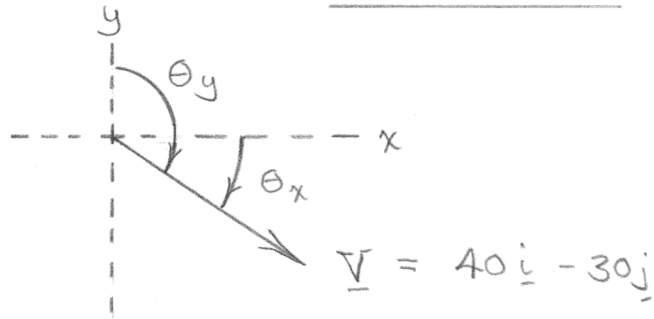


$$|\underline{V}| = \sqrt{V_x^2 + V_y^2} = \sqrt{40^2 + 30^2} = 50$$

$$\underline{n} = \frac{\underline{V}}{|\underline{V}|} = \frac{40\underline{i} - 30\underline{j}}{50} = \underline{0.8i - 0.6j}$$

$$\cos \theta_x = 0.8, \quad \theta_x = 36.9^\circ$$

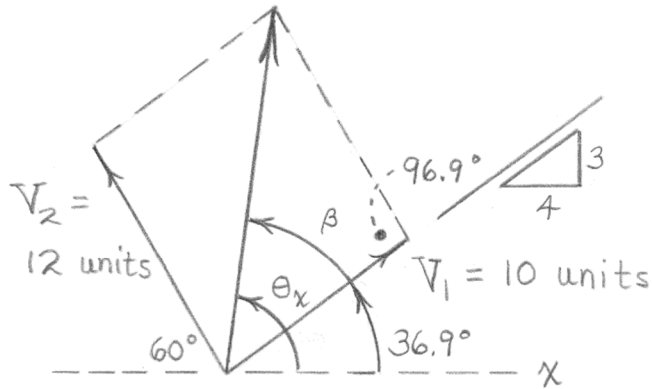
$$\cos \theta_y = -0.6, \quad \theta_y = 126.9^\circ$$



WILEY

1/2

$$\underline{V} = \underline{V}_1 + \underline{V}_2 \quad (\text{But } V \neq V_1 + V_2 !!)$$



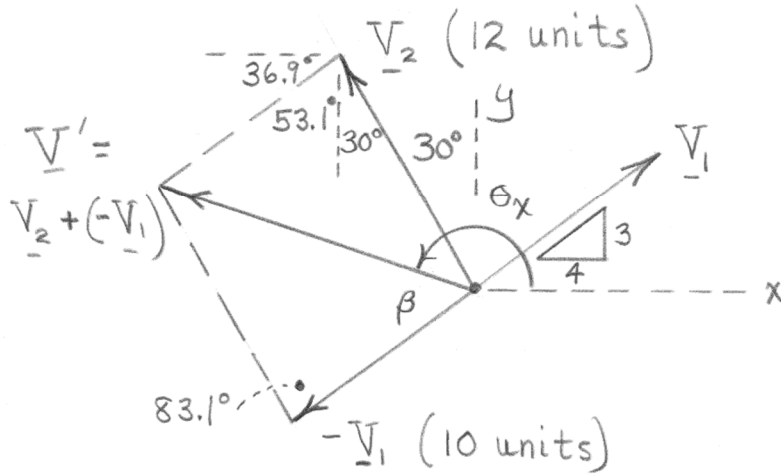
Graphically,  $V = 16.4$  units,  $\theta_x = 83^\circ$

$$\text{Algebraically, } V^2 = 10^2 + 12^2 - 2(10)(12)\cos 96.9^\circ$$

$$\underline{V = 16.51 \text{ units}}$$

$$\frac{\sin \beta}{12} = \frac{\sin 96.9^\circ}{16.51} \quad \beta = 46.2^\circ$$

$$\theta_x = \beta + 36.9^\circ = 46.2^\circ + 36.9^\circ = \underline{83.0^\circ}$$



Graphically,  $\underline{V}' = 14.7$  units,  $\theta_x = 163^\circ$

Algebraically,  $V'^2 = 10^2 + 12^2 - 2(10)(12)\cos 83.1^\circ$

$$\underline{V' = 14.67 \text{ units}}$$

$$\frac{\sin \beta}{12} = \frac{\sin 83.1^\circ}{14.67} \quad \beta = 54.3^\circ$$

$$\begin{aligned} \theta_x &= (180^\circ + 36.9^\circ) - \beta = 180^\circ + 36.9^\circ - 54.3^\circ \\ &= \underline{162.6^\circ} \end{aligned}$$

