

For principal stresses, $I_{\tau} = \tau_1 + \tau_2 + \tau_3$, and

$$II_{\tau} = \tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3. \text{ Then}$$

$$9\tau_{\text{oct}}^2 = 2\tau_1^2 + 2\tau_2^2 + 2\tau_3^2 - 2II_{\tau}$$

$$= 2(\tau_1 + \tau_2 + \tau_3)^2 - 4\tau_1\tau_2 - 4\tau_1\tau_3 - 4\tau_2\tau_3 - 2II_{\tau}$$

$$\therefore 9\tau_{\text{oct}}^2 = 2(I_{\tau})^2 - 6II_{\tau}.$$

1.9

The Green tensor is defined as (Eq. (1.32a)):

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial \delta_k}{\partial X_i} \frac{\partial \delta_k}{\partial X_j} - \delta_{ij} \right).$$

Then by straightforward substitution, the following strain tensor arrays are found:

(a) Simple dilatation:
$$\epsilon_{ij} = \begin{pmatrix} \frac{1}{2}(\lambda^2 - 1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) Pure deformation:
$$\epsilon_{ij} = \begin{pmatrix} \frac{1}{2}(\lambda_1^2 - 1) & 0 & 0 \\ 0 & \frac{1}{2}(\lambda_2^2 - 1) & 0 \\ 0 & 0 & \frac{1}{2}(\lambda_3^2 - 1) \end{pmatrix}$$

(c) Cubical dilatation:
$$\epsilon_{ij} = \frac{1}{2}(\lambda^2 - 1) \delta_{ij}.$$

(d) Simple shear:
$$\epsilon_{ij} = \begin{pmatrix} 0 & \frac{1}{2}\Gamma & 0 \\ \frac{1}{2}\Gamma & \frac{1}{2}\Gamma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Note in (d) the normal strain ϵ_{22} that occurs in the nonlinear theory for simple shear. Expressed in terms of stresses, this is often called the "normal stress effect".

1.10

Here $u_i = \xi_i - x_i = u_i(x_j)$, and $2 \overset{\text{linear}}{\epsilon}_{ij} = (u_{i,j} + u_{j,i})$

(a) Simple dilatation:
$$\epsilon_{ij}^{\text{linear}} = \begin{pmatrix} \lambda - 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) Pure deformation:
$$\epsilon_{ij}^{\text{linear}} = \begin{pmatrix} \lambda_1 - 1 & 0 & 0 \\ 0 & \lambda_2 - 1 & 0 \\ 0 & 0 & \lambda_3 - 1 \end{pmatrix}$$

(c) Cubical dilatation:
$$\epsilon_{ij}^{\text{linear}} = (\lambda - 1) \delta_{ij}$$

(d) Simple shear:
$$\epsilon_{ij}^{\text{linear}} = \begin{pmatrix} 0 & \Gamma/2 & 0 \\ \Gamma/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For physical interpretations, consider first simple dilatation. Then

$$\frac{d\xi_1}{dx_1} = \lambda, \quad \frac{d\xi_2}{dx_2} = \frac{d\xi_3}{dx_3} = 1. \quad \text{Thus we may note that}$$

$$\lambda = \frac{\text{deforming length}}{\text{initial length}} \equiv \text{stretch ratio}$$

Also, equally obviously, from case (d), we can identify Γ as the shear angle.

1.11

The initial volume of an element V is $dx_1 dx_2 dx_3$. The deformed volume is $V^* = d\xi_1 d\xi_2 d\xi_3$. Hence

$$V^* = (\lambda_1 dx_1)(\lambda_2 dx_2)(\lambda_3 dx_3) = \lambda_1 \lambda_2 \lambda_3 V, \quad \text{using the}$$

information of Prob. 9, part (b), for pure deformation.

