Chapter 2: The Logic of Compound Statements

The ability to reason using the principles of logic is essential for solving problems in abstract mathematics and computer science and for understanding the reasoning used in mathematical proof and disproof. Because a significant number of students who come to college have had limited opportunity to develop this ability, a primary aim of Chapters 2 and 3 is to help students develop an inner voice that speaks with logical precision. Consequently, the various rules used in logical reasoning are developed both symbolically and in the context of their somewhat limited but very important use in everyday language. Exercise sets for Sections 2.1–2.3 and 3.1–3.4 contain sentences for students to negate, write the contrapositive for, and so forth. Virtually all students benefit from doing these exercises. Another aim of Chapters 2 and 3 is to teach students the rudiments of symbolic logic as a foundation for a variety of upper-division courses. Symbolic logic is used in, among others, the study of digital logic circuits, relational databases, artificial intelligence, and program verification.

Suggestions

1. In Section 2.1 a surprising number of students apply De Morgan's law to write the negation of, for example, "$1 < x \leq 3$" as "$1 \geq x > 3$." You may find that it takes some effort to teach them to avoid making this mistake.

2. In Sections 2.1 and 2.4, students have more difficulty than you might expect simplifying statement forms and circuits. Only through trial and error can you learn the extent to which this is the case at your institution. If it is, you might either assign only the easier exercises or build in extra time to teach students how to do the more complicated ones. Discussion of simplification techniques occurs again in Chapter 6 in the context of set theory. At this later point in the course most students are able to deal with it successfully.

3. In ordinary English, the phrase "only if" is often used as a synonym for "if and only if." But it is possible to find informal sentences for which the intuitive interpretation is the same as the logical definition. It is helpful to give examples of such statements when you introduce the logical definition. For instance, it is not hard to get students to agree that "The team will win the championship only if it wins the semifinal game" means the same as "If the team does not win the semifinal game then it will not win the championship." Once students see this, you can suggest that they remember this example when they encounter more abstract "only if"statements.

Through guided discussion, students also come to agree that the statement "Winning the semifinal game is a necessary condition for winning the championship" translates to "If the team does not win the semifinal game then it will not win the championship." They can be encouraged to use this (or a similar statement) as a reference to help develop intuition for general statements of the form "A is a necessary condition for B."

With students who have weaker backgrounds, you may find yourself tying up excessive amounts of class time discussing "only if" and "necessary and sufficient conditions." You might just assign the easier exercises, or you might assign exercises on these topics to be done for extra credit (putting corresponding extra credit problems on exams) and use the results to help distinguish A from B students. It is probably best not to omit these topics altogether, though, because the language of "only if" and "necessary and sufficient conditions" is a standard part of the technical vocabulary of textbooks used in upper-division courses, as well as occurring regularly in non-mathematical writing.

4. In Section 2.3, many students mistakenly conclude that an argument is valid if, when they compute the truth table, they find a single row in which both the premises and the conclusion are true. The source of students' difficulty appears to be their tendency to ignore quantification and to misinterpret if-then statements as "and" statements. Since the definition of validity includes both a universal quantifier and if-then, it is helpful to go back over the definition and the procedures for testing for validity and invalidity after discussing the general topic of universal conditional statements in Section 3.1. As a practical measure to help students assess validity and invalidity correctly, the first example in Section 2.3 is of an invalid argument whose truth table has eight rows, several of which have true premises and a true conclusion. To further focus students' attention on the situations where all the premises are true, the truth values for the conclusions of arguments are omitted when at least one premise is false.

5. In Section 2.3, you might suggest that students just familiarize themselves with, but not memorize, the various forms of valid arguments covered in Section 2.3. It is wise, however, to have them learn the terms modus ponens and modus tollens (because these are used in some upper-division computer science courses) and converse and inverse errors (because these errors are so common).

Section 2.1

1. Common form: If p then q.
   p
   Therefore, q

   $(a + 2b)(a^2 - b)$ can be written in prefix notation. All algebraic expressions can be written in prefix notation.

2. Common form: If p then q.
   $\sim q$
   Therefore, $\sim p$

   All prime numbers are odd. 2 is odd

3. Common form: $p \vee q$
   $\sim p$
   Therefore, q

   My mind is shot. Logic is confusing.

4. Common form: If p then q.
   If q then r.
   Therefore, If p then r.

   Has 4 vertices and 6 edges. Is complete; Any two of its vertices can be joined by a path

5. *a.* It is a statement because it is a true sentence. 1,024 is a perfect square because $1{,}024 = 32^2$, and the next smaller perfect square is $31^2 = 961$, which has fewer than four digits.

   b. The truth or falsity of this sentence depends on the reference for the pronoun "she." Considered on its own, the sentence cannot be said to be either true or false, and so it is not a statement.

   c. This sentence is false; hence it is a statement.

   d. This is not a statement because its truth or falsity depends on the value of x.