---

$$\frac{1}{4} \quad F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ N}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \quad \underline{\theta_x = 42.0^\circ}$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \quad \underline{\theta_y = 68.2^\circ}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \quad \underline{\theta_z = 123.9^\circ}$$

WILEY

---

$$\boxed{1/5} \quad m = \frac{W}{g} = \frac{3000}{32.174} = \underline{93.2 \text{ slugs}}$$

$$m = 93.2 \text{ slugs} \left( \frac{14.594 \text{ kg}}{\text{slug}} \right) = \underline{1361 \text{ kg}}$$

↑ from Table D/5

---

To illustrate the sensitivity of such calculations to significant-figure issues,

we now use  $g = 32.2 \text{ ft/sec}^2$ :

$$m = \frac{W}{g} = \frac{3000}{32.2} = 93.2 \text{ slugs} \checkmark$$

$$m = 93.2 (14.594) = 1360 \text{ kg} !$$

The value of  $g = 32.2 \text{ ft/sec}^2$  will normally, but not always, suffice.

WILEY

1/6

$$F = W = \frac{Gm_1m_2}{r^2}$$

where  $G = 6.673 (10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$

$$m_1 = 85 \text{ kg}$$

$$m_2 = 5.976 (10^{24}) \text{ kg}$$

and  $r = (6371 + 250) (10^3) \text{ m}$

Substitute these numbers  $\frac{1}{1}$  obtain  $\underline{W = 773 \text{ N}}$

U.S. units :  $W = 773 \text{ N} \left( \frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{173.8 \text{ lb}}$

# WILEY

---

$$\frac{1}{7} \quad W = (125 \text{ lb}) \left( \frac{4.4482 \text{ N}}{\text{lb}} \right) = \underline{556 \text{ N}}$$

$$m = \frac{W}{g} = \frac{125}{32.2} = \underline{3.88 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{556}{9.81} = \underline{56.7 \text{ kg}}$$

WILEY

---

$$\frac{1}{8} \quad A = 8.67, \quad B = 1.429$$

$$(A+B) = 8.67 + 1.429 = \underline{10.10}$$

$$(A-B) = 8.67 - 1.429 = \underline{7.24}$$

$$(AB) = (8.67)(1.429) = \underline{12.39}$$

$$(A/B) = 8.67/1.429 = \underline{6.07}$$

WILEY

1/9

$$F = \frac{G m_e m_m}{d^2} = \frac{6.673(10^{-11})(5.976 \cdot 10^{24})^2(1)(0.0123)}{(384\,398 \cdot 10^3)^2}$$

$$= \underline{1.984(10^{20}) \text{ N}}$$

$$F = 1.984(10^{20}) \text{ N} \left( \frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{4.46(10^{19}) \text{ lb}}$$

WILEY

---

$$\frac{1}{10} \quad \underline{F} = \underline{F}_n = F \left( \frac{-4\underline{i} - 2\underline{j}}{\sqrt{4^2 + 2^2}} \right),$$

$$\text{where } F = \frac{G m_{cu} m_{st}}{d^2}$$

$$= \frac{G \left( \rho_{cu} \frac{4}{3} \pi r^3 \right) \left( \rho_{st} \frac{4}{3} \pi \left( \frac{r}{2} \right)^3 \right)}{(4r)^2 + (2r)^2}$$

$$= \frac{1}{90} G \rho_{cu} \rho_{st} \pi^2 r^4$$

$$= \frac{1}{90} (6.673 \cdot 10^{-11}) (8910) (7830) \pi^2 0.050^4$$

$$= 3.19 (10^{-9}) \text{ N}$$

$$\text{Then } \underline{F} = 3.19 (10^{-9}) \left[ \frac{-4\underline{i} - 2\underline{j}}{\sqrt{20}} \right]$$
$$= (-2.85\underline{i} - 1.427\underline{j}) 10^{-9} \text{ N}$$

