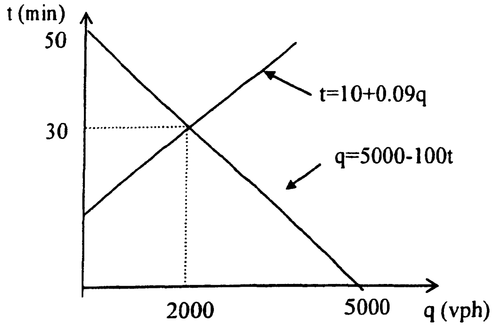
CHAPTER 2
TRANSPORTATION ECONOMICS

<2.1>

$t = 10 + 0.09q$
$q = 5000 - 100t$



a)

$t = 10 + 0.01q \Rightarrow \quad q = \dfrac{t-10}{0.01} = 100t - 1000$

$100t^* - 1000 = 5000 - 100t^*$

$200t^* = 6000$

$$\boxed{\begin{array}{l} t^* = 30\min \\ q^* = 5000 - 100 \times 30 = 2000 veh/hr \end{array}}$$

b) L = 22.5 miles, where L = length in miles.

$t = 30\min$

$$\boxed{v = 22.5/30 \times 60 = 45mph}$$

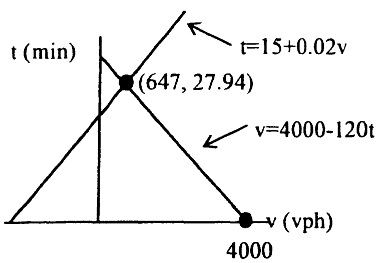
c) t* . 10 + 0.005q ⇒ q* = 200t - 2000
200t* - 2000 = 5000 - 100t*
300t* = 7000

$$\boxed{\begin{array}{l} t^* = 23.33\min \\ q^* = (5000 - 100) \times 23.33 = 2667 \ \ veh/hr \end{array}}$$

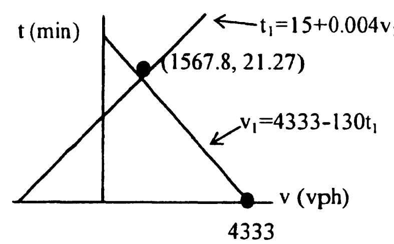
$$v = \frac{22.5}{23.3} \times 60 = 58mph$$

<2.2>

a)



Existing Situation



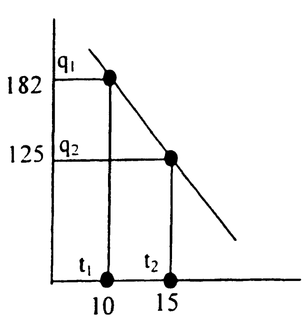
Proposed Situation

b)
$$v = 4333 - 130(15 + 0.004v)$$
$$v = 4333 - 1950 - 0.52v$$
$$1.52v = 2383$$
$$v = 1567.8 \ veh / hr.$$
$$t = 15 + (0.004 \times 1567.8) = 21.27 \ \text{min.}$$

c)
$$l = 20 \text{ miles}$$
$$t = 21.27 \text{ min}$$
$$v = \frac{l}{t} \cdot 60 = {}^{20}\!/_{21.27} \times 60 = 56.42 \ mph$$

<2.3>

Assume that the demand function is valid in the time range between t = 10 & t = 15 min



$$\begin{aligned} \Delta veh - \min s &= q_1 t_1 - q_2 t_2 \\ &= q_1 t_1 - q_2 t_2 \\ &= (125)(15) - (182)(10) \\ &= 55 \text{ veh - mins are lost due to congestion} \end{aligned}$$

<2.4>

Assuming the demand function to be linear we get the following equations:

$2000 = \alpha - 1.5\,\beta$

$1000 = \alpha - 2.0\,\beta$

Hence $\alpha = 5000$, and $\beta = 2000$ $\quad q = 5000 - 2000\,p$

a) when the fare is 50c, $q = 5000 - (2000 \times 0.5) = 4000$

b) If the transit system were free, p = 0, and q = 5000, which is the latent demand.