6. *a.* $s \wedge i$ *b.* $\sim s \wedge \sim i$

7. $m \wedge \sim c$

8. *a.* $(h \wedge w) \wedge \sim s$ *b.* $\sim w \wedge (h \wedge s)$ *c.* $\sim w \wedge \sim h \wedge \sim s$
   d. $(\sim w \wedge \sim s) \wedge h$ *e.* $w \wedge \sim (h \wedge s)$ ($w \wedge (\sim h \vee \sim s)$ is also acceptable)

9. *a.* $p \vee q$ **b.** $r \wedge p$ **c.** $r \wedge (p \vee q)$

10. *a.* $p \wedge q \wedge r$ *b.* $p \wedge \sim q$ **c.** $p \wedge (\sim q \vee \sim r)$ **d.** $(\sim p \wedge q) \wedge \sim r$ **e.** $\sim p \vee (q \wedge r)$

11. Inclusive or. For instance, a team could win the playoff by winning games 1, 3, and 4 and losing game 2. Such an outcome would satisfy both conditions.

12.

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

13.

p	q	$p \wedge q$	$p \vee q$	$\sim (p \wedge q)$	$\sim (p \wedge q) \vee (p \vee q)$
T	T	T	T	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	F	T	T

14.

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

15.

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

16.

p	q	$p \wedge q$	$p \vee (p \wedge q)$	p
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	F	F	F

same truth values

The truth table shows that $p \vee (p \wedge q)$ and p always have the same truth values. Thus they are logically equivalent. (This proves one of the absorption laws.)

17.

p	q	$p \wedge q$	$\sim p$	$\sim q$	$\sim (p \wedge q)$	$\sim p \wedge \sim q$	
T	T	T	F	F	F	F	
T	F	F	F	T	T	F	$\leftarrow$
F	T	F	T	F	T	F	$\leftarrow$
F	F	F	T	T	T	T	

different truth values in rows 2 and 3

The truth table shows that $\sim (p \wedge q)$ and $\sim p \wedge \sim q$ do not always have the same truth values. Therefore they are not logically equivalent.

18.

p	$\mathbf{t}$	$p \vee \mathbf{t}$
T	T	T
F	T	T

same truth values

The truth table shows that $p \vee \mathbf{t}$ and $\mathbf{t}$ always have the same truth values. Thus they are logically equivalent. (This proves one of the universal bound laws.)

19.

p	$\mathbf{t}$	$p \wedge \mathbf{t}$	p
T	T	T	T
F	T	F	F

same truth values

The truth table shows that $p \wedge\ \mathbf{t}$ and p always have the same truth values. Thus they are logically equivalent. This proves the identity law for $\wedge$.

20.

p	$\mathbf{c}$	$p \wedge \mathbf{c}$	$p \vee \mathbf{c}$	
T	F	F	T	$\leftarrow$
F	F	F	F	

different truth values in row 1

The truth table shows that $p \wedge\ \mathbf{c}$ and $p \vee \mathbf{c}$ do not always have the same truth values. Thus they are not logically equivalent.

21.

p	q	$p \quad q \quad r$	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ always have the same truth values. Thus they are logically equivalent. (This proves the associative law for $\wedge$.)

22.

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

same truth values

The truth table shows that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ always have the same truth values. Therefore they are logically equivalent. This proves the distributive law for $\wedge$ over $\vee$.

23.

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \vee r$	$p \wedge (q \vee r)$	
T	T	T	T	T	T	T	
T	T	F	T	T	T	T	
T	F	T	F	T	T	T	
T	F	F	F	F	F	F	
F	T	T	F	T	T	F	$\leftarrow$
F	T	F	F	T	F	F	
F	F	T	F	T	T	F	$\leftarrow$
F	F	F	F	F	F	F	

different truth values in rows 5 and 7

The truth table shows that $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ have different truth values in rows 5 and 7. Thus they are not logically equivalent. (This proves that parentheses are needed with $\wedge$ and $\vee$.)

24.

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$	
T	T	T	T	T	T	T	
T	T	F	T	F	T	F	$\leftarrow$
T	F	T	T	T	T	T	$\leftarrow$
T	F	F	T	F	T	F	
F	T	T	T	F	T	T	
F	T	F	T	F	T	F	$\leftarrow$
F	F	T	F	F	F	F	
F	F	F	F	F	F	F	

different truth values in rows 2, 3, and 6

The truth table shows that $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$ have different truth values in rows 2, 3, and 6. Hence they are not logically equivalent.

25. Hal is not a math major or Hal's sister is not a computer science major.

26. Sam is not an orange belt or Kate is not a red belt.

27. The connector is not loose and the machine is not unplugged.

28. The train is not late and my watch is not fast.

29. This computer program does not have a logical error in the first ten lines and it is not being run with an incomplete data set.

30. The dollar is not at an all-time high or the stock market is not at a record low.

31. **a.** 01, 02, 11, 12 **b.** 21, 22 **c.** 11, 10, 21, 20

32. $-2 \geq x$ or $x \geq 7$

33. $-10 \geq x$ or $x \geq 2$

34. $2 \leq x \leq 5$

35. $x > -1$ and $x \leq 1$

36. $1 \leq x$ or $x < -3$

37. $0 \leq x$ or $x < -7$

38. This statement's logical form is $(p \wedge q) \vee r$, so its negation has the form $\sim((p \wedge q) \vee r) \equiv \sim(p \wedge q) \wedge \sim r \equiv (\sim p \vee \sim q) \wedge \sim r$. Thus a negation for the statement is ($num\_orders \leq 100$ or $num\_instock > 500$) and $num\_instock \geq 200$.

