

1.1. Since the plastic strain is one quarter of the elastic strain, the total strain at the yield point is

$$\epsilon = \epsilon^e + \epsilon^p = \frac{5}{4} \epsilon^e = \frac{5Y}{4E}$$

and the stress/strain equation gives

$$Y = \frac{E}{180} \left(\frac{5Y}{4E} \right)^{0.25}$$

or
$$\left(\frac{Y}{E} \right)^{0.75} = \frac{1}{180} \left(\frac{5}{4} \right)^{0.25}$$

or
$$\frac{Y}{E} = \left(\frac{1}{180} \right)^{4/3} \left(\frac{5}{4} \right)^{1/3} = \frac{1}{180} \left(\frac{1}{144} \right)^{1/3} \approx \frac{1}{943} .$$

The true strain at instability is $\epsilon = 0.25$, while the true and nominal stresses at instability are given by

$$\frac{\sigma}{Y} = \frac{E}{Y} \frac{(0.25)^{0.25}}{180} = (144)^{1/3} (0.25)^{0.25} \approx 3.71$$

$$\frac{s}{Y} = \frac{\sigma}{Y} \left(\frac{A}{A_0} \right) = \frac{\sigma}{Y} e^{-\epsilon} = 3.71 e^{-0.25} \approx 2.89 .$$

1.2. The engineering strain at the onset of instability is given by the instability condition

$$\frac{d\sigma}{de} = \frac{d}{de} (C e^{n}) = \frac{n\sigma}{e} = \frac{\sigma}{1+e}$$

or $(1+e)n = e$, or $e = n/(1-n)$

and $\epsilon = \ln(1+e) = \ln \left(\frac{1}{1-n} \right)$, which exceeds $\ln \left(\frac{1}{1-e} \right)$ when $e < n$.

Since the magnitude of the total true strain at instability must be equal to $\ln\{1/(1-n)\}$,

$$\ln \frac{1}{1-e} + \ln \frac{l_2}{l_1} = \ln \left(\frac{1}{1-n} \right)$$

where l_1 is the length of the bar at the end of compression, and l_2 the length at instability. Then

$$l_2 = l_1 \left(\frac{1-e}{1-n} \right) = l_0(1-e) \left(\frac{1-e}{1-n} \right) = l_0 \frac{(1-e)^2}{1-n}$$

where l_0 is the initial length of the bar.

1.3. According to the Voce equation, the instability condition is

$$\frac{d\sigma}{d\epsilon} = \frac{d}{d\epsilon} \{ C(1 - m e^{-n\epsilon}) \} = C m n e^{-n\epsilon} = n(C - \sigma)$$

and the onset of instability corresponds to

$$n(C - \sigma) = \sigma, \quad \text{or} \quad \sigma = Cn/(1+n).$$

Hence,
$$e^{n\epsilon} = \frac{Cm}{C-\sigma} = m \left(1 - \frac{n}{1+n} \right) = m(1+n)$$

and
$$\epsilon = \frac{1}{n} \ln [m(1+n)], \quad m(1+n) \geq 0.$$

When $m(1+n) < 1$, the instability strain is zero for a rigid/plastic bar. Introducing ϵ^* ,

$$\begin{aligned} \sigma &= C(1 - m e^{-n\epsilon}) = C \left\{ 1 - m \left(\frac{l_0}{l} \right)^n \right\} \\ &= C \left\{ (1-m) + m \left[1 - \left(\frac{l_0}{l} \right)^n \right] \right\} = C(1-m + mn\epsilon^*). \end{aligned}$$

The stress-strain curve is thus linearized. Note that when $n=0$, $\epsilon^* = n(l/l_0) = \epsilon$.