---

WILEY

---

$$\frac{1}{11} \quad E = 3 \sin^2 \theta \tan \theta \cos \theta$$

$$\text{Exact: } E = 3 \sin^2 2^\circ \tan 2^\circ \cos 2^\circ \\ = \underline{1.275 (10^{-4})}$$

$$\text{Approx: } E_{ap} = 3(\theta^2)(\theta)(1) \\ = 3\theta^3 \quad (\theta \text{ in rad})$$

$$E_{ap} = 3 \left[ 2 \frac{\pi}{180} \right]^3 = \underline{1.276 (10^{-4})}$$

WILEY

$$\frac{1}{12} \quad \text{SI: } [\varphi] = (1)(\text{kg})(\text{m}^2)/\text{s}^2 \\ = \text{kg} \cdot \text{m}^2/\text{s}^2$$

$$\text{U.S.: } [\varphi] = (1)(\text{slug})(\text{ft}^2)/\text{sec}^2 \\ = \left(\frac{\text{lb} \cdot \text{sec}^2}{\text{ft}}\right)(\text{ft})^2/\text{sec}^2 = \underline{\text{lb} \cdot \text{ft}}$$

Note: The SI units reduce to

$(\text{kg} \cdot \text{m}/\text{s}^2) \text{m} = \text{N} \cdot \text{m}$ , but N is not a base unit.

WILEY

---

$$\begin{cases} F_x = 600 \cos 40^\circ = \underline{460 \text{ N}} \\ F_y = -600 \sin 40^\circ = \underline{-386 \text{ N}} \end{cases}$$

---

$$\underline{\underline{F = 460\mathbf{i} - 386\mathbf{j} \text{ N}}}$$

WILEY

2/2

$$\underline{F} = 400 (-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$
$$= -346 \underline{i} + 200 \underline{j} \text{ N}$$

Scalar components:  $\begin{cases} F_x = -346 \text{ N} \\ F_y = 200 \text{ N} \end{cases}$

Vector components:  $\begin{cases} \underline{F}_x = -346 \underline{i} \text{ N} \\ \underline{F}_y = 200 \underline{j} \text{ N} \end{cases}$

WILEY

2/3

$$\underline{F} = 6.5 \left( -\frac{12}{13} \underline{i} - \frac{5}{13} \underline{j} \right)$$
$$= -6 \underline{i} - 2.5 \underline{j} \text{ kN}$$

(Note: Writing 6, rather than 6.00, indicates an exact result.)

WILEY

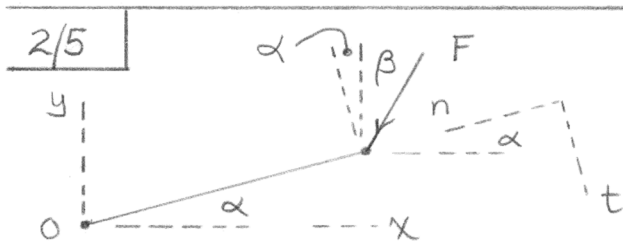
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$$\underline{2/4} \quad \underline{F} = F \underline{n}_{AB} = 34 \left[ \frac{15\underline{i} + 8\underline{j}}{\sqrt{15^2 + 8^2}} \right]$$
$$= 30\underline{i} + 16\underline{j} \text{ kN}$$

Scalar components:  $\begin{cases} F_x = 30 \text{ kN} \\ F_y = 16 \text{ kN} \end{cases}$

---

WILEY

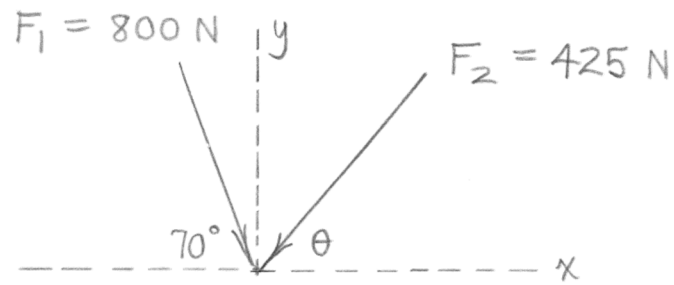


$$\begin{cases} F_x = -F \sin \beta \\ F_y = -F \cos \beta \end{cases}$$

$$\begin{cases} F_n = F \sin(\alpha + \beta) \\ F_t = F \cos(\alpha + \beta) \end{cases}$$

WILEY

2/6



$$R_x = \sum F_x = 800 \cos 70^\circ - 425 \cos \theta = 0$$
$$\theta = \underline{49.9^\circ}$$

$$R_y = \sum F_y = -800 \sin 70^\circ - 425 \sin 49.9^\circ$$
$$= -1077 \text{ N}$$

$$\text{So } R = \underline{1077 \text{ N}}$$

# WILEY

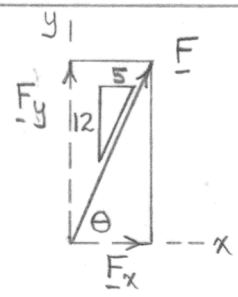
$$\begin{cases} \underline{R} = (500 + 350 \cos 60^\circ) \underline{i} + 350 \sin 60^\circ \underline{j} \\ \underline{R} = 675 \underline{i} + 303 \underline{j} \text{ N} \end{cases}$$

$$R = \sqrt{675^2 + 303^2} \longrightarrow \underline{R = 740 \text{ N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{675}{740}\right) \longrightarrow \underline{\theta_x = 24.2^\circ \text{ ABOVE } +x \text{ AXIS}}$$