39. The statement's logical form is $(p \wedge q) \vee ((r \wedge s) \wedge t)$, so its negation has the form

$$\begin{aligned} \sim ((p \wedge q) \vee ((r \wedge s) \wedge t)) &\equiv \sim (p \wedge q) \wedge \sim ((r \wedge s) \wedge t)) \\ &\equiv (\sim p \vee \sim q) \wedge (\sim (r \wedge s) \vee \sim t)) \\ &\equiv (\sim p \vee \sim q) \wedge ((\sim r \vee \sim s) \vee \sim t)). \end{aligned}$$

Thus a negation is ($num\_orders \geq 50$ or $num\_instock \leq 300$) and (($50 > num\_orders$ or $num\_orders \geq 75$) or $num\_instock \leq 500$).

40.

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T

Since all the truth values of $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ are T, $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.

41.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q)(p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Since all the truth values of $(p \wedge \sim q) \wedge (\sim p \vee q)$ are F, $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

42.

p	q	r	$\sim p$	$\sim q$	$\sim p \wedge q$	$q \wedge r$	$((\sim p \wedge q) \wedge (q \wedge r))$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	F	F	F

all $F's$

Since all the truth values of $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ are F, $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ is a contradiction.

43.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

all $T's$

44. **a.** No real numbers satisfy this inequality

b. No real numbers satisfy this inequality.

45. Let b be "Bob is a double math and computer science major," m be "Ann is a math major," and a be "Ann is a double math and computer science major." Then the two statements can be symbolized as follows: a. $(b \wedge m) \wedge \sim a$ and **b.** $\sim (b \wedge a) \wedge (m \wedge b)$. *Note*: The entries in the truth table assume that a person who is a double math and computer science major is also a math major and a computer science major.

b	m	a	$\sim a$	$b \wedge m$	$m \wedge b$	$b \wedge a$	$\sim (b \wedge a)$	$(b \wedge m) \wedge \sim a$	$\sim (b \wedge a) \wedge (m \wedge b)$
T	T	T	F	T	T	T	F	F	F
T	T	F	T	F	T	F	T	T	T
T	F	T	F	T	F	T	F	F	F
T	F	F	T	F	F	F	T	F	F
F	T	T	F	F	F	F	T	F	F
F	T	F	T	F	F	F	T	F	F
F	F	T	F	F	F	F	T	F	F
F	F	F	T	F	F	F	T	F	F

same truth values

The truth table shows that $(b \wedge m) \wedge \sim a$ and $\sim (b \wedge a) \wedge (m \wedge b)$ always have the same truth values. Hence they are logically equivalent.

46. **a.** *Solution 1:* Construct a truth table for $p \oplus p$ using the truth values for *exclusive or.*

p	$p \oplus p$
T	F
F	F

because an *exclusive or* statement is false when both components are true and when both components are false, and the two components in $p \oplus p$ are both p.

Since all its truth values are false, $p \oplus p \equiv \mathbf{c}$, a contradiction.

Solution 2: Replace q by p in the logical equivalence $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$, and simplify the result.

$$\begin{aligned} p \oplus p &\equiv (p \vee p) \wedge \sim(p \wedge p) && \text{by definition of } \oplus \\ &\equiv p \wedge \sim p && \text{by the identity laws} \\ &\equiv \mathbf{c} && \text{by the negation law for } \wedge \end{aligned}$$

b. Yes.

p	q	r	$p \oplus q$	$q \oplus r$	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	F	T	T
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \oplus r$ and $p \oplus (q \oplus r)$ always have the same truth values. So they are logically equivalent.

c. Yes.

p	q	r	$p \oplus q$	$p \wedge r$	$q \wedge r$	$(p \oplus q) \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	T	T	F	F
T	T	F	F	F	F	F	F
T	F	T	T	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

same truth values

The truth table shows that $(p \oplus q) \wedge r$ and $(p \wedge r) \oplus (q \wedge r)$ always have the same truth values. So they are logically equivalent.

47. There is a famous story about a philosopher who once gave a talk in which he observed that whereas in English and many other languages a double negative is equivalent to a positive, there is no language in which a double positive is equivalent to a negative. To this, another philosopher, Sidney Morgenbesser, responded sarcastically, "Yeah, yeah."

[Strictly speaking, sarcasm functions like negation. When spoken sarcastically, the words "Yeah, yeah" are not a true double positive; they just mean "no."]

48. **a.** the distributive law **b.** the commutative law for $\vee$
c. the negation law for $\vee$ **d.** the identity law for $\wedge$

49. **a.** the commutative law for $\vee$ **b.** the distributive law
c. the negation law for $\wedge$ **d.** the identity law for $\vee$

50.
$$\begin{aligned} (p \wedge \sim q) \vee p &\equiv p \vee (p \wedge \sim q) && \text{by the commutative law for } \vee \\ &\equiv p && \text{by the absorption law (with } \sim q \text{ in place of } q) \end{aligned}$$

51. *Solution 1*:
$$\begin{aligned} p \wedge (\sim q \vee p) &\equiv p \wedge (p \vee \sim q) && \text{commutative law for } \vee \\ &\equiv p && \text{absorption law} \end{aligned}$$

Solution 2: $p \wedge (\sim q \vee p) \equiv (p \wedge \sim q) \vee (p \wedge p)$ distributive law
$\equiv (p \wedge \sim q) \vee p$ identity law for $\wedge$
$\equiv p$ by exercise 50.