<2.5>

$\rho_o = 50^{¢}$ / ride $\qquad q_0 = 500{,}000$ per / day

$\rho_1 = 60^{¢}$ / ride $\qquad q_1 = 470{,}000$ per / day

a)
$$\in = \frac{(Q_1 - Q_0)}{(\rho_1 - \rho_0)} \frac{(\rho_1 + \rho_2)/2}{(Q_1 + Q_2)/2} = \frac{(470-500)(50+60)/2}{(60-50)(500+470)/2} = -0.34 \text{ (inelastic)}$$

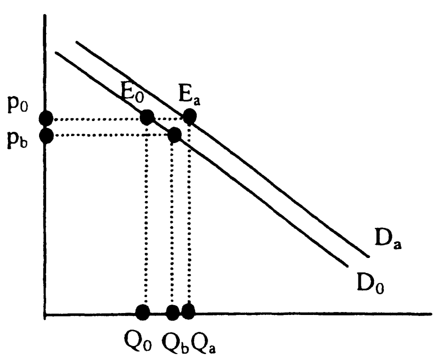
b)
$0.50 \times 500{,}000 = \$250{,}000 / day$

$0.60 \times 470{,}000 = \$282{,}000 / day$

Total gain $= \$32{,}000 / day$

<2.6>

Assume (1) that the demand function remains unchanged, and (2) that the cost of improving the service is ignored.



a) Initial demand function (D_o)

$Q_o = 2125 - 1000\,P_o$

Option (a), demand function (D_a)

$Q_a = 2150 - 1000\,P_o$

Additional Revenue ΔTR_a

$= P_oQ_a - P_oQ_o$

$= 1.30\,[(2150 - 1000P_o) - (2125 - 1000P_o)]$

$= \$32.50$

b) New price reduced to P_b = \$1.00/ride

$\Delta TR_b = 1.00\,[2125 - 1000\,(1.00)] - 1.30\,[2125 - 1000\,(1.30)]$

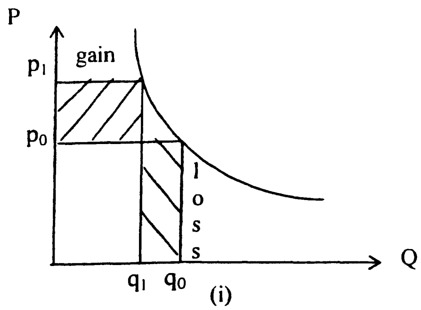
$= \$52.50$

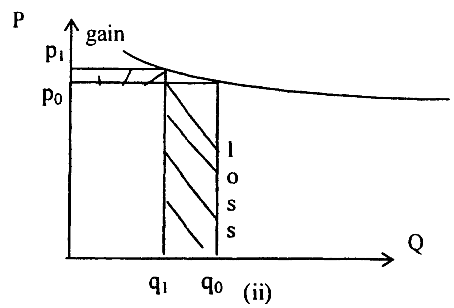
Since $\Delta TR_b > \Delta TR_a$, option b is better.

<2.7>

(i) Price elasticity = - 0.4 (inelastic) $\therefore$ price rise would lead to a fall in patronage but a total revenue gain. So fears are not justified.

(ii) In this case, $\in_p = -1.3$ (elastic) in which case a rise in fare not only leads to a total loss of revenue, but a decrease in patronage. It would therefore not be advisable to raise the fare.





<2.8>

Using arc-price elasticity
$P_1 = \$100$; $P_2 = \$120$; $Q_1 = 5000$; and $Q_2 = ?$. Also e = - 1.2
Notice that the price is elastic, and in general when price is elastic (-1.2), raising the price will result in total loss, but lowering the price will result in total gain.

$$-1.2 = \left(\frac{5000 - Q_2}{100 - 120}\right)\left(\frac{100 + 120}{5000 + Q_2}\right) \qquad \text{therefore, } Q_2 = 4016$$

Total difference in revenue = (100 x 5000) - (120 x 4016)
Total loss = \$ -18,080

This loss should come as no surprise to the airlines and should have been taken into account before raising the fare.

<2.9>

This problem could be solved in several ways. The simplest is to follow classic economic style which states that price elasticity is the % change in quantity demanded which accompanies a 1% change in price.

$$\therefore e_p = \frac{10\%}{40\%} = 0.25 \text{ (inelastic)}$$

If scooter - sellers raise prices of scooters by 50% then the number of scooters sold will decrease by (0.25 x 50%) = 12.5%; but total revenue will rise by (1 - 0.25) x 50% = 37.5%.

This problem can also be solved by using arc-price elasticity and the results will be about the same.