1.4. The total compressive load at any stage is

$$P = \sigma A = \sigma A_0 \left(\frac{h_0}{h} \right) = \sigma A_0 / (1 - e).$$

Since $d\epsilon = de/(1-e)$ in compression, the differentiation of the above equation gives

$$\begin{aligned} \frac{dP}{de} &= \frac{A_0}{(1-e)^2} \frac{d\sigma}{d\epsilon} + \frac{\sigma A_0}{(1-e)^2} = \frac{A_0}{(1-e)^2} \left(\frac{d\sigma}{d\epsilon} + \sigma \right) \\ \frac{d^2P}{de^2} &= \frac{A_0}{(1-e)^3} \left(\frac{d^2\sigma}{d\epsilon^2} + \frac{d\sigma}{d\epsilon} \right) + \frac{2A_0}{(1-e)^3} \left(\frac{d\sigma}{d\epsilon} + \sigma \right). \end{aligned}$$

Hence, at the point of inflection ($d^2P/de^2 = 0$),

$$\frac{d^2\sigma}{d\epsilon^2} + 3 \frac{d\sigma}{d\epsilon} + 2\sigma = 0.$$

The empirical stress-strain equation $\sigma = C\epsilon^n$ gives

$$\frac{d\sigma}{d\epsilon} = \frac{n\sigma}{\epsilon}, \quad \frac{d^2\sigma}{d\epsilon^2} = \frac{n}{\epsilon} \frac{d\sigma}{d\epsilon} - \frac{n\sigma}{\epsilon^2} = n(n-1) \frac{\sigma}{\epsilon^2}.$$

Hence the point of inflection corresponds

$$\left\{ \frac{n(n-1)}{\epsilon^2} + \frac{3n}{\epsilon} + 2 \right\} \sigma = 0$$

or
$$2\epsilon^2 + 3n\epsilon - n(1-n) = 0$$

or
$$\epsilon = \frac{1}{4} [-3n + \sqrt{n(8+n)}]$$

Thus, $\epsilon > n$ corresponds to $\sqrt{n(8+n)} > 7n$, or $8n + n^2 > 49n^2$, or $n < 1/6$.

1.5. The nominal compressive stress at any stage is

$$s = \frac{P}{A_0} = \frac{P}{A} \left(\frac{h_0}{h} \right) = \sigma \exp(\epsilon) .$$

By successive differentiation, we get

$$\frac{ds}{d\epsilon} = \left(\frac{d\sigma}{d\epsilon} + \sigma \right) \exp(\epsilon)$$

$$\frac{d^2s}{d\epsilon^2} = \left(\frac{d^2\sigma}{d\epsilon^2} + 2 \frac{d\sigma}{d\epsilon} + \sigma \right) \exp(\epsilon) .$$

The point of inflection corresponds to $d^2s/d\epsilon^2 = 0$, or

$$\frac{d^2\sigma}{d\epsilon^2} + 2 \frac{d\sigma}{d\epsilon} + \sigma = \left(\frac{d}{d\epsilon} + 1 \right)^2 \sigma = 0$$

$$\text{or} \quad \left\{ \frac{n(n-1)}{\epsilon^2} + \frac{2n}{\epsilon} + 1 \right\} \sigma = 0$$

$$\text{or} \quad \epsilon^2 + 2n\epsilon - n(1-n) = 0, \quad \text{or} \quad \epsilon = \sqrt{n-n} > n.$$

The inflection strain will exceed the uniaxial instability strain if $\sqrt{n-n} > n$, or $n < 0.25$.

1.6. The differentiation of the Ramberg-Osgood equation furnishes

$$\frac{d\epsilon}{d\sigma} = \frac{1}{E} + \frac{3}{7nE} \left(\frac{\sigma}{\sigma_0} \right)^{(1-n)/n} = \frac{1}{\sigma}$$

at the onset of instability in simple tension.

Since $E \gg \sigma$, we have

$$\frac{3\sigma_0}{7nE} \left(\frac{\sigma}{\sigma_0} \right)^{1/n} \cong 1, \quad \text{or} \quad \frac{\sigma}{\sigma_0} \cong \left(\frac{7nE}{3\sigma_0} \right)^n$$

$$\text{and} \quad \epsilon \cong \frac{\sigma_0}{E} \left(\frac{7nE}{3\sigma_0} \right)^n + \frac{3\sigma_0}{7E} \left(\frac{7nE}{3\sigma_0} \right)$$

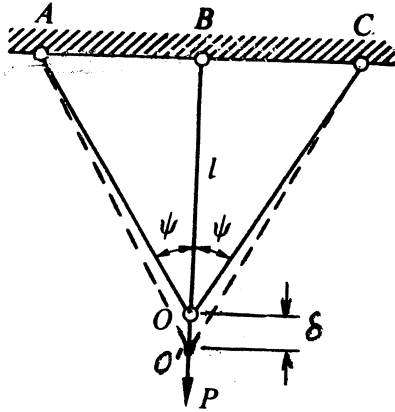
$$\text{or} \quad \epsilon = n + \left(\frac{7n}{3} \right)^n \left(\frac{\sigma_0}{E} \right)^{1-n}$$

When $n = 0.05$ and $\sigma_0/E = 0.002$, the instability strain is

$$\epsilon = 0.05 + \left(\frac{0.35}{3} \right)^{0.05} (0.002)^{0.95} \cong 0.0525$$

the percentage error being $0.25/(0.0525) \cong 4.8\%$.

