1.1 (a) THERE 8 POSSIBLE 3-bit CODEWORDS:

$$\left\{\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right\}$$

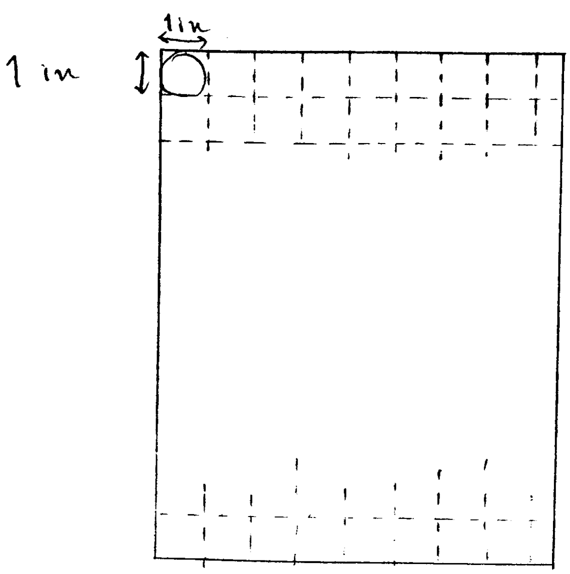
$$\Rightarrow \mathcal{C} = \left\{\begin{array}{cccc} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{array}\right\}$$

(b) CONSIDER THE (7, 4) HAMMING CODE:

$$\begin{array}{ccccccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{array}$$

IT HAS MINIMUM DISTANCE 3.

1.2 (a) A $8.5 \times 11 \text{ in}^2$ PAPER SHEET CAN ALLOCATE $8 \times 11 = 88$ SQUARES OF SIZE 1 inch. EACH OF THIS SQUARES CAN ENCLOSE A CIRCLE OF 1 inch DIAMETER. HENCE WE CAN CUT THE 88 CIRCLES:



WHY IS NOT POSSIBLE TO HAVE 120 CIRCLES (OR MORE) OF 1-INCH DIAMETER ?

BECAUSE THE TOTAL SURFACE OF 120 CIRCLES OF 1 in DIAMETER IS :

$$S = 120 \times \frac{\pi d^2}{4}$$
$$= 30\ \pi\ in^2$$
$$= 94.2477\ in^2$$
$$> 93.5 = 8.5 \times 11\ in^2$$

WHICH IS THE SPACE AVAILABLE IN THE SHEET.

NOTICE THAT IF WE ADD THE SURFACE OF 119 1-IN DIAMETER CIRCLE, WE GET :

$$S = 119 \times \frac{\pi d^2}{4} = 93.46\ in^2 < 93.5\ in^2 = 8.5 \times 11\ in^2$$

So THEORETICALLY, THERE IS ROOM FOR 119 CIRCLES.

b. THERE ARE $2^8 = 256$ DISTINCT 8-bit WORDS. IF WE WANT TO FIND THE NUMBER OF CODEWORDS WITH MINIMUM DISTANCE 3, WE MUST COUNT THE NUMBER OF DECODING SPHERES WITH RADIUS 1.5, BECAUSE ALL THESE ARE DISJOINT.

EACH DECODING SPHERE WILL CONTAIN 9 ELEMENTS : THE CODEWORD IT'SELF, PLUS 8 VECTORS OBTAINED MODIFYING ONE BIT OF THE CODEWORD.

THE TOTAL NUMBER OF DECODING SPHERES IS

$$\frac{256}{9} \approx 28.44$$

THERE EXISTS 28 CODEWORDS WITH DISTANCE 3 OR MORE FROM EACH OTHER.

(1.3) a. WE HAVE $n = 2^m - 1$, $k = 2^m - 1 - m$

$\Rightarrow$ $m = 3$, $n = 7$, $k = 4$

$m = 4$, $n = 15$, $k = 11$

$m = 5$, $n = 31$, $k = 26$

$m = 6$, $n = 63$, $k = 57$

$m = 7$, $n = 127$, $k = 120$.

b. RATE OF THE CODE $R = \frac{k}{n}$

$m = 3$, $R = \frac{4}{7} = 0.57$

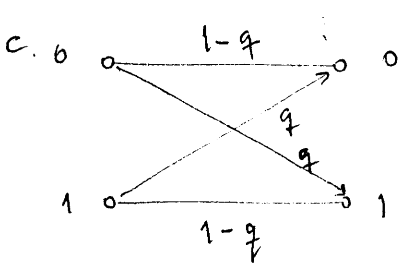
$m = 4$, $R = \frac{11}{15} = 0.73$

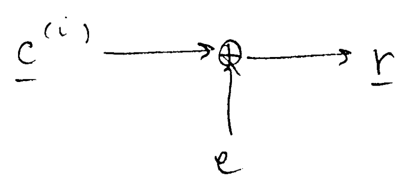
$m = 5$, $R = \frac{26}{31} = 0.84$

$m = 6$, $R = \frac{57}{63} = 0.90$

$m = 7$, $R = \frac{120}{127} = 0.94$.

c.





WE HAVE:

$$\Pr\{\text{"decod. error"}\} = \Pr\{ wt(\underline{e}) = wt(\underline{c}^{(i)} + \underline{r}) \geq 2 \}$$

$$= \sum_{j=2}^{n} \binom{n}{j} (1-q)^{n-j} q^j$$

$$= 1 - (1-q)^n - n(1-q)^{n-1} q$$

AS $n \to \infty$ $\Pr\{\text{"decod. error"}\} \to 1$.