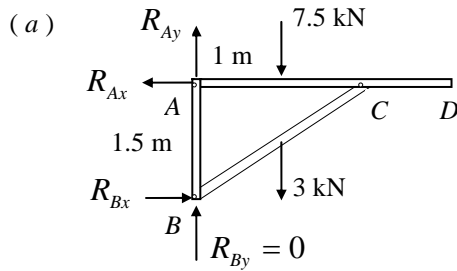


## CHAPTER 1

### SOLUTION (1.1)



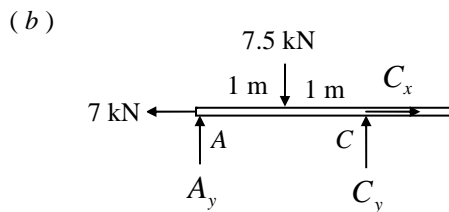
Entire Structure

$$\sum M_A = 0: 7.5(1) + 3(1) = 1.5R_{Bx}$$

or  $R_{Bx} = 7 \text{ kN}$

$$\sum F_x = 0: R_{Ax} = -7 \text{ kN}$$

$$\sum F_y = 0: R_{Ay} = 10.5 \text{ kN}$$

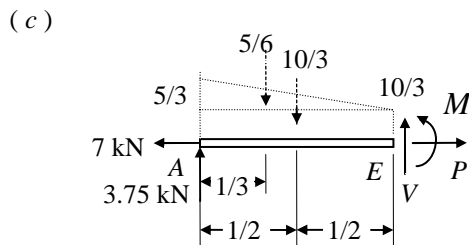


Member AD

$$\sum M_A = 0: C_y(2) = 7.5(1); C_y = 3.75 \text{ kN}$$

$$\sum F_y = 0: A_y = 7.5 - 3.75 = 3.75 \text{ kN}$$

$$\sum F_x = 0: C_x = 7 \text{ kN}$$



Segment AE

$$\sum F_x = 0: P = 7 \text{ kN}$$

$$\sum F_y = 0: V = 3.75 - \frac{5}{6} - \frac{10}{3} = 0.417 \text{ kN}$$

$$\sum M_E = 0: M = 3.75(1) - \frac{5}{6}(\frac{2}{3}) - \frac{10}{3}(\frac{1}{2}) = 1.528 \text{ kN} \cdot \text{m}$$

### SOLUTION (1.2)

Refer to Fig. P1.2:  $\sum M_A = 0: R_B = \frac{2}{3} pa \uparrow$

(a)  $\sum M_0 = 0: M = \frac{2}{3} pax - \frac{1}{2} px^2$  (a) ◀

Thus

$$\frac{dM}{dx} = 0: \frac{2}{3} pa - px = 0; x = \frac{2}{3} a$$

Equation (a), for  $x = \frac{2}{3} a$ ;  $M_{\max} = \frac{2}{9} pa^2$  ◀

(b) Equation (a), for  $x = \frac{3}{2} a$ :

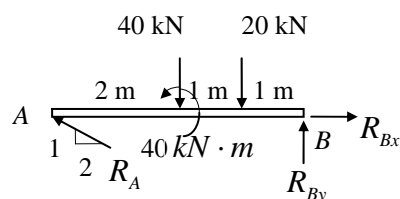
$$M = \frac{2}{3} pa(\frac{3}{2} a) - \frac{1}{2} p(\frac{3}{2} a)^2 = -\frac{1}{8} pa^2 = \frac{1}{8} pa^2 \curvearrowright$$

Shear force at  $x = \frac{3}{2} a$ :

$$V = -\frac{2}{3} pa + \frac{3}{2} pa = \frac{5}{6} pa \uparrow$$

### SOLUTION (1.3)

(a)



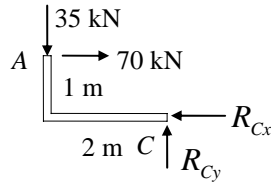
Member AB

$$\sum M_B = 0: R_A = 35\sqrt{5} \text{ kN}$$

$$\sum F_x = 0: R_{Bx} = 70 \text{ kN}$$

$$\sum F_y = 0: R_{By} = 25 \text{ kN} \quad (\text{CONT.})$$

(1.3 CONT.)