From the definition of the Green strain tensor,

$$(ds^*)^2 / (ds)^2 = 1 + [2 \epsilon_{ij} dx_i dx_j / (ds)^2]$$

Let $ds = dx_1 \therefore (ds^*)^2 / (dx_1)^2 = \lambda_1^2$. Thus

$$\lambda_1^2 = 1 + 2 \epsilon_{11} \rightarrow \lambda_1 = \sqrt{1 + 2 \epsilon_{11}}$$

$$\therefore \lambda_i = \sqrt{1 + 2 \epsilon_{ii}} \quad , \quad i = 1, 2, 3; \quad (\text{no sum!})$$

Then, from above,

$$V^*/V = \sqrt{1 + 2 \epsilon_{11}} \sqrt{1 + 2 \epsilon_{22}} \sqrt{1 + 2 \epsilon_{33}}$$

If the strains are infinitesimal, $2 \epsilon_{ij} = u_{i,j} + u_{j,i} \ll 1$.

Then

$$V^*/V \approx (1 + \epsilon_{11})(1 + \epsilon_{22})(1 + \epsilon_{33}) \approx 1 + \epsilon_{11} + \epsilon_{22} + \epsilon_{33}$$

$$\therefore V^*/V - 1 = \epsilon_{ii} = \nabla \cdot \vec{U}$$

[Note, can also get $\lambda_i^2 = 1 + 2 \epsilon_{ii}$ result from Problem (9), part (b).]

1.12

Clearly $\epsilon_{ij} = 0$ implies $u_{i,j} + u_{j,i} = 0$. Thus

$$\frac{\partial u}{\partial x} = 0 \rightarrow u = u_1(y, z); \quad \frac{\partial v}{\partial y} = 0 \rightarrow v = v_1(x, z); \quad \frac{\partial w}{\partial z} = 0 \rightarrow w = w_1(x, y).$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \rightarrow \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} = 0 \quad \textcircled{1}; \quad \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \rightarrow \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} = 0 \quad \textcircled{2}$$

$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \rightarrow \frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial y} = 0 \quad \textcircled{3}$$

The implications of the vanishing normal strains can then be used with equations $\textcircled{1} - \textcircled{3}$ as follows:

$$\textcircled{1} \rightarrow u_1 = -y \frac{\partial v_1}{\partial x} + f(z); \quad \textcircled{2} \rightarrow u_1 = -z \frac{\partial w_1}{\partial x} + g(y)$$

$$\textcircled{1} \rightarrow v_1 = -x \frac{\partial u_1}{\partial y} + h(z); \quad \textcircled{3} \rightarrow v_1 = -z \frac{\partial w_1}{\partial y} + i(x)$$

$$\textcircled{2} \rightarrow w_1 = -x \frac{\partial u_1}{\partial z} + j(y); \quad \textcircled{3} \rightarrow w_1 = -y \frac{\partial v_1}{\partial z} + k(x)$$

$$\text{Then } \frac{\partial u_1}{\partial x} = 0 \quad \frac{\partial^2 v_1}{\partial x^2} = \frac{\partial^2 w_1}{\partial x^2} = 0.$$

$$\therefore v_1 = A(z)x + B(z), \quad w_1 = C(y)x + D(y)$$

$$\text{Also } \frac{\partial v_1}{\partial y} = 0 \rightarrow u_1 = E(z)y + F(z), \quad w_1 = G(x)y + H(x).$$

$$\text{And } \frac{\partial w_1}{\partial z} = 0 \rightarrow u_1 = I(y)z + J(y), \quad v_1 = K(x)z + L(x).$$

$$\begin{aligned} \therefore u_1(y,z) &= E(z)y + F(z) = I(y)z + J(y) \\ v_1(x,z) &= K(x)z + L(x) = A(z)x + B(z) \\ w_1(x,y) &= G(x)y + H(x) = C(y)x + D(y) . \end{aligned}$$