1.7.



$$\text{or } e^{2n_1} - 1 = (e^{2n_2} - 1) \sec^2 \psi$$

$$\text{or } \cos \psi = \sqrt{\frac{\exp(2n_2) - 1}{\exp(2n_1) - 1}}$$

1.8. During a small deformation produced by a vertical load P at O , the strains in the vertical and inclined bars are

$$\epsilon_1 = \frac{\delta}{l}, \quad \epsilon_2 = \frac{\delta \cos \psi}{l \sec \psi} = \frac{\delta}{l} \cos^2 \psi$$

respectively. When all the bars are plastic, the corresponding stresses are

$$\sigma_1 = Y \left(\frac{E\delta}{Yl} \right)^n, \quad \sigma_2 = Y \left(\frac{E\delta}{Yl} \cos^2 \psi \right)^n$$

Hence the applied load is

$$P = A(\sigma_1 + 2\sigma_2 \cos \psi)$$

$$\text{or } \frac{P}{AY} = \left(\frac{E\delta}{Yl} \right)^n (1 + 2 \cos^{2n+1} \psi)$$

which holds for $\epsilon_2 \geq Y/E$, or $E\delta/Yl \geq \sec^2 \psi$. When only the vertical bar is plastic, the stresses are

$$\sigma_1 = Y \left(\frac{E\delta}{Yl} \right)^n, \quad \sigma_2 = \frac{E\delta}{l} \cos^2 \psi$$

$$\text{Hence } \frac{P}{AY} = \left(\frac{E\delta}{Yl} \right)^n + 2 \left(\frac{E\delta}{Yl} \right) \cos^3 \psi$$

which holds for $\epsilon_1 \geq Y/E$ and $\epsilon_2 \leq Y/E$. Therefore, the restriction is $1 \leq E\delta/Yl \leq \sec^2 \psi$. The results corresponding to $\psi = \pi/4$ and $n = 0.25$ are

If plastic instability occurs simultaneously in the three bars, the true strain is n_1 in the central bar and n_2 in the inclined bars at instability. If O is displaced to O' at the point of necking, then from geometry,