WILEY

2/8



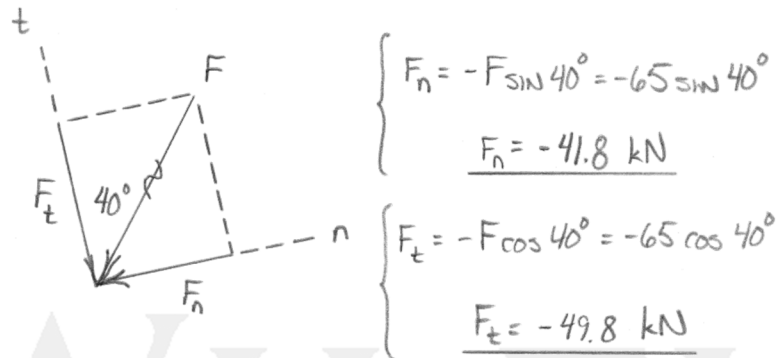
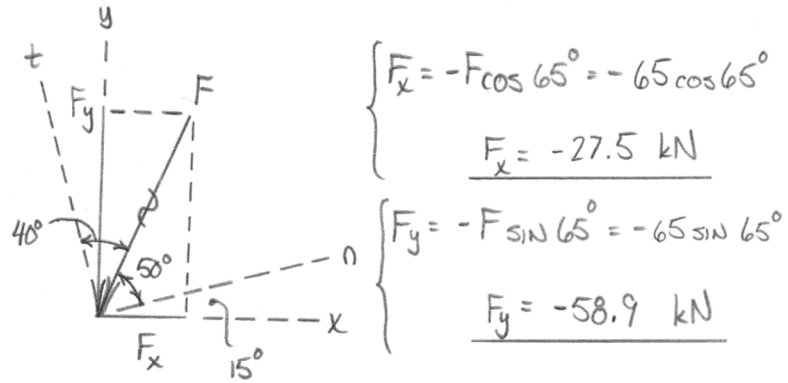
$$\cos \theta = \frac{5}{13}, \quad \sin \theta = \frac{12}{13}$$

$$F_y = F \sin \theta = F \frac{12}{13} = 320 \text{ N}$$

$$F = 347 \text{ N}$$

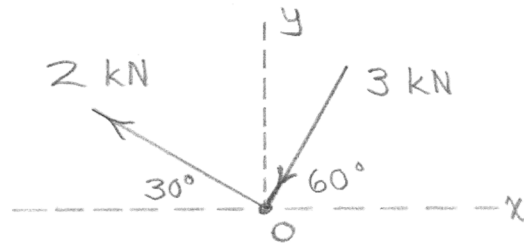
$$F_x = F \cos \theta = 347 \left( \frac{5}{13} \right) = \underline{133.3 \text{ N}}$$

WILEY



WILEY

2/10



$$R_x = \sum F_x = -2 \cos 30^\circ - 3 \cos 60^\circ = -3.23 \text{ kN}$$

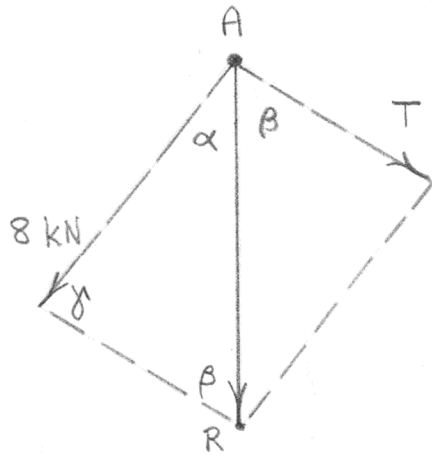
$$R_y = \sum F_y = 2 \sin 30^\circ - 3 \sin 60^\circ = -1.598 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.61 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = \underline{206^\circ}$$