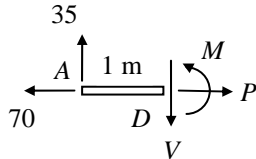
Member AC

$$\sum M_A = 0: R_{Cx} = 2R_{Cy}$$

$$\sum F_y = 0: R_{Cy} = 35 \text{ kN}$$

$$\sum F_x = 0: R_{Cx} = 70 \text{ kN}$$

(b)



Segment AD

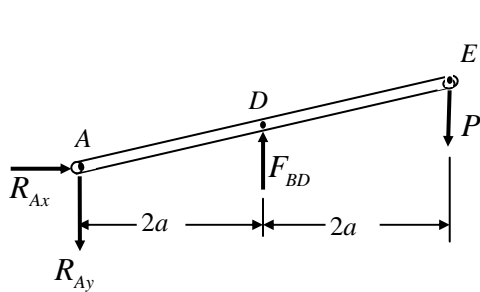
$$\sum F_x = 0: P = 70 \text{ kN}$$

$$\sum F_y = 0: V = 35 \text{ kN}$$

$$\sum M_D = 0: M = 35 \text{ kN} \cdot \text{m}$$

**SOLUTION (1.4)**

Link BD is a two-force member and hence the direction of  $F_{BD}$  is known.



(a) Free body-Member ADE

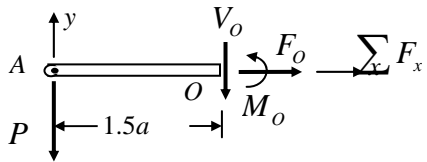
$$\sum M_A = 0:$$

$$-P(4a) + F_{BD}(2a) = 0, \quad F_{BD} = 2P \uparrow$$

$$\sum F_x = 0: R_{Ax} = 0$$

$$\sum F_y = 0:$$

$$-R_{Ay} + F_{BD} - P = 0, \quad R_{Ay} = P \downarrow$$



(b) Free Body-Part AO

$$\sum F_x = 0: F_o = 0$$

$$\sum F_y = 0: V_o = P \uparrow$$

$$\sum M_o = 0: M_o = 1.5Pa \curvearrowright$$

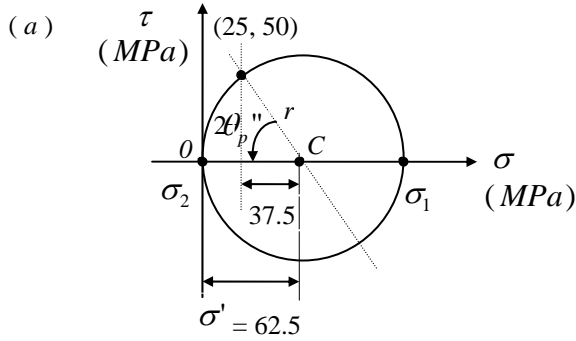
**SOLUTION (1.5)**

We have  $\theta = 115^\circ$ . Apply Eqs. ( 1.11 ):

$$\begin{aligned} \sigma_{x'n} &= \frac{1}{2}(-50 + 40) + \frac{1}{2}(-50 - 40) \cos 230^\circ - 20 \sin 230^\circ \\ &= -5 + 28.92 + 15.32 = 39.2 \text{ MPa} \end{aligned}$$

$$\tau_{x'y'} = -\frac{1}{2}(-50 - 40) \sin 230^\circ - 20 \cos 230^\circ = -21.6 \text{ MPa}$$