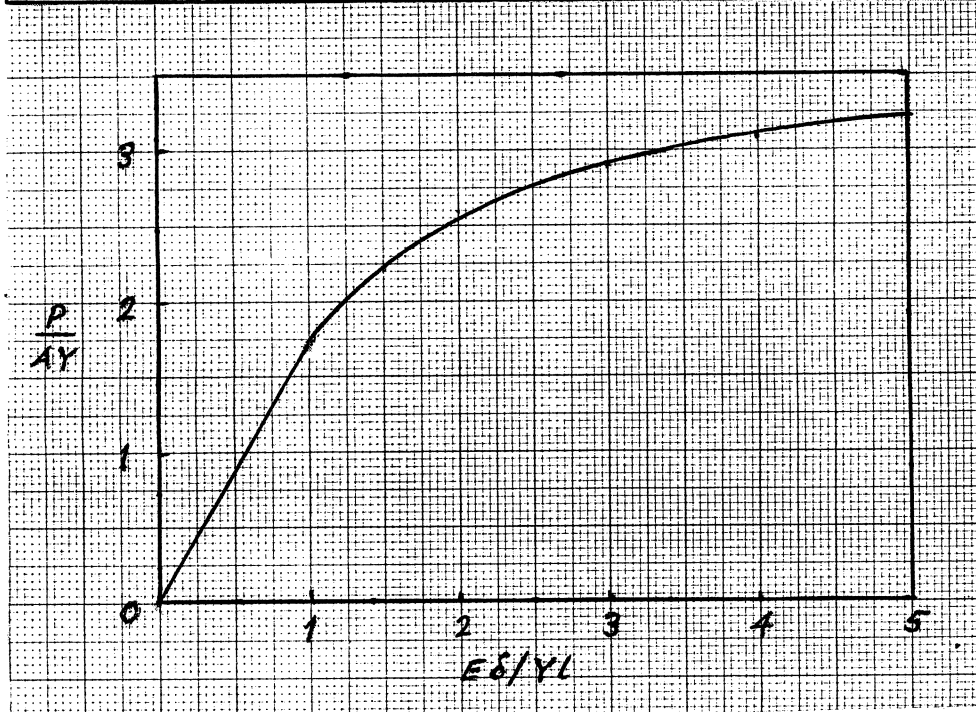
$$O'B = l e^{n_1},$$

$$O'A = O'C = l e^{n_2} \sec \psi.$$

From geometry,

$$\begin{aligned} l^2 \tan^2 \psi &= AB^2 - O'A^2 - O'B^2 \\ &= l^2 (e^{2n_2} \sec^2 \psi - e^{2n_1}) \end{aligned}$$

$E\delta/YL$	1.0	1.5	2.0	3.0	4.0	5.0
P/AY	1.707	2.167	2.603	2.881	3.096	3.276



1.9. Equating the total work corresponding to the stress-strain equations,

$$\int_0^{\epsilon_0} (Y + H\epsilon) d\epsilon = C \int_0^{\epsilon_0} \epsilon^n d\epsilon$$

$$\text{or } Y \epsilon_0 + \frac{1}{2} H \epsilon_0 = \frac{C}{1+n} \epsilon_0^{1+n} = \frac{\sigma_0 \epsilon_0}{1+n}$$

$$\text{or } 2Y + H \epsilon_0 = 2 \sigma_0 / (1+n)$$

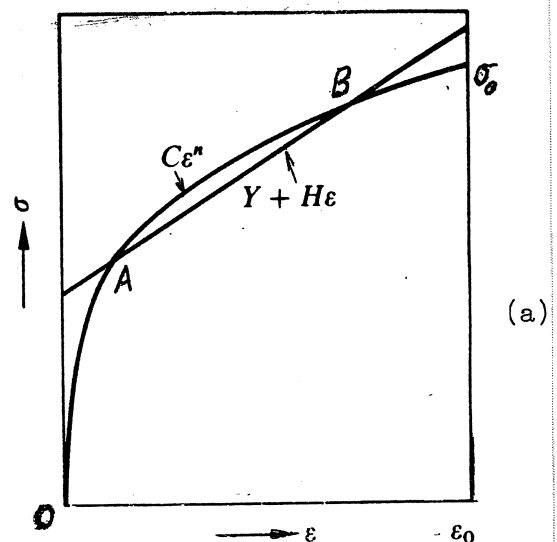
The second condition furnishes the relation

$$(Y + H\epsilon_0) - C \epsilon_0^n = 2 \left\{ C \left(\frac{\epsilon_0}{2} \right)^n - \left(Y + \frac{H\epsilon_0}{2} \right) \right\}$$

$$\text{or } 3Y + 2H \epsilon_0 = C \epsilon_0^n (1 + 2^{1-n}) = \sigma_0 (1 + 2^{1-n}) \quad (b)$$

Solving (a) and (b) for Y and H, we have

$$Y = \sigma_0 \left(\frac{3-n}{1+n} - 2^{1-n} \right), \quad H \epsilon_0 = 2 \sigma_0 \left(2^{1-n} - \frac{2-n}{1+n} \right).$$



The maximum percentage error in the linear approximation over the range AB corresponds to

$$\frac{d}{d\varepsilon} \left(1 - \frac{Y + H\varepsilon}{C\varepsilon^n} \right) = \frac{-H + (Y + H\varepsilon)(n/\varepsilon)}{C\varepsilon^n} = 0$$

or
$$\varepsilon = \left(\frac{n}{1-n} \right) \frac{Y}{H} .$$

Hence
$$Y + H\varepsilon = \frac{Y}{1-n} = \frac{\sigma_0}{1-n} \left(\frac{3-n}{1+n} - 2^{1-n} \right)$$

and
$$C\varepsilon^n = \sigma_0 \left(\frac{\varepsilon}{\varepsilon_0} \right)^n = \sigma_0 \left(\frac{n}{1-n} \right)^n \left(\frac{Y}{H \varepsilon_0} \right)^n$$

$$= \sigma_0 \left(\frac{n}{1-n} \right)^n \left(\frac{3-n}{1+n} - 2^{1-n} \right)^n \left\{ 2 \left(2^{1-n} - \frac{2-n}{1+n} \right) \right\}^n .$$