52. $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv (\sim p \wedge \sim (\sim q)) \vee (\sim p \wedge \sim q)$ De Morgan's law
$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ double negative law
$\equiv \sim p \wedge (q \vee \sim q)$ distributive law
$\equiv \sim p \wedge \mathbf{t}$ negation law for $\vee$
$\equiv \sim p$ identity law for $\wedge$

53. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv \sim[\sim p \wedge (q \vee \sim q)] \vee (p \wedge q)$ by the distributive law
$\equiv \sim(\sim p \wedge \mathbf{t}) \vee (p \wedge q)$ by the negation law for $\vee$
$\equiv \sim(\sim p) \vee (p \wedge q)$ by the identity law for $\wedge$
$\equiv p \vee (p \wedge q)$ by the double negative law
$\equiv p$ by the absorption law

54. $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv (p \wedge (\sim (\sim p) \wedge \sim q)) \vee (p \wedge q)$ De Morgan's law
$\equiv (p \wedge (p \wedge \sim q)) \vee (p \wedge q)$ double negative law
$\equiv ((p \wedge p) \wedge \sim q)) \vee (p \wedge q)$ associative law for $\wedge$
$\equiv (p \wedge \sim q)) \vee (p \wedge q)$ idempotent law for $\wedge$
$\equiv p \wedge (\sim q \vee q)$ distributive law
$\equiv p \wedge (q \vee \sim q)$ commutative law for $\vee$
$\equiv p \wedge \mathbf{t}$ negation law for $\vee$
$\equiv p$ identity law for $\wedge$

Section 2.2

1. If this loop does not contain a **stop** or a **go to**, then it will repeat exactly N times.
2. If I catch the 8:05 bus, then I am on time for work.
3. If you do not freeze, then I'll shoot.
4. If you don't fix my ceiling, then I won't pay my rent.
5.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim p \vee q \to \sim q$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

6.

p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \vee (\sim p \wedge q)$	$(p \vee q) \vee (\sim p \wedge q) \to q$
T	T	F	F	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	F	F	F	T

7.

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge \sim q \to r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

8.

p	q	r	$\sim p$	$\sim p \vee q$	$\sim p \vee q \to r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	F

9.

p	q	r	$\sim r$	$p \wedge \sim r$	$q \wedge r$	$p \wedge \sim r \leftrightarrow q \vee r$
T	T	T	F	F	T	F
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	T	F	T	F	T	F
F	F	T	F	F	T	F
F	F	F	T	F	F	T

10.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

11.

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$p \wedge q \to r$	$(p \to (q \to r)) \leftrightarrow (p \wedge q \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

12. If $x > 2$ then $x^2 > 4$, and if $x < -2$ then $x^2 > 4$.

13. **a.**

p	q	$\sim p$	$p \to q$	$\sim p \wedge q$
T	T	F	T	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

same truth values

The truth table shows that $p \to q$ and $\sim p \vee q$ always have the same truth values. Hence they are logically equivalent.

b.

p	q	$\sim q$	$p \to q$	$\sim (p \to q)$	$p \wedge \sim q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

same truth values

The truth table shows that $\sim (p \to q)$ and $p \wedge \sim q$ always have the same truth values. Hence they are logically equivalent.

14. **a.**

p	q	r	$\sim q$	$\sim r$	$q \vee r$	$p \wedge \sim q$	$p \wedge \sim r$	$p \to q \vee r$	$p \wedge \sim q \to r$	$p \wedge \sim r \to q$
T	T	T	F	F	T	F	F	T	T	T
T	T	F	F	T	T	F	T	T	T	T
T	F	T	T	F	T	T	F	T	T	T
T	F	F	T	T	F	T	T	F	F	F
F	T	T	F	F	T	F	F	T	T	T
F	T	F	F	T	T	F	F	T	T	T
F	F	T	T	F	T	F	F	T	T	T
F	F	F	T	T	F	F	F	T	T	T

same truth values

The truth table shows that the three statement forms $p \to q \vee r$, $p \wedge \sim q \to r$, and $p \wedge \sim r \to q$ always have the same truth values. Thus they are all logically equivalent.

b. If n is prime and n is not odd, then n is 2.
And: If n is prime and n is not 2, then n is odd.

15.

p	q	r	$q \to r$	$p \to q$	$p \to (q \to r)$	$(p \to q) \to r$	
T	T	T	T	T	T	T	
T	T	F	F	T	F	F	
T	F	T	T	F	T	T	
T	F	F	T	F	T	T	
F	T	T	T	T	T	T	
F	T	F	F	T	T	F	$\leftarrow$
F	F	T	T	T	T	F	$\leftarrow$
F	F	F	T	T	T	F	$\leftarrow$

different truth values

The truth table shows that $p \to (q \to r)$ and $(p \to q) \to r$ do not always have the same truth values. (They differ for the combinations of truth values for p, q, and r shown in rows 6, 7, and 8.) Therefore they are not logically equivalent.