**SOLUTION (1.6)**



$$\theta_p'' = \frac{1}{2} \tan^{-1} \frac{50}{37.5} = 26.6^\circ$$

$$r = (50^2 + 37.5^2)^{1/2} = 62.5$$

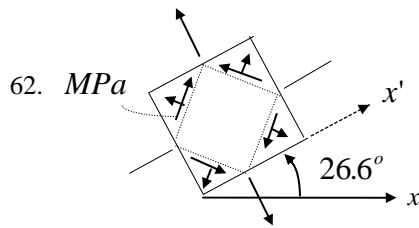
Thus,

$$\sigma_1 = 62.5 + 62.5 = 125 \text{ MPa}$$

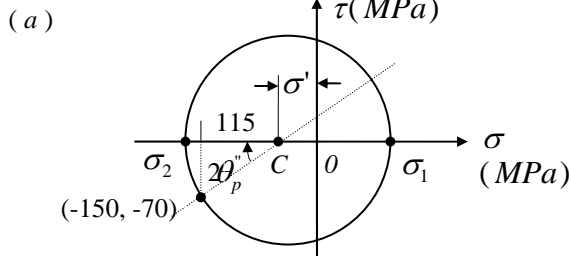
$$\sigma_2 = 0$$

(b) 125 MPa

$$\tau_{\max} = r = 62.5 \text{ MPa}$$



**SOLUTION (1.7)**



$$\sigma' = \frac{1}{2}(-150 + 80) = -35 \text{ MPa}$$

$$\theta_p'' = \frac{1}{2} \tan^{-1} \frac{70}{115} = 15.66^\circ$$

$$r = (115^2 + 70^2)^{1/2} = 134.6$$

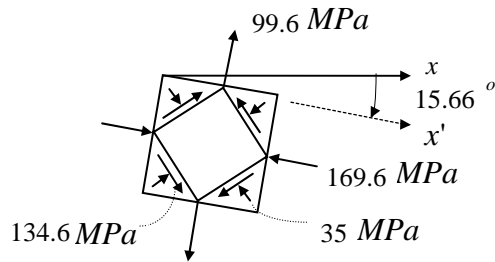
Thus,

$$\sigma_1 = -35 + 134.6 = 99.6 \text{ MPa}$$

$$\sigma_2 = -169.6 \text{ MPa}$$

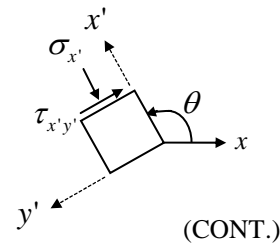
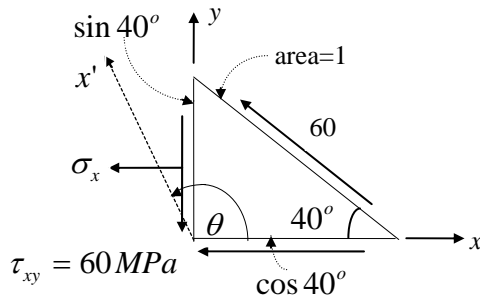
(b)

$$\tau_{\max} = r = 134.6 \text{ MPa}$$



**SOLUTION (1.8)**

(a)



(1.8 CONT.)

We have  $\sigma_y = 0$  and  $\tau_{xy} = 60 \text{ MPa}$ .

$$\sum F_x = 0: \quad \sigma_x \sin 40^\circ = 60 \cos 40^\circ + 60 \cos 40^\circ$$

or  $\sigma_x = 143 \text{ MPa (comp.)}$  ◀

Apply Eqs. (1.11) with  $\theta = 90 + 25 = 115^\circ$ :

$$\sigma_{x'} = -\frac{143}{2} - \frac{143}{2} \cos 230^\circ + 60 \sin 230^\circ = -71.5 \text{ MPa}$$

$$\tau_{x'y'} = 71.5 \sin 230^\circ + 60 \cos 230^\circ = -93.34 \text{ MPa} \quad \blacktriangleleft$$

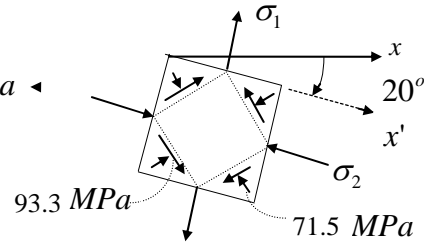
(b)

$$\tau_{\max} = [(-71.5)^2 + 60^2]^{1/2} = 93.34 \text{ MPa} \quad \blacktriangleleft$$

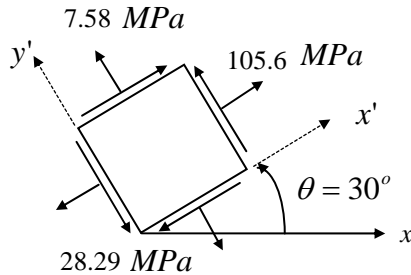
$$\sigma' = -71.5 \text{ MPa}$$

It may be seen from a sketch of Mohr's Circle that

$$\theta_p'' = \frac{1}{2} \tan^{-1} \frac{60}{71.5} = 20^\circ \quad \curvearrowright$$