Setting $n = 0.3$, we get

$$Y + H\varepsilon = \frac{\sigma_0}{0.7} \left(\frac{2.7}{1.3} - 2^{0.7} \right) = \frac{0.4524}{0.7} \sigma_0 = 0.6463 \sigma_0$$

$$C\varepsilon^n = \sigma_0 \left(\frac{3}{7} \right)^{0.3} (0.4524)^{0.3} \left\{ 2 \left(2^{0.7} - \frac{1.7}{1.3} \right) \right\}^{0.3}$$

$$= \sigma_0 (0.3060)^{0.3} = 0.7010 \sigma_0$$

and the percentage error is $\left(1 - \frac{0.6463}{0.7010} \right) (100) = 7.8\% .$

1.10. Let r denote the current mean radius of the ring and t the current thickness. When the angular velocity is ω , the centrifugal force acting in an element of the ring is

$$F = \rho \omega^2 r (t r d\theta) = \rho \omega^2 r^2 t d\theta .$$

For radial equilibrium, $F = t \sigma d\theta$, or

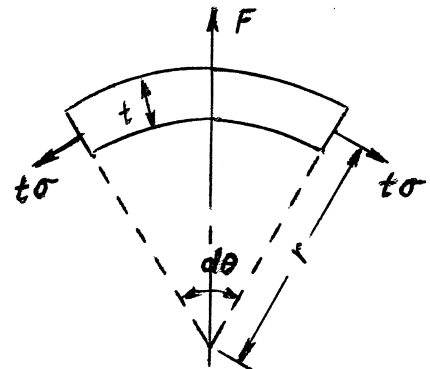
$$\sigma = \rho \omega^2 r^2 .$$

Hence
$$\frac{d\sigma}{\sigma} = 2 \left(\frac{d\omega}{\omega} + \frac{dr}{r} \right) .$$

Since $d\omega = 0$ at the onset of instability, we have

$$\frac{d\sigma}{\sigma} = 2 d\varepsilon \quad , \quad \text{or} \quad \frac{d\sigma}{d\varepsilon} = 2\sigma$$

where σ is the uniaxial stress acting in the ring circumferentially, and ε the corresponding strain. Since $d\sigma/d\varepsilon = n\sigma/\varepsilon$ by the given power law, $\varepsilon = n/2$



at instability. The maximum angular velocity is given by

$$\rho \omega^2 r_0^2 = \sigma (r_0/r)^2 = C \epsilon^n e^{-2\epsilon} = C \left(\frac{n}{2}\right)^n e^{-n}$$

1.11. In a thin spherical shell under an internal pressure p , the non-zero principal stresses are each equal to σ . Thus

$$\sigma_\theta = \sigma_\phi = \sigma = \frac{pr}{2t}$$

where r is the current mean radius and t the current thickness. The components of the strain increment are

$$d\epsilon_r = \frac{dt}{t} = -d\epsilon, \quad d\epsilon_\theta = d\epsilon_\phi = \frac{dr}{r} = \frac{1}{2} d\epsilon$$

Since $dp=0$ at the onset of instability,

$$\frac{d\sigma}{\sigma} = \frac{dp}{p} + \frac{dr}{r} - \frac{dt}{t} = \frac{3}{2} d\epsilon$$

and the instability condition becomes

$$\frac{d\sigma}{d\epsilon} = \frac{3}{2} \sigma$$

The empirical equation $\sigma = C\epsilon^n$ gives $n \sigma/\epsilon = 3 \sigma/2$, or $\epsilon = \frac{2}{3} n$ at instability, and the corresponding thickness and radius are

$$t = t_0 e^{-\epsilon} = t_0 \exp\left(-\frac{2}{3} n\right)$$

$$r = r_0 e^{-\epsilon/2} = r_0 \exp\left(\frac{1}{3} n\right)$$

Hence, the bursting pressure is given by

$$\frac{p}{C} = \frac{2\sigma t}{Cr} = \frac{2\sigma t_0}{Cr_0} e^{-n} = \frac{2t_0}{r_0} \left(\frac{2}{3} n\right)^n e^{-n}$$