WILEY

2/11



$$\begin{cases} \alpha = \tan^{-1} \frac{40}{50} = 38.7^\circ \\ \beta = \tan^{-1} \frac{50}{30} = 59.0^\circ \end{cases}$$

$$\begin{aligned} \gamma &= 180^\circ - \alpha - \beta \\ &= 82.3^\circ \end{aligned}$$

$$\frac{\sin \beta}{8} = \frac{\sin \alpha}{T}$$

$$\underline{T = 5.83 \text{ kN}}$$

$$\frac{\sin \beta}{8} = \frac{\sin \gamma}{R},$$

$$\underline{R = 9.25 \text{ kN}}$$

WILEY

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2/12

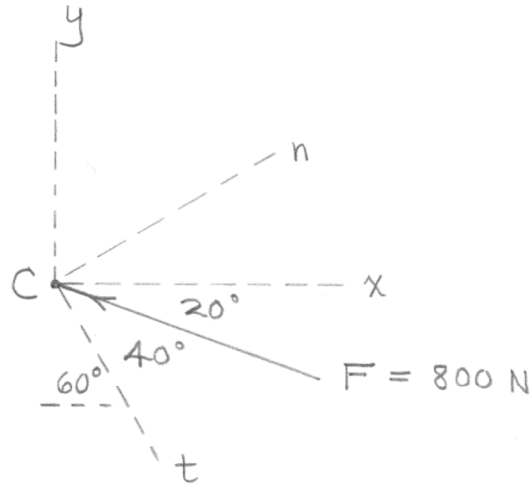
$$R_x = \sum F_x = 400 + 400 \cos 60^\circ = 600 \text{ N}$$

$$R_y = \sum F_y = 400 \sin 60^\circ = 346 \text{ N}$$

$$\Rightarrow \underline{R} = \underline{600i + 346j} \text{ N}$$

$$R = \sqrt{600^2 + 346^2} = \underline{693 \text{ N}}$$

WILEY

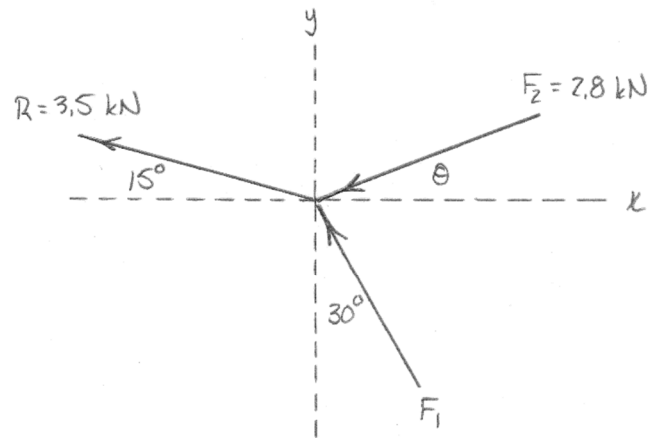


$$\left\{ \begin{array}{l} F_x = -800 \cos 20^\circ = -752\text{ N} \\ F_y = 800 \sin 20^\circ = \underline{274\text{ N}} \end{array} \right.$$

$$\left\{ \begin{array}{l} F_n = -800 \sin 40^\circ = \underline{-514\text{ N}} \\ F_t = -800 \cos 40^\circ = \underline{-613\text{ N}} \end{array} \right.$$

WILEY

2/14



$$\begin{cases} R_x = \sum F_x: & -3.5 \cos 15^\circ = -F_1 \sin 30^\circ - 2.8 \cos \theta & \textcircled{1} \\ R_y = \sum F_y: & 3.5 \sin 15^\circ = F_1 \cos 30^\circ - 2.8 \sin \theta & \textcircled{2} \end{cases}$$

Solving ① AND ②...

$$\begin{cases} F_1 = 1.165 \text{ kN} \\ \theta = 2.11^\circ \end{cases}$$

OR

$$\begin{cases} F_1 = 3.78 \text{ kN} \\ \theta = 57.9^\circ \end{cases}$$

2/15

$$L^2 = (r - r \sin \theta)^2 + (r + r \cos \theta)^2$$

$$= r^2 - 2r^2 \sin \theta + r^2 \sin^2 \theta + r^2 + 2r^2 \cos \theta + r^2 \cos^2 \theta$$

$$= r^2 (3 + 2 \cos \theta - 2 \sin \theta)$$

So  $\cos \beta = \frac{r(1 + \cos \theta)}{r \sqrt{3 + 2 \cos \theta - 2 \sin \theta}}$