16. Let p represent "You paid full price" and q represent "You didn't buy it at Crown Books." Thus, "If you paid full price, you didn't buy it at Crown Books" has the form $p \to q$. And "You didn't buy it at Crown Books or you paid full price" has the form $q \vee p$.

p	q	$p \to q$	$q \vee p$	
T	T	T	T	
T	F	F	T	←
F	T	T	T	←
F	F	T	F	

different truth values

These two statements are not logically equivalent because their forms have different truth values in rows 2 and 4.
(An alternative representation for the forms of the two statements is $p \to \sim q$ and $\sim q \vee p$. In this case, the truth values differ in rows 1 and 3.)

17. Let p represent "2 is a factor of n," q represent "3 is a factor of n," and r represent "6 is a factor of n." The statement "If 2 is a factor of n and 3 is a factor of n, then 6 is a factor of n" has the form $p \wedge q \to r$. And the statement "2 is not a factor of n or 3 is a not a factor of n or 6 is a factor of n" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge q \to r$	$\sim p \vee \sim q \vee r$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	F	F
T	F	T	F	F	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $\sim p \vee \sim q \vee r$ always have the same truth values. Therefore they are logically equivalent.

18. *Part 1*: Let p represent "It walks like a duck," q represent "It talks like a duck," and r represent "It is a duck." The statement "If it walks like a duck and it talks like a duck, then it is a duck" has the form $p \wedge q \to r$. And the statement "Either it does not walk like a duck or it does not talk like a duck or it is a duck" has the form $\sim p \vee \sim q \vee r$.

p	q	r	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \vee \sim q$	$p \wedge q \to r$	$(\sim p \vee \sim q) \vee r$
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	F	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

same truth values

The truth table shows that $p \wedge q \to r$ and $(\sim p \vee \sim q) \vee r$ always have the same truth values. Thus the following statements are logically equivalent: "If it walks like a duck and it talks like a duck, then it is a duck" and "Either it does not walk like a duck or it does not talk like a duck or it is a duck."

Part 2: The statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck" has the form $\sim p \wedge \sim q \rightarrow \sim r$.

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \wedge q$	$\sim p \wedge \sim q$	$p \wedge q \rightarrow r$	$(\sim p \wedge \sim q) \rightarrow \sim r$	
T	T	T	F	F	F	T	F	T	T	
T	T	F	F	F	T	T	F	F	T	←
T	F	T	F	T	F	F	F	T	T	
T	F	F	F	T	T	F	F	T	T	
F	T	T	T	F	F	F	F	T	T	
F	T	F	T	F	T	F	F	T	T	
F	F	T	T	T	F	F	T	T	F	←
F	F	F	T	T	T	F	T	T	T	

different truth values

The truth table shows that $p \wedge q \rightarrow r$ and $(\sim p \wedge \sim q) \rightarrow \sim r$ do not always have the same truth values. (They differ for the combinations of truth values of p, q, and r shown in rows 2 and 7.) Thus they are not logically equivalent, and so the statement "If it walks like a duck and it talks like a duck, then it is a duck" is not logically equivalent to the statement "If it does not walk like a duck and it does not talk like a duck then it is not a duck." In addition, because of the logical equivalence shown in Part 1, we can also conclude that the following two statements are not logically equivalent: "Either it does not walk like a duck or it does not talk like a duck or it is a duck" and "If it does not walk like a duck and it does not talk like a duck then it is not a duck."

19. False. The negation of an if-then statement is not an if-then statement. It is an *and* statement.

20. **a.** *Negation*: P is a square and P is not a rectangle.

b. *Negation*: Today is New Year's Eve and tomorrow is not January.

c. *Negation*: The decimal expansion of r is terminating and r is not rational.

d. *Negation*: n is prime and both n is not odd and n is not 2. Or: n is prime and n is neither odd nor 2.

e. *Negation*: x is nonnegative and x is not positive and x is not 0.
Or: x is nonnegative but x is not positive and x is not 0.
Or: x is nonnegative and x is neither positive nor 0.

f. *Negation*: Tom is Ann's father and either Jim is not her uncle or Sue is not her aunt.

g. *Negation*: n is divisible by 6 and either n is not divisible by 2 or n is not divisible by 3.

21. By assumption, $p \rightarrow q$ is false. By definition of a conditional statement, the only way this can happen is for the hypothesis, p, to be true and the conclusion, q, to be false.

a. The only way $\sim p \rightarrow q$ can be false is for $\sim p$ to be true and q to be false. But since p is true, $\sim p$ is false. Hence $\sim p \rightarrow q$ is not false and so it is true.

b. Since p is true, then $p \vee q$ is true because if one component of an *and* statement is true, then the statement as a whole is true.

c The only way $q \rightarrow p$ can be false is for q to be true and p to be false. Thus, since q is false, $q \rightarrow p$ is not false and so it is true.

22. **a.** *Contrapositive*: If P is not a rectangle, then P is not a square.

b. *Contrapositive*: If tomorrow is not January, then today is not New Year's Eve.

c. *Contrapositive*: If r is not rational, then the decimal expansion of r is not terminating.

d. *Contrapositive*: If n is not odd and n is not 2, then n is not prime.

e. *Contrapositive*: If x is not positive and x is not 0, then x is not nonnegative.
Or: If x is neither positive nor 0, then x is negative.

f. *Contrapositive*: If either Jim is not Ann's uncle or Sue is not her aunt, then Tom is not her father.

g. *Contrapositive*: If n is not divisible by 2 or n is not divisible by 3, then n is not divisible by 6.