1.12. Let the longitudinal tensile stresses existing in the inner and outer cylinders, having cross-sectional areas A_1 and A_2 respectively, be denoted by σ_1 and σ_2 . Then the resultant axial tension is

$$P = A_1 \sigma_1 + A_2 \sigma_2$$

$$\text{Hence } \frac{dP}{d\epsilon} = \left[A_1 \frac{d\sigma_1}{d\epsilon} + \sigma_1 \frac{dA_1}{d\epsilon} \right] + \left[A_2 \frac{d\sigma_2}{d\epsilon} + \sigma_2 \frac{dA_2}{d\epsilon} \right]$$

where ϵ is the longitudinal strain in the composite bar of length l . Since $A_1 l$ and $A_2 l$ are constants by the constancy of volume,

$$\frac{dA_1}{A_1} = \frac{dA_2}{A_2} = - \frac{d\ell}{\ell} = - d\varepsilon$$

and A_2/A_1 remains constant during the deformation. At the load maximum, $dP = 0$, giving

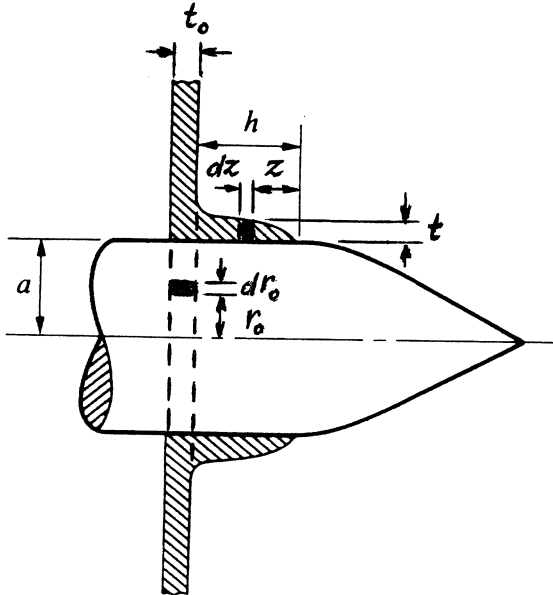
$$\begin{aligned} 0 &= A_1 \left(\frac{d\sigma_1}{d\varepsilon} - \sigma_1 \right) + A_2 \left(\frac{d\sigma_2}{d\varepsilon} - \sigma_2 \right) \\ &= A_1 \sigma_1 \left(\frac{n_1}{\varepsilon} - 1 \right) + A_2 \sigma_2 \left(\frac{n_2}{\varepsilon} - 1 \right) \end{aligned}$$

$$\text{or } \varepsilon = \frac{A_1 \sigma_1 n_1 + A_2 \sigma_2 n_2}{A_1 \sigma_1 + A_2 \sigma_2} = \frac{n_1 + n_2}{2}$$

if $A_1 \sigma_1 = A_2 \sigma_2$ at instability. Then

$$\frac{A_2}{A_1} = \frac{\sigma_1}{\sigma_2} = \frac{C_1}{C_2} \varepsilon^{n_1 - n_2} = \frac{C_1}{C_2} \left(\frac{n_1 + n_2}{2} \right)^{n_1 - n_2}$$

1.13.



Let r_0 be initial radius to an element that is currently at a distance z from the outer edge of the lip. In view of the incompressibility of the material,

$$2\pi r_0 t_0 dr_0 = 2\pi a t dz$$

$$\text{or } dz/dr_0 = r_0 t_0 / at$$

Since the state of stress is uniaxial, the thickness strain is one half in magnitude of the hoop strain. Hence

$$\ln \left(\frac{t}{t_0} \right) = - \frac{1}{2} \ln \left(\frac{a}{r_0} \right), \text{ or } \frac{t}{t_0} = \sqrt{\frac{r_0}{a}}$$

which gives

$$\frac{dz}{dr_0} = \frac{r_0}{a} \sqrt{\frac{a}{r_0}} = \sqrt{\frac{r_0}{a}}$$

$$\text{or } \frac{z}{a} = a^{-3/2} \int_0^{r_0} \sqrt{r_0} dr_0 = \frac{2}{3} \left(\frac{r_0}{a} \right)^{3/2}$$