**SOLUTION (1.9)**



$$\sigma_x = \sigma_y = 80 \cos 45^\circ = 56.57 \text{ MPa}$$

$$\tau_{xy} = 80 \sin 45^\circ = 56.57 \text{ MPa}$$

Apply Eqs. (1.11):

$$\sigma_{x'} = \frac{1}{2} (56.57 + 56.57) + 0 + 56.57 \sin 60^\circ = 105.6 \text{ MPa}$$

$$\sigma_{y'} = 56.57 - 48.99 = 7.58 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{x'y'} = -0 + 56.57 \cos 60^\circ = 28.29 \text{ MPa}$$

**SOLUTION (1.10)**

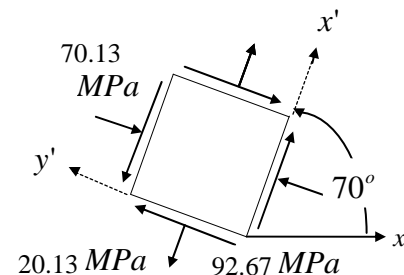
We have  $\sigma_x = 0$ ,  $\sigma_y = -50 \text{ MPa}$ ,  $\tau_{xy} = 100 \text{ MPa}$ ,  $\theta = 70^\circ$

Apply Eqs. (1.11):

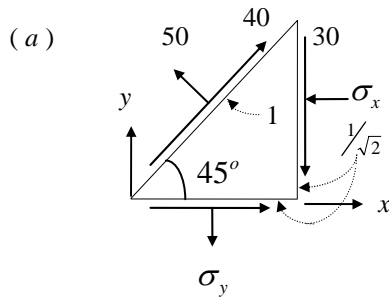
$$\begin{aligned} \sigma_{x'} &= -\frac{50}{2} + \frac{50}{2} \cos 140^\circ + 100 \sin 140^\circ \\ &= 20.13 \text{ MPa} \end{aligned}$$

$$\sigma_{y'} = -25 + 19.15 - 64.28 = -70.13 \text{ MPa} \quad \blacktriangleleft$$

$$\begin{aligned} \tau_{x'y'} &= -25 \sin 140^\circ + 100 \cos 140^\circ \\ &= -92.67 \text{ MPa} \end{aligned}$$



**SOLUTION (1.11)**



$$\tau_{xy} = -30 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}$$

$$\sum F_x = 0: \quad \sigma_x \left(\frac{1}{\sqrt{2}}\right) = 30\left(\frac{1}{\sqrt{2}}\right) + 40\left(\frac{1}{\sqrt{2}}\right) - 50\left(\frac{1}{\sqrt{2}}\right)$$

or  $\sigma_x = 20 \text{ MPa (comp.)}$

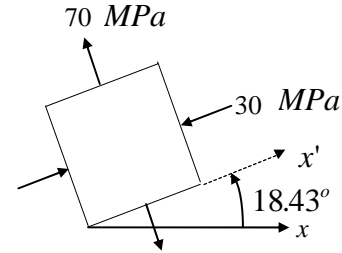
(b) It may be seen from a sketch of Mohr's circle that

$$\theta_p'' = \frac{1}{2} \tan^{-1} \frac{3}{4} = 18.43^\circ$$

$$\sigma_{1,2} = \frac{-20+60}{2} \pm \left[ \left(\frac{-20-60}{2}\right)^2 + (-30)^2 \right]^{1/2}$$

$$= 20 \pm 50$$

or  $\sigma_1 = 70 \text{ MPa}, \quad \sigma_2 = -30 \text{ MPa}$



**SOLUTION (1.12)**

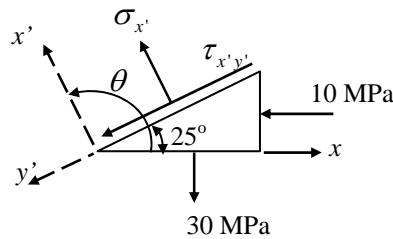
We have

$$\theta = 25 + 90 = 115^\circ$$

$$\sigma_x = -10 \text{ MPa}$$

$$\sigma_y = 30 \text{ MPa}$$

$$\tau_{xy} = 0$$



(a)  $\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta$

$$= \frac{1}{2}(-10 + 30) + \frac{1}{2}(-10 - 30) \cos 230^\circ = 22.86 \text{ MPa}$$