23. **a.** *Converse*: If P is a rectangle, then P is a square.
Inverse: If P is not a square, then P is not a rectangle.

b. *Converse*: If tomorrow is January, then today is New Year's Eve.
Inverse: If today is not New Year's Eve, then tomorrow is not January.

c. *Converse*: If r is rational then the decimal expansion of r is terminating.
Inverse: If the decimal expansion of r is not terminating, then r is not rational.

d. *Converse*: If n is odd or n is 2, then n is prime.
Inverse: If n is not prime, then n is not odd and n is not 2.

e. *Converse*: If x is positive or x is 0, then x is nonnegative.
Inverse: If x is not nonnegative, then both x is not positive and x is not 0.
Or: If x is negative, then x is neither positive nor 0.

f. *Converse*: If Jim is Ann's uncle and Sue is her aunt, then Tom is her father.
Inverse: If Tom is not Ann's father, then Jim is not her uncle or Sue is not her aunt

g. *Converse*: If n is divisible by 2 and n is divisible by 3, then n is divisible by 6
Inverse: If n is not divisible by 6, then n is not divisible by 2 or n is not divisible by 3.

24.

p	q	$p \to q$	$q \to p$	
T	T	T	T	
T	F	F	T	←
F	T	T	F	←
F	F	T	T	

different truth values

The truth table shows that $p \to q$ and $q \to p$ have different truth values in the second and third rows. Hence they are not logically equivalent.

25.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	
T	T	F	F	T	T	
T	F	F	T	F	T	←
F	T	T	F	T	F	←
F	F	T	T	T	T	

different truth values

The truth table shows that $p \to q$ *and* $\sim p \to \sim q$ have different truth values in rows 2 and 3, so they are not logically equivalent. Thus a conditional statement is not logically equivalent to its inverse.

26.

p	q	$\sim p$	$\sim q$	$\sim q \to \sim p$	$p \to q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

same truth values

The truth table shows that $\sim q \to \sim p$ and $p \to q$ always have the same truth values and thus are logically equivalent. It follows that a conditional statement and its contrapositive are logically equivalent to each other.

27.

p	q	$\sim p$	$\sim q$	$q \to p$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	T	T

same truth values

The truth table shows that $q \to p$ and $\sim p \to \sim q$ always have the same truth values and thus are logically equivalent. It follows that the converse and inverse of a conditional statement are logically equivalent to each other.

28. The if-then form of "I say what I mean" is "If I mean something, then I say it."
The if-then form of "I mean what I say" is "If I say something, then I mean it."
Thus "I mean what I say" is the converse of "I say what I mean," and so the two statements are not logically equivalent.

29. The corresponding tautology is $(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$

p	q	r	$\sim q$	$q \vee r$	$p \wedge \sim q$	$p \to (q \vee r)$	$p \wedge \sim q \to r$	$(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$
T	T	T	F	T	F	T	T	T
T	T	F	F	T	F	T	T	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	F	F	T	T	T

The truth table shows that $(p \to (q \vee r)) \leftrightarrow ((p \wedge \sim q) \to r)$ is a tautology because all of its truth values are T.

30. The corresponding tautology is $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \leftrightarrow$ $(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

all T's

The truth table shows that $p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$ is always true. Hence it is a tautology.

31. The corresponding tautology is $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$.

p	q	r	$q \to r$	$p \wedge q$	$p \to (q \to r)$	$(p \wedge q) \to r)$	$p \to (q \to r) \leftrightarrow (p \wedge q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

all T's

The truth table shows that $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$ is always true. Hence it is a tautology.

32. If this quadratic equation has two distinct real roots, then its discriminant is greater than zero, and if the discriminant of this quadratic equation is greater than zero, then the equation has two real roots.

33. If this integer is even, then it equals twice some integer, and if this integer equals twice some integer, then it is even.

34. If the Cubs do not win tomorrow's game, then they will not win the pennant.
If the Cubs win the pennant, then they will have won tomorrow's game.

35. If Sam is not an expert sailor, then he will not be allowed on Signe's racing boat.
If Sam is allowed on Signe's racing boat, then he is an expert sailor.

36. The Personnel Director did not lie. By using the phrase "only if," the Personnel Director set forth conditions that were necessary but not sufficient for being hired: if you did not satisfy those conditions then you would not be hired. The Personnel Director's statement said nothing about what would happen if you did satisfy those conditions.

37. If a new hearing is not granted, payment will be made on the fifth.

38. If it doesn't rain, then Ann will go.

39. If a security code is not entered, then the door will not open.

40. If I catch the 8:05 bus, then I am on time for work.

41. If this triangle has two $45°$ angles, then it is a right triangle.

42. If this number is not divisible by 3, then it is not divisible by 9.
If this number is divisible by 9, then it is divisible by 3.

43. If Jim does not do his homework regularly, then Jim will not pass the course.

If Jim passes the course, then he will have done his homework regularly.

44. If Jon's team wins the rest of its games, then it will win the championship.

45. If this computer program produces error messages during translation, then it is not correct.

If this computer program is correct, then it does not produce error messages during translation.