Since $z = h$ when $r_0 = a$, we get $h = \frac{2}{3} a$. Also, the thickness variation is given by

$$\frac{t}{t_0} = \left(\frac{r_0}{a} \right)^{1/2} = \left(\frac{3z}{2a} \right)^{1/3}$$

The plastic work per unit volume of a typical element is

$$E = \int \sigma d\varepsilon = C \int_0^{\ln(a/r_0)} \varepsilon^n d\varepsilon = \frac{C}{1+n} \left(\ln \frac{a}{r_0} \right)^{1+n}$$

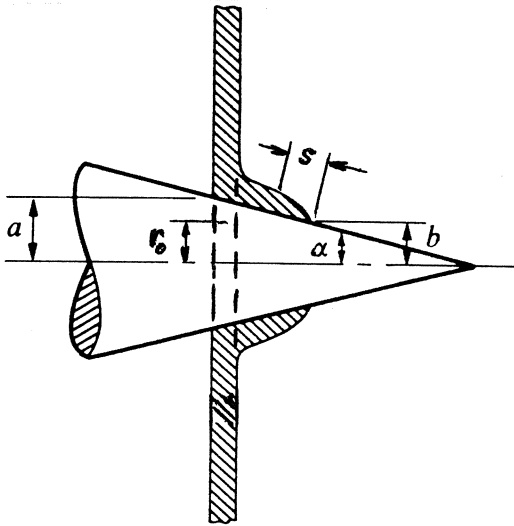
The total work done during the perforation is

$$\begin{aligned}
 W &= 2\pi a \int_0^h E t dz = \frac{2\pi C t_0}{1+n} \int_0^a \left(\ln \frac{a}{r_0} \right)^{1+n} r_0 dr_0 \\
 &= \frac{\pi C t_0 a^2}{(1+n)2^{1+n}} \int_0^\infty e^{-x} x^{1+n} dx \left(x = 2 \ln \frac{a}{r_0} \right) \\
 \text{or } W &= \frac{\pi C t_0 a^2}{(1+n)2^{1+n}} \Gamma(2+n) = \frac{\pi C t_0 a^2}{2^{1+n}} \Gamma(1+n) .
 \end{aligned}$$

When $n = 0.5$, we have

$$W = \frac{\pi C t_0 a^2}{2^{5/2}} \Gamma\left(\frac{1}{2}\right) = \frac{\pi \sqrt{\pi} C t_0 a^2}{4 \sqrt{2}} \approx 0.984 a^2 t_0 C .$$

1.14.



Let s denote the inclined distance of a typical element that was initially at a radial distance r_0 from the axis of symmetry. The incompressibility of the material requires

$$2\pi r_0 t_0 dr_0 = 2\pi (b + s \sin \alpha) t ds .$$

Since the state of stress is uniaxial,

$$t_0 \sqrt{r_0} = t \sqrt{b + s \sin \alpha} .$$

Hence $\sqrt{b + s \sin \alpha} (ds/dr_0) = r_0 \sqrt{r_0}$.

The integration of this equation gives

$$(b + s \sin \alpha)^{3/2} - b^{3/2} = r_0^{3/2} \sin \alpha .$$

Since $b + s \sin \alpha = a$ when $r_0 = a$, we have

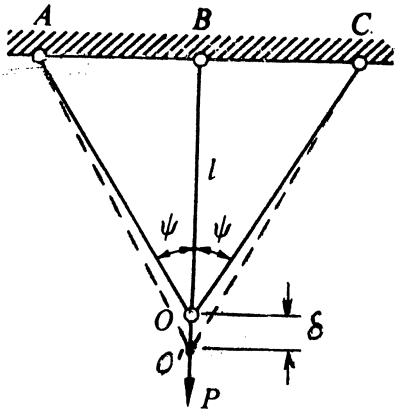
$$a^{3/2} - b^{3/2} = a^{3/2} \sin \alpha, \quad \text{or} \quad b = a(1 - \sin \alpha)^{2/3} .$$

$$\text{Also, } \left(1 + \frac{s}{b} \sin \alpha \right)^{3/2} = 1 + \left(\frac{r_0}{b} \right)^{3/2} \sin \alpha .$$

The thickness variation of the perforated plate is therefore given by

$$\frac{t}{t_0} = \sqrt{\frac{r_0}{b}} \left(1 + \frac{s}{b} \sin \alpha \right)^{-1/2} = \sqrt{\frac{r_0}{b}} \left\{ 1 + \left(\frac{r_0}{b} \right)^{3/2} \sin \alpha \right\}^{-1/3} .$$

1.15.