From the equations for $u_1(y,z)$, $E(z)y - I(y)z = J(y) - F(z)$

$$\therefore E(z) = E_1 = \text{const.}, \quad I(y) = I_1 = \text{const.}$$

$$\therefore u_1(y,z) = E_1 y + I_1 z ; \text{ similarly,}$$

$$v_1(x,z) = K_1 z + A_1 x ;$$

$$w_1(x,y) = C_1 x + G_1 y .$$

$$\text{But } \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} = E_1 + A_1 = 0 \rightarrow E_1 = -A_1 .$$

$$\text{Similarly, } C_1 = -I_1, \quad G_1 = -K_1 .$$

$$\therefore u(x,y,z) = 0 + E_1 y + I_1 z$$

$$v(x,y,z) = -E_1 x + 0 + K_1 z$$

$$w(x,y,z) = -I_1 x - K_1 y + 0 .$$

$$\text{If we write } \vec{\omega} = \omega_1 \hat{i} + \omega_2 \hat{j} + \omega_3 \hat{k} = -K_1 \hat{i} + I_1 \hat{j} - E_1 \hat{k}, \text{ and}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \text{ we can easily see that}$$

$$\vec{u} = \vec{\omega} \times \vec{r} .$$

A constant translational displacement can always be added. If it is denoted as \vec{u}_0 , with $\partial \vec{u}_0 / \partial x_i = 0$ for all i , the most general rigid body displacement can then be written as

$$\vec{u} = \vec{u}_0 + \vec{\omega} \times \vec{r} .$$

1.13 We have just shown (Problem 11), that the volumetric change is calculated as

$$V^*/V - 1 = \epsilon_{ii} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} .$$

Thus in terms of the deviator, $(V^*/V - 1)_{\text{Deviator}} = \bar{\epsilon}_{ii}$, or

$$(\Delta V/V)_{\text{Deviator}} = \epsilon_{ii} - \frac{1}{3} e \delta_{ii} = \epsilon_{ii} - \frac{3}{3} e \equiv 0 . \text{ Hence}$$

the dilatation of the deviator is identically zero.

To determine the invariants, consider the array of principal strains, since $e \delta_{ij}$ affects only the diagonal terms. Thus the principal deviator strains are (with $e = \frac{1}{3}(\epsilon_1 + \epsilon_2 + \epsilon_3)$)

$$\bar{\epsilon}_{ij} = \begin{pmatrix} (\epsilon_1 - \frac{1}{3}e) & 0 & 0 \\ 0 & (\epsilon_2 - \frac{1}{3}e) & 0 \\ 0 & 0 & (\epsilon_3 - \frac{1}{3}e) \end{pmatrix} = \begin{pmatrix} \bar{\epsilon}_1 & 0 & 0 \\ 0 & \bar{\epsilon}_2 & 0 \\ 0 & 0 & \bar{\epsilon}_3 \end{pmatrix}$$

Then the invariants are determined from $|\bar{\epsilon}_{ij} - \lambda \delta_{ij}| = 0$.

$$\text{Then } \lambda^3 - \text{I}_{\bar{\epsilon}} \lambda^2 + \text{II}_{\bar{\epsilon}} \lambda - \text{III}_{\bar{\epsilon}} = 0$$

$$\therefore \text{I}_{\bar{\epsilon}} = \bar{\epsilon}_1 + \bar{\epsilon}_2 + \bar{\epsilon}_3 = \epsilon_1 + \epsilon_2 + \epsilon_3 - 3e \equiv 0.$$

$$\begin{aligned} \text{II}_{\bar{\epsilon}} &= \bar{\epsilon}_1 \bar{\epsilon}_3 + \bar{\epsilon}_2 \bar{\epsilon}_3 + \bar{\epsilon}_1 \bar{\epsilon}_2 = (\epsilon_1 - e)(\epsilon_3 - e) \\ &\quad + (\epsilon_2 - e)(\epsilon_3 - e) + (\epsilon_1 - e)(\epsilon_2 - e) \\ &= -\frac{1}{6} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_1 - \epsilon_3)^2 + (\epsilon_2 - \epsilon_3)^2] \end{aligned}$$

$$\text{III}_{\bar{\epsilon}} = \bar{\epsilon}_1 \bar{\epsilon}_2 \bar{\epsilon}_3 = (\epsilon_1 - \frac{e}{3})(\epsilon_2 - \frac{e}{3})(\epsilon_3 - \frac{e}{3})$$