46. **a.** This statement is the converse of the given statement, and so it is not necessarily true. For instance, if the actual boiling point of compound X were 200°C, then the given statement would be true but this statement would be false.

b. This statement must be true. It is the contrapositive of the given statement.

c. must be true **d.** not necessarily true **e.** must be true **f.** not necessarily true

Note: To solve this problem, it may be helpful to imagine a compound whose boiling point is greater than 150° C. For concreteness, suppose it is 200° C. Then the given statement would be true for this compound, but statements a, d, and f would be false.

47. **a.** $p \wedge \sim q \to r \equiv \sim(p \wedge \sim q) \vee r$

b. $p \wedge \sim q \to r \equiv \sim(p \wedge \sim q) \vee r$ by the identity for $\to$ shown in the directions *[an acceptable answer]*

$\equiv \sim[\sim(\sim(p \wedge \sim q)) \wedge \sim r]$ by De Morgan's law *[another acceptable answer]*

$\equiv \sim[(p \wedge \sim q) \wedge \sim r]$ by the double negative law *[another acceptable answer]*

Any of the expressions in part (b) would also be acceptable answers for part (a).

48. **a.** $p \vee \sim q \to r \vee q \equiv \sim(p \vee \sim q) \vee (r \vee q)$ by the identity for $\to$ shown in the directions *[an acceptable answer]*

$\equiv (\sim p \wedge \sim(\sim q)) \vee (r \vee q)$ by De Morgan's law *[another acceptable answer]*

$\equiv (\sim p \wedge q) \vee (r \vee q)$ by the double negative law *[another acceptable answer]*

b. $p \vee \sim q \to r \vee q \equiv (\sim p \wedge q) \vee (r \vee q)$ by part (a)

$\equiv \sim(\sim(\sim p \wedge q) \wedge \sim(r \vee q))$ by De Morgan's law

$\equiv \sim(\sim(\sim p \wedge q) \wedge (\sim r \wedge \sim q))$ by De Morgan's law

Any of the expressions in part (b) would also be acceptable answers for part (a).

49. **a.** $(p \to r) \leftrightarrow (q \to r) \equiv (\sim p \vee r) \leftrightarrow (\sim q \vee r)$

$\equiv [\sim(\sim p \vee r) \vee (\sim q \vee r)] \wedge [\sim(\sim q \vee r) \vee (\sim p \vee r)]$
by the identity for $\leftrightarrow$ shown in the directions *[an acceptable answer]*

$\equiv [(p \wedge \sim r) \vee (\sim q \vee r)] \wedge [(q \wedge \sim r) \vee (\sim p \vee r)]$
by De Morgan's law *[another acceptable answer]*

b. $(\sim p \vee r) \leftrightarrow (\sim q \vee r) \equiv \sim[\sim(p \wedge \sim r) \wedge \sim(\sim q \vee r)] \wedge \sim[\sim(q \wedge \sim r) \wedge \sim(\sim p \vee r)]$
by De Morgan's law

$\equiv \sim[\sim(p \wedge \sim r) \wedge (q \wedge \sim r)] \wedge \sim[\sim(q \wedge \sim r) \wedge (p \wedge \sim r)]$
by De Morgan's law

Any of the expressions in part (b) would also be acceptable answers for part (a).

50. **a.** $$\begin{aligned}(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) &\equiv [\sim p \vee (q \to r)] \leftrightarrow [\sim (p \wedge q) \vee r]\\ &\equiv [\sim p \vee (\sim q \vee r)] \leftrightarrow [\sim (p \wedge q) \vee r]\\ &\equiv \sim [\sim p \vee (\sim q \vee r)] \vee [\sim (p \wedge q) \vee r]\\ &\qquad \wedge \sim [\sim (p \wedge q) \vee r] \vee [\sim p \vee (\sim q \vee r)]\end{aligned}$$

b. By part (a), De Morgan's law, and the double negative law,

$$\begin{aligned}(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r) &\equiv \sim [\sim p \vee (\sim q \vee r)] \vee [\sim (p \wedge q) \vee r]\\ &\qquad \wedge \sim [\sim (p \wedge q) \vee r] \vee [\sim p \vee (\sim q \vee r)]\\ &\equiv \sim [\sim p \vee (\sim q \vee r)] \wedge \sim [\sim (p \wedge q) \vee r]\\ &\qquad \wedge \sim\sim [(p \wedge q) \wedge \sim r] \wedge \sim [\sim p \vee (\sim q \vee r)]\\ &\equiv \sim\sim [p \wedge \sim (\sim q \vee r)] \wedge [(p \wedge q) \wedge \sim r]\\ &\qquad \wedge \sim\sim [(p \wedge q) \wedge \sim r] \wedge [p \wedge \sim (\sim q \vee r)]\\ &\equiv \sim\sim [p \wedge (q \wedge \sim r)] \wedge [(p \wedge q) \wedge \sim r]\\ &\qquad \wedge \sim\sim [(p \wedge q) \wedge \sim r] \wedge [p \wedge (q \wedge \sim r)].\end{aligned}$$

Any of the expressions in the right-hand column would also be acceptable answers for part (a).