In the purely elastic range, the stresses are

$$\sigma_1 = E \epsilon_1 = \frac{E \delta}{l}$$

$$\sigma_2 = E \epsilon_2 = \frac{E \delta}{l} \cos^2 \psi = \sigma_1 \cos^2 \psi$$

and the applied load is

$$P = A(\sigma_1 + 2 \sigma_2 \cos \psi) = A \sigma_1 (1 + 2 \cos^3 \psi).$$

Hence the elastic stresses become

$$\sigma_1 = \frac{P/A}{1 + 2 \cos^3 \psi}, \quad \sigma_2 = \frac{(P/A) \cos^2 \psi}{1 + 2 \cos^3 \psi}.$$

Subtracting these stresses from the fully plastic stresses of Prob. 1.8, we obtain the residual stresses. Thus

$$\frac{\sigma_1'}{Y} = \left(\frac{E \delta}{Yl} \right)^n - \frac{P/AY}{1 + 2 \cos^3 \psi} = \left(\frac{E \delta}{Yl} \right)^n \left\{ 1 - \frac{1 + 2 \cos^{2n+1} \psi}{1 + 2 \cos^3 \psi} \right\}$$

$$\text{or } \frac{\sigma_1'}{2Y} \sec \psi = - \left(\frac{E \delta}{Yl} \right)^n \left(\frac{\cos^{2n} \psi - \cos^2 \psi}{1 + 2 \cos^3 \psi} \right).$$

Since $\sigma_1 + 2 \sigma_2' \cos \psi = 0$ for the unloaded structure,

$$\sigma_2' / Y = -(\sigma_1 / 2Y) \sec \psi.$$

The residual deflection δ' of point O is obtained by subtracting $(\sigma_1/E)l$ from the plastic deflection δ . Thus

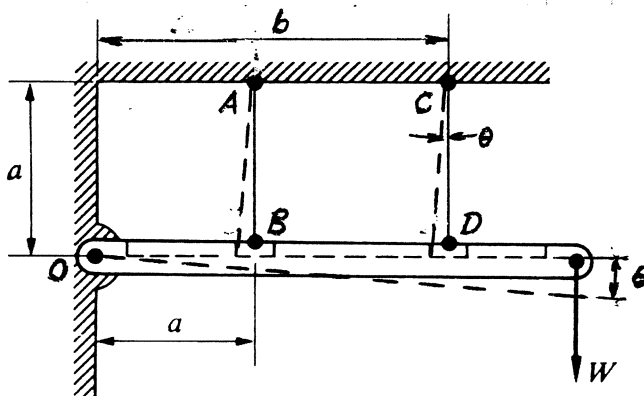
$$\frac{E \delta'}{Yl} = \frac{E \delta}{Yl} - \left(\frac{E \delta}{Yl} \right)^n \left(\frac{1 + 2 \cos^{2n+1} \psi}{1 + 2 \cos^3 \psi} \right).$$

When $n = 0.25$, $\psi = \pi/4$ and $E\delta/Yl = 3$, we get

$$\frac{\sigma_1'}{Y} = -\sqrt{2} (3)^{0.25} \left\{ \frac{(0.5)^{0.25} - 0.5}{1 + 0.707} \right\} = -0.372.$$

$$\frac{E \delta'}{Yl} = 3 - (3)^{0.25} \left\{ \frac{1 + \sqrt{2} (0.5)^{0.25}}{1 + 0.707} \right\} = 1.312.$$

1.16



At the onset of instability, the horizontal rigid bar is inclined at an angle θ . Since the groove is smooth, the stretched wires AB' and CD' must remain perpendicular to the center line of the bar. From geometry, we have

$$AB' = a(\cos \theta + \sin \theta)$$

$$CD' = a \cos \theta + b \sin \theta.$$