1.14

The transformation law for a second-order tensor is

$$A'_{ij} = a_{ip} a_{jq} A_{pq}$$

Then

$$\lambda \delta'_{ij} = a_{ip} a_{jq} \lambda \delta_{pq}$$

or

$$\lambda \delta'_{ij} = \lambda a_{ip} a_{jp} \equiv \lambda \delta_{ij}$$

Thus the equation of a second-order, isotropic tensor has been identified.

1.15

For rotation about the x_3 axis,

$$a_{ij} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \left\{ \begin{array}{l} \text{Note, isotropy transformations:} \\ A'_{ij} = a_{ip} a_{jq} A_{pq} \equiv A_{ij} \end{array} \right\}$$

$$\therefore A_{23} = a_{22} a_{33} A_{23} = -A_{23} \rightarrow A_{23} = 0$$

$$A_{32} = a_{33} a_{22} A_{32} = -A_{32} \rightarrow A_{32} = 0$$

$$A_{13} = a_{11} a_{33} A_{13} = -A_{13} \longrightarrow A_{13} = 0$$

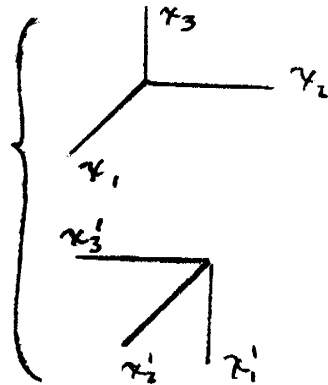
$$A_{31} = a_{33} a_{11} A_{31} = -A_{31} \longrightarrow A_{31} = 0.$$

Similarly for the rotation about the x_1 axis,

$$a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \quad \begin{aligned} A_{12} &= -A_{12} = 0 \\ A_{21} &= -A_{21} = 0 \end{aligned}$$

Now a rotation about the x_1 axis of 90° , followed by a rotation of 90° about the x_2 axis, produces

$$a_{ij} = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$



Then

$$\left. \begin{aligned} A_{11} &= a_{13} a_{13} A_{33} = A_{33} \\ A_{22} &= a_{21} a_{21} A_{11} = A_{11} \\ A_{33} &= a_{32} a_{32} A_{22} = A_{22} \end{aligned} \right\} \longrightarrow A_{11} = A_{22} = A_{33}$$

$$\text{Thus } A_{11} = A_{22} = A_{33} = \lambda.$$

$\therefore A_{ij} = \lambda \delta_{ij}$ is the most general second order isotropic tensor.

1.16

For $D_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \beta \delta_{ik} \delta_{jl} + \gamma \delta_{il} \delta_{jk}$, carry out the fourth order transformation-

$$\begin{aligned} D'_{ijkl} &= \lambda a_{im} a_{jn} a_{ko} a_{lp} \delta_{mn} \delta_{op} + \beta a_{im} a_{ko} a_{jn} a_{lp} \delta_{mo} \delta_{np} \\ &\quad + \gamma a_{im} a_{lp} a_{jn} a_{ko} \delta_{mp} \delta_{no} \\ &= \lambda a_{im} a_{jm} a_{kp} a_{lp} + \beta a_{im} a_{km} a_{jn} a_{ln} + \gamma a_{im} a_{lm} a_{jn} a_{kn} \\ &= \lambda \delta_{ij} \delta_{kl} + \beta \delta_{ik} \delta_{jl} + \gamma \delta_{il} \delta_{jk} \equiv D_{ijkl} \cdot \text{QED.} \end{aligned}$$