51. Yes. As in exercises 47-50, the following logical equivalences can be used to rewrite any statement form in a logically equivalent way using only $\sim$ and $\wedge$:

$$\begin{aligned}&p \to q \equiv \sim p \vee q && p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)\\ &p \vee q \equiv \sim (\sim p \wedge \sim q) && \sim (\sim p) \equiv p\end{aligned}$$

The logical equivalence $p \wedge q \equiv \sim (\sim p \vee \sim q)$ can then be used to rewrite any statement form in a logically equivalent way using only $\sim$ and $\vee$.

Section 2.3

1. $\sqrt{2}$ is not rational.

2. $1 - 0.99999...$ is less than every positive real number.

3. Logic is not easy.

4. This graph cannot be colored with two colors.

5. They did not telephone.

6.

		premises		*conclusion*	
p	q	$p \to q$	$p \to q$	$p \vee q$	
T	T	T	T	T	← *critical row*
T	F	F	T		
F	T	T	F		
F	T	T	T	F	← *critical row*

Rows 2 and 4 of the truth table are the critical rows in which all the premises are true, but row 4 shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

7.

				premises			conclusion	
p	q	r	$\sim q$	p	$p \to q$	$\sim q \vee r$	r	
T	T	T	F	T	T	T	T	← *critical row*
T	T	F	F	T	T	F		
T	F	T	T	T	F	T		
T	F	F	T	T	F	T		
F	T	T	F	F	T	T		
F	T	F	F	F	T	F		
F	F	T	T	F	T	T		
F	F	F	T	F	T	T		

This row describes the only situation in which all the premises are true. Because the conclusion is also true here, the argument form is valid.

8.

					premises		conclusion	
p	q	r	$\sim q$	$p \vee q$	$p \to \sim q$	$p \to r$	r	
T	T	T	F	T	F	T		
T	T	F	F	T	F	F		
T	F	T	T	T	T	T	T	← *critical row*
T	F	F	T	T	T	F		
F	T	T	F	T	T	T	T	← *critical row*
F	T	F	F	T	T	T	F	← *critical row*
F	F	T	T	F	T	T		
F	F	F	T	F	T	T		

This row shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

9.

							premises		conclusion	
p	q	r	$\sim q$	$\sim r$	$p \wedge q$	$p \wedge q \to \sim r$	$p \vee \sim q$	$\sim q \to p$	$\sim r$	
T	T	T	F	F	T	F	T	T		
T	T	F	F	T	T	T	T	T	T	← *critical row*
T	F	T	T	F	F	T	T	T	F	← *critical row*
T	F	F	T	T	F	T	T	T	T	← *critical row*
F	T	T	F	F	F	T	F	T		
F	T	F	F	T	F	T	F	T		
F	F	T	T	F	F	T	T	F		
F	F	F	T	T	F	T	T	F		

Rows 2, 3, and 4 of the truth table are the critical rows in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

10.

									premise	conclusion	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p \wedge \sim q$	$p \vee q$	r	$p \vee q \to r$	$\sim r \to \sim p \wedge \sim q$	
T	T	T	F	F	F	F	T	T	T	T	← *critical row*
T	T	F	F	F	T	F	T	F	F	F	
T	F	T	F	T	F	F	T	T	T	T	← *critical row*
T	F	F	F	T	T	F	T	F	F	F	
F	T	T	T	F	F	F	T	T	T	T	← *critical row*
F	T	F	T	F	T	F	T	F	F	F	
F	F	T	T	T	F	T	F	T	T	T	← *critical row*
F	F	F	T	T	T	T	F	F	T	T	← *critical row*

This form of argument has just one premise. Rows 1, 3, 5, 7, and 8 of the truth table represent all the situations in which the premise is true, and in each of these rows the conclusion is also true. Therefore, the argument form is valid.

11.

							premises		*conclusion*	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \vee r$	$p \to q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim r$	
T	T	T	F	F	F	T	T	F		
T	T	F	F	F	T	T	T	T	T	← *critical row*
T	F	T	F	T	F	T	T	T	F	← *critical row*
T	F	F	F	T	T	F	F	T		
F	T	T	T	F	F	T	T	F		
F	T	F	T	F	T	T	T	T	T	← *critical row*
F	F	T	T	T	F	T	T	T	T	← *critical row*
F	F	F	T	T	T	F	T	T	T	← *critical row*

Rows 2, 3, 6, 7, and 8 of the truth table represent the situations in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

12. **a.**

		premises		*conclusion*	
p	q	$p \to q$	q	p	
T	T	T	T	T	← *critical row*
T	F	F	F		
F	T	T	T	F	← *critical row*
F	T	T	F		

Rows 1 and 3 of the truth table represent the situations in which all the premises are true, but row 3 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

b.

		premises		*conclusion*	
p	q	$p \to q$	$\sim p$	$\sim q$	
T	T	T	F		
T	F	F	T	T	← *critical row*
F	T	T	F		
F	T	T	T	F	← *critical row*

Rows 2 and 4 of the truth table represent the situations in which all the premises are true, but row 4 shows that it is possible for an argument of this form to have true premises and a false conclusion. Hence the argument form is invalid.

13.

		premises		*conclusion*	
p	q	$p \to q$	$\sim q$	$\sim p$	
T	T	T	F		
T	F	F	T		
F	T	T	F		
F	F	T	T	T	← *critical row*

Row 4 of the truth table represents the only situation in which all the premises are true, and in this row the conclusion is also true. Therefore, the argument form (modus tollens) is valid.