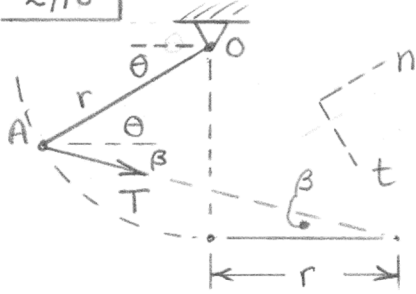
$$= \frac{1 + \cos \theta}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}$$

$$\sin \beta = \frac{1 - \sin \theta}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}$$

$$T_x = T \cos \beta = \frac{T(1 + \cos \theta)}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}$$

$$T_y = -T \sin \beta = \frac{T(\sin \theta - 1)}{\sqrt{3 + 2 \cos \theta - 2 \sin \theta}}$$

2/16



From solution to previous problem:

$$\beta = \tan^{-1} \left[ \frac{1 - \sin \theta}{1 + \cos \theta} \right]$$

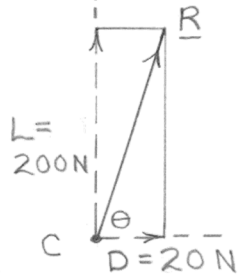
$$\begin{cases} T_n = T \cos(\theta + \beta) \\ T_t = T \sin(\theta + \beta) \end{cases}$$

For  $T = 100 \text{ N}$  and  $\theta = 35^\circ$ :

$$\beta = 13.19^\circ$$

$$\begin{cases} T_n = 66.7 \text{ N} \\ T_t = 74.5 \text{ N} \end{cases}$$

2/17



$$\frac{L}{D} = \frac{200}{20} = 10, \quad D = 20 \text{ N}$$

$$R = \sqrt{L^2 + D^2} = \sqrt{(200)^2 + (20)^2}$$

$$= \underline{201 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{200}{20}\right)$$

$$= \underline{84.3^\circ}$$

WILEY

2/18

Using the coordinates of the problem figure:

$$\begin{aligned} R_x = \Sigma F_x &= 200 \cos 35^\circ - 150 \sin 30^\circ \\ &= 88.8 \text{ N} \end{aligned}$$

$$\begin{aligned} R_y = \Sigma F_y &= 200 \sin 35^\circ + 150 \cos 30^\circ \\ &= 245 \text{ N} \end{aligned}$$

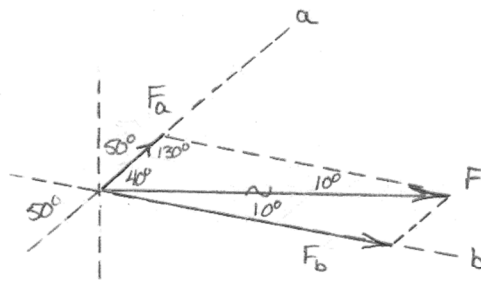
$$\therefore \underline{R} = 88.8 \underline{i} + 245 \underline{j} \text{ N}$$

WILEY

2/19

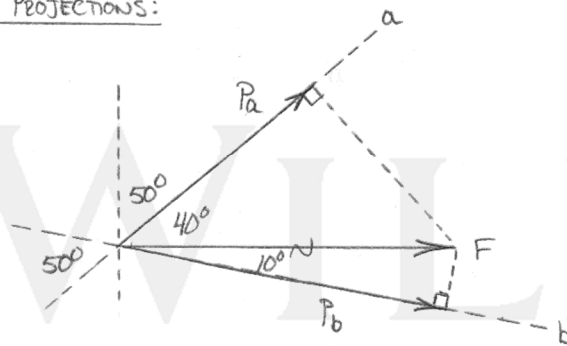
$F = 2,5 \text{ kN}$

• COMPONENTS:



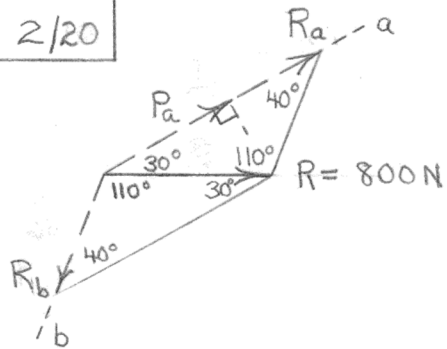
$$\frac{F}{\sin 130^\circ} = \frac{F_a}{\sin 10^\circ} = \frac{F_b}{\sin 40^\circ} \rightarrow \begin{cases} F_a = 0,567 \text{ kN} \\ F_b = 2,10 \text{ kN} \end{cases}$$

• PROJECTIONS:



$$\begin{cases} P_a = F \cos 40^\circ = 2,5 \cos 40^\circ \rightarrow P_a = 1,915 \text{ kN} \\ P_b = F \cos 10^\circ = 2,5 \cos 10^\circ \rightarrow P_b = 2,46 \text{ kN} \end{cases}$$