Thus,

$$\sigma_w = \sigma_{x'} = 22.86 \text{ MPa}$$

(b)  $\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$

$$= -\frac{1}{2}(-10 - 30) \sin 230^\circ = -15.32 \text{ MPa}$$

So

$$\tau_w = \tau_{x'y'} = -15.32 \text{ MPa}$$



**SOLUTION (1.13)**

(a)  $\epsilon_x = \alpha \Delta T = 11.7(10^{-6})(80^\circ) = 936 \mu$

Thus  $\delta = \epsilon_x L = 936(10^{-6})250 = 0.234 \text{ mm} \rightarrow$

(b)  $\epsilon_x = \alpha \Delta T = 11.7(10^{-6})\left(\frac{80x}{L}\right) = 9.36(10^{-4}) \frac{x}{L}$

Hence  $u = \int \epsilon_x dx = 4.68(10^{-4}) \frac{x^2}{L} + c$  (a)

We have  $u=0$  at  $x=0$ ;  $c=0$ . Equation (a) for  $x=250 \text{ mm}$  gives  $\delta = 0.117 \text{ mm} \rightarrow$

---

**SOLUTION (1.14)**

From solution of Prob. 1.11:

$$\sigma_x = -20 \text{ MPa}, \quad \sigma_y = 60 \text{ MPa}, \quad \tau_{xy} = -30 \text{ MPa}$$

$$\text{Thus, } \varepsilon_x = \frac{1}{70(10^3)} [-20 - 0.3(60)] = -543 \mu, \quad \varepsilon_y = \frac{1}{70(10^3)} [60 + 0.3(20)] = 943 \mu \quad \blacktriangleleft$$

$$\gamma_{xy} = -\frac{30}{70(10^3)^{2.6}} = -1114 \mu$$

(a) Equations (1.25) with  $\theta = 180 - 60 = 120^\circ$ :

$$\begin{aligned} \varepsilon_{x'} &= \frac{-543+943}{2} + \frac{-543-943}{2} \cos 240^\circ - 1114 \sin 240^\circ \\ &= 200 + 371.5 + 964.7 = 1536 \mu \end{aligned} \quad \blacktriangleleft$$

$$(b) \quad \gamma_{\max} = 2\left[\left(\frac{-543-943}{2}\right)^2 + \left(-\frac{1114}{2}\right)^2\right]^{1/2} = 1857 \mu \quad \blacktriangleleft$$

---

**SOLUTION (1.15)**

$$(a) \quad \varepsilon_{1,2} = \frac{50+250}{2} \pm \left[\left(\frac{50-250}{2}\right)^2 + \left(-\frac{150}{2}\right)^2\right]^{1/2} = 150 \pm 125$$

$$\text{or } \varepsilon_1 = 275 \mu, \quad \varepsilon_2 = 25 \mu \quad \blacktriangleleft$$

(b) Apply Hooke's Law with  $\sigma_z = 0$ :

$$275(10^{-6}) = \frac{1}{210(10^9)} (\sigma_1 - 0.3\sigma_2); \quad 57.75 = \sigma_1 - 0.3\sigma_2$$

$$25(10^{-6}) = \frac{1}{210(10^9)} (\sigma_2 - 0.3\sigma_1); \quad 5.25 = \sigma_2 - 0.3\sigma_1$$

$$\text{Solving, } \sigma_1 = 65.19 \text{ MPa}, \quad \sigma_2 = 24.81 \text{ MPa} \quad \blacktriangleleft$$

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**SOLUTION (1.16)**

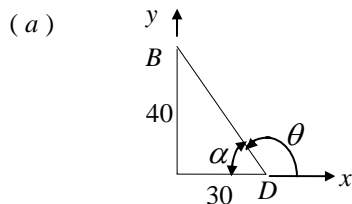
We have

$$(a) \quad \varepsilon_x = \frac{0.006}{50} = 120 \mu, \quad \varepsilon_y = -\frac{0.004}{25} = -160 \mu$$

$$\gamma_{xy} = -1000 - 50 = -1500 \mu \text{ rad} \quad \blacktriangleleft$$

$$(b) \quad \gamma_{\max} = 2\mu\left[\left(\frac{120+160}{2}\right)^2 + \left(-\frac{1500}{2}\right)^2\right]^{1/2} = 1526 \mu \quad \blacktriangleleft$$