<2.10>

A. Complements

B. Complements

C. Substitutes

D. Substitutes

E. Complements

F. Complements (if sold together)

<2.11>

$Q = \alpha P^{\beta}$

$12500 = \alpha(50)^{-0.75}$

$\alpha \quad = 235{,}038$

An increase in fare from 50¢ to 70¢ will attract $Q \quad = 235{,}038(70)^{-0.75}$

$= 9712$ passengers

Revenue-wise we have
At 50¢/ride x 12500 = $6250
At 70¢/ride x 9712 = $6798
Gain (total) = $548

Advice to management would be to raise fare to 70¢/ride

[Since $\in = -0.75$, elastic, it is obvious what the conclusion would be]

<2.12>

(a)

$$\frac{\Delta Q/Q}{\Delta A/A} = -2.2 \Rightarrow 1\% \text{ reduction in travel time by auto}$$

will result in a 2.2% increase in automobile trips

$$\frac{\Delta Q/Q}{\Delta B/B} = 0.13 \Rightarrow$$ 1% reduction of travel time by bus will result in a 0.13% reduction in auto trips

$$\frac{\Delta Q/Q}{\Delta C/C} = -0.4 \Rightarrow$$ 1% reduction in the avg cost of travel by auto will result in a 0.4% increase in auto trips

$$\frac{\Delta Q/Q}{\Delta D/D} = 0.75 \Rightarrow$$ 1% reduction in the avg cost of travel by bus will result in a 0.75% reduction in auto trips

The signs are justified.

b) A = 20% $\nearrow$
B = 10% $\nearrow$
C = 5% $\nearrow$
D = 15% $\searrow$

$$Q_0 = aA^{-2.2}B^{-0.4}C^{-0.4}D^{0.75}$$

$$Q_1 = a(1.20A)^{-2.2}(1.10B)^{0.13}(1.05C)^{-0.4}(0.85D)^{0.75}$$

$$Q_1 = 0.589aA^{-2.2}B^{0.13}C^{-0.4}D^{0.75}$$

$$\frac{Q_1 - Q_0}{Q_0} = (0.589 - 1) = -0.411 \Rightarrow 41.1\% \text{ decrease in automobile trip}$$

c) B = 10% $\searrow$
D = 10% $\nearrow$

$$Q_1 = aA^{-2.2}(0.9B)^{0.13}\ C^{-0.4}\ (1.10D)^{0.75} = 1.06$$

$$\frac{Q_1 - Q_0}{Q_0} = (1.06 - 1) = 0.06 \Rightarrow 6\% \text{ increase in automobile trip}$$

<2.13>

The cross elasticity coefficient indicates that the % change of express-bus riders (Q) will equal 2 times the % change in the price (P) of the ordinary bus

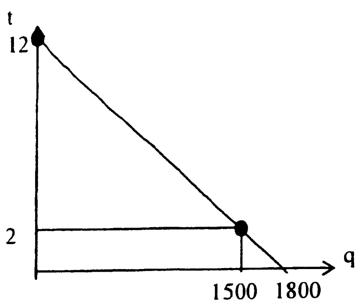
$$\in = \frac{\%\Delta Q \text{ of express bus riders}}{\%\Delta P \text{ of ordinary bus riders}} = 2$$

$$\therefore\ \%\Delta Q \text{ of express bus riders} = 2 \times \%\Delta P \text{ of ordinary bus riders} = 2\left[\frac{(P_2 - P_1)100}{(P_2 + P_1)/2}\right]$$

$$= 2\left[\frac{(0.5 - 0.75)100}{(0.5 + 0.75)/2}\right] = -80\%$$

which means that express bus riders and revenue will decline by 80%, if the price of express bus service remains the same.

<2.14>



$$q = 1800 - 150t$$

$$\epsilon_t = \left(\frac{dq}{dt}\right)\left(\frac{t}{q}\right) = (-150)\left(\frac{2}{1500}\right) = -0.2 \text{ (inelastic)}$$

<2.15>

$$\frac{\partial Q}{Q}\Big/\frac{\partial P}{P} = -0.75 \text{ (inelastic) or } \frac{20\%}{X\%} = -0.75, \; X\% = -26.7\%$$

a 26.7% decrease in fare = 73.3 cents and a new seating arrangement of 2400. Change in Consumer's surplus $= \frac{2000 + 2400}{2} \times 0.267 = \$587.4$ per hour

Existing Revenue $= (2000)(1) = 2000$

Revised revenue $= (2400)(0.73) = 1760$

Loss $= \$240$

Obviously, if the demand is sufficient, there is no need to decrease the fare.

<2.16>

Assume that the given price elasticity is valid in the range of the 10% change in the number of buses:

A 10% increase in the number of buses implies a 10% increase in the number of seats.

Price elasticity of - 0.3 means

$$\frac{\Delta Q}{Q}\Big/\frac{\Delta P}{P} = -0.3$$

or a 3% increase in Q results from a 10% decrease in P. $\therefore$ a 10% increase in Q would result in a 33.33% decrease in fare (P).

Original capacity: 50 x 55 = 2750   New capacity: 2750 + 275 = 3025