2/20



Law of sines :

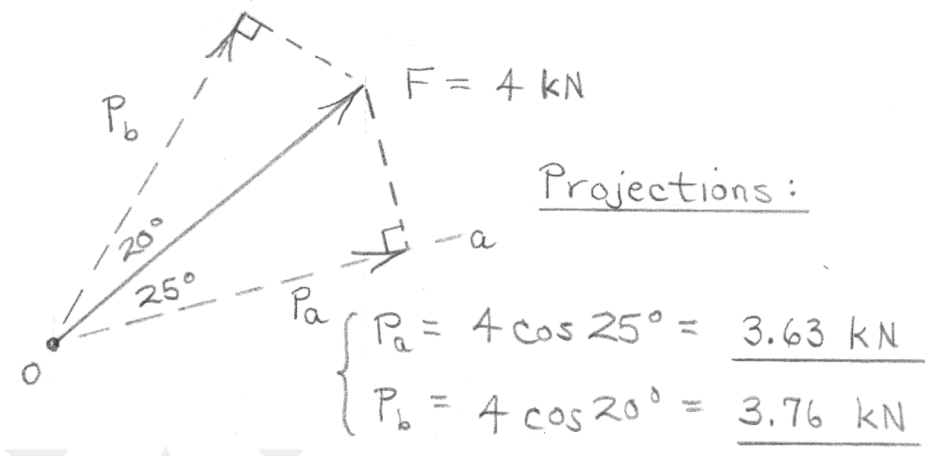
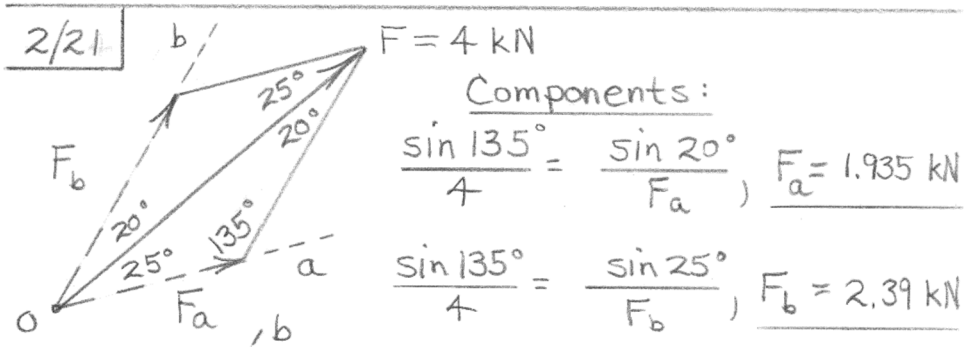
$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$$\underline{R_a = 1170 \text{ N}}$$

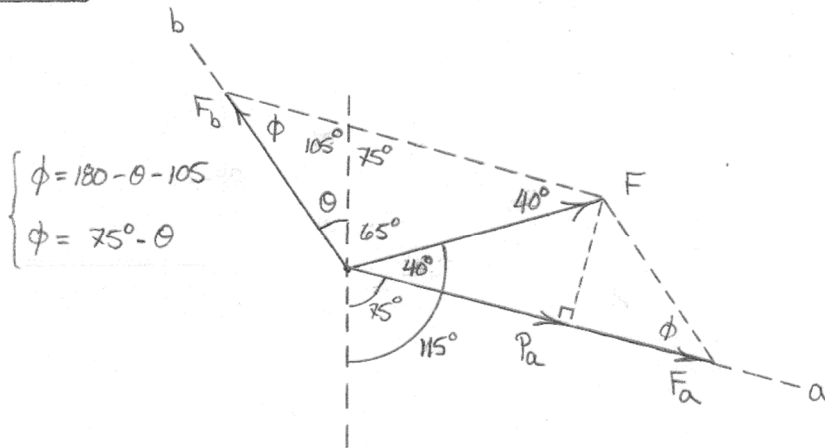
$$\underline{R_b = 622 \text{ N}}$$

$$\text{Projection } R_a = R \cos 30^\circ = 800 \cos 30^\circ = \underline{693 \text{ N}}$$