---

**SOLUTION (1.17)**

$$\alpha = \tan^{-1} \frac{4}{3} = 53.13^\circ,$$

$$\theta = 126.87^\circ$$

Apply Eq. (1.25);

$$\varepsilon_{x'} = \frac{400+800}{2} + \frac{400-800}{2} \cos 2(126.87^\circ) + \frac{200}{2} \sin 2(126.87^\circ) = 560 \mu \quad \blacktriangleleft$$

$$(b) \quad \varepsilon_{1,2} = \frac{400+800}{2} \pm \left[\left(\frac{400-800}{2}\right)^2 + \left(\frac{200}{2}\right)^2\right]^{1/2} = 600 \pm 224$$

$$\text{or } \varepsilon_1 = 824 \mu, \quad \varepsilon_2 = 376 \mu \quad \blacktriangleleft$$

It may be seen from a sketch of Mohr's Circle that;

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{1}{2}\right) = 13.28^\circ \quad \curvearrowright$$

---

**SOLUTION (1.18)**

$$(a) \quad \varepsilon_x = \frac{1}{100(10^3)}(150 + \frac{90}{3}) = 1800 \mu, \quad \varepsilon_y = \frac{1}{100(10^3)}(-90 - \frac{150}{3}) = -1400 \mu$$

$$\varepsilon_z = -\frac{\frac{1}{2}}{100(10^3)}(150 - 90) = -200 \mu$$

$$\Delta a = 1800\mu(100) = 180 \mu m, \quad \Delta b = -1400\mu(50) = -70 \mu m,$$

$$\Delta t = -200\mu(10) = -2 \mu m$$

Thus,

$$a' = 1000.18 \text{ mm}, \quad b' = 49.93 \text{ mm}, \quad t' = 9.998 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad A'C' = [100.8^2 + 49.93^2]^{\frac{1}{2}} = 111.933 \text{ mm} \quad \blacktriangleleft$$

---

**SOLUTION (1.19)**

(a) We have  $\sigma_x = \sigma_y = \sigma_z = -p$ ,  $AC = 111.8034 \text{ mm}$ .

Equation (1.34),

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = -\frac{p}{E}(1 - 2\nu) = -\frac{120}{100(10^3)}(\frac{1}{3}) = -400 \mu \quad (a)$$

and

$$\Delta a = -400\mu(100) = -40 \mu m, \quad \Delta b = -400\mu(50) = -20 \mu m$$

$$\Delta t = -400\mu(10) = -4 \mu m$$

Hence

$$a' = 99.96 \text{ mm}, \quad b' = 49.98 \text{ mm}, \quad t' = 9.996 \text{ mm} \quad \blacktriangleleft$$

$$(b) \quad A'C' = [99.96^2 + 49.98^2]^{\frac{1}{2}} = 111.7587 \text{ mm} \quad \blacktriangleleft$$

---

**SOLUTION (1.20)**

$$\angle DAC = \theta = \tan^{-1} \frac{3}{4} = 36.9^\circ$$

$$L_{AC} = L_{BD} = \sqrt{90^2 + 120^2} = 150 \text{ mm} \quad G = \frac{E}{2(1+\frac{1}{3})} = \frac{3}{8} E$$

$$\varepsilon_x = \frac{1}{E}(150 + \frac{75}{3}) = \frac{175(10^6)}{E}$$

$$\varepsilon_y = \frac{1}{E}(75 + \frac{150}{3}) = \frac{125(10^6)}{E}$$

$$\gamma_{xy} = \frac{150}{G} = \frac{400(10^6)}{3E}$$

We have

$$\theta_1 = 180 - 36.9 = 143.1^\circ$$

Taking  $x'$  along  $BD$

$$\varepsilon_{x'} = (25 + 150 \cos 286.2^\circ + 66.67 \sin 286.2^\circ) \frac{10^6}{E}$$

$$= (25 + 41.85 - 64) \frac{10^6}{E} = 2.85(10^6)/E$$

$$\Delta L_{BD} = 100 \frac{2.85(10^6)}{210(10^9)} = 1.35 \times 10^{-3} \text{ mm} \quad \blacktriangleleft$$

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