WILEY



WILEY



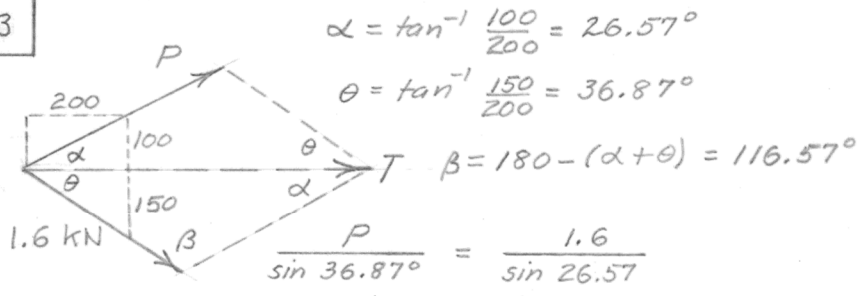
$$\begin{cases} \phi = 180 - \theta - 105 \\ \phi = 75 - \theta \end{cases}$$

$$P_a = F \cos 40^\circ \rightarrow 325 = F \cos 40^\circ \rightarrow \underline{F = 424 \text{ N}}$$

$$\left\{ \frac{F}{\sin \phi} = \frac{F_b}{\sin 40^\circ} \rightarrow \frac{424}{\sin(75^\circ - \theta)} = \frac{325}{\sin 40^\circ} \right.$$

SOLVING THIS YIELDS...  $\theta = 17.95^\circ$  OR  $-48.0^\circ$

2/23



$$\alpha = \tan^{-1} \frac{100}{200} = 26.57^\circ$$

$$\theta = \tan^{-1} \frac{150}{200} = 36.87^\circ$$

$$\beta = 180 - (\alpha + \theta) = 116.57^\circ$$

$$\frac{P}{\sin 36.87^\circ} = \frac{1.6}{\sin 26.57^\circ}$$

$$P = 1.6 \frac{0.6}{0.4472} = \underline{2.15 \text{ kN}}$$

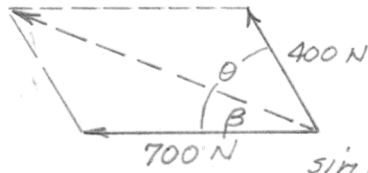
$$\frac{T}{\sin 116.57^\circ} = \frac{1.6}{\sin 26.57^\circ} \quad T = 1.6 \frac{0.8944}{0.4472} = \underline{3.20 \text{ kN}}$$

WILEY

2/24 Law of cosines:  $1000^2 = 400^2 + 700^2 + 2(400)(700)\cos\theta$

$\cos\theta = 0.6250, \theta = 51.3^\circ$

$R = 1000\text{ N}$



Law of sines:

$$\frac{1000}{\sin(180^\circ - 51.3^\circ)} = \frac{400}{\sin\beta}$$

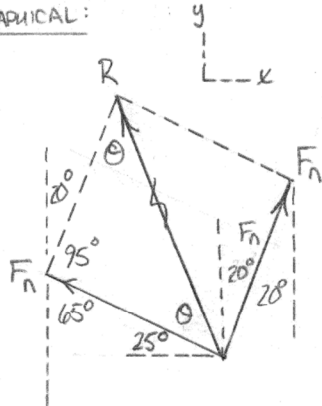
$$\sin\beta = \frac{400}{1000} \cdot 0.7806 = 0.3122$$

$\beta = 18.19^\circ$

WILEY

2/25

• GRAPHICAL:



$$\begin{cases} R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 \cos 95^\circ} \\ R = \sqrt{2(5500)^2 [1 - \cos 95^\circ]} \\ R = 8110 \text{ N} \end{cases}$$

$$\theta = \frac{180^\circ - 95^\circ}{2} \rightarrow \theta = 42.5^\circ$$

$R = 8110 \text{ N @ } 112.5^\circ \text{ CCW FROM } +x \text{ AXIS}$

• VECTORS:

$$\underline{R} = (F_1 \sin 20^\circ - F_2 \sin 65^\circ) \underline{i} + (F_1 \cos 20^\circ + F_2 \cos 65^\circ) \underline{j}$$

$$\underline{R} = 5500 [(\sin 20^\circ - \sin 65^\circ) \underline{i} + (\cos 20^\circ + \cos 65^\circ) \underline{j}]$$

$$\underline{R} = -3100 \underline{i} + 7490 \underline{j} \text{ N}$$

$$R = \sqrt{3100^2 + 7490^2} \rightarrow \underline{R = 8110 \text{ N}}$$

$$\theta_x = \cos^{-1} \left( \frac{R_x}{R} \right) = \cos^{-1} \left( \frac{-3100}{8110} \right) \rightarrow \underline{\theta_x = 112.5^\circ \text{ CCW FROM } +x \text{ AXIS}}$$