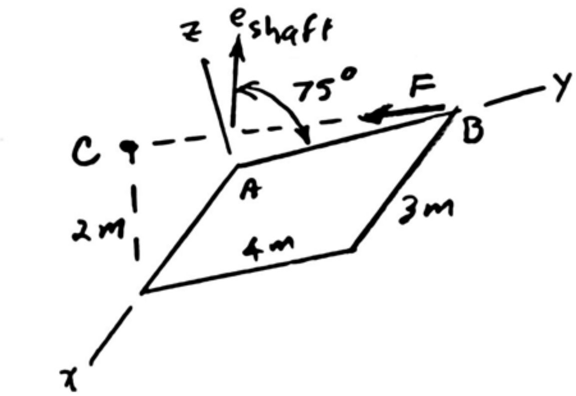
Exercise 1.1



Given $F = 5000\,N$

Find (a) components of $\bar{F}$ relative to xyz in sketch,

(b) moment of $\bar{F}$ about corner A,

(c) moment of $\bar{F}$ about the shaft.

Solution: $\bar{F} = 5000\,\bar{e}_{c/B}$

where $\bar{e}_{c/B} = \dfrac{\bar{r}_{C/A} - \bar{r}_{B/A}}{|\bar{r}_{C/A} - \bar{r}_{B/A}|}$

$\bar{r}_{C/A} = 2\bar{i} + 2(\cos 75^\circ \bar{j} + \sin 75^\circ \bar{k})$ & $\bar{r}_{B/A} = 4\bar{j}$

so $\bar{e}_{c/B} = 0.6017\bar{i} - 0.6984\bar{j} + 0.3875\bar{k}$

$\bar{F} = 3009\bar{i} - 3492\bar{j} + 1937\bar{k}$ N $\triangleleft$

Then $\bar{M}_A = \bar{r}_{B/A} \times \bar{F} = 7749\bar{i} - 12034\bar{k}$ N-m $\triangleleft$

$\bar{M}_{shaft} = \bar{M}_A \cdot \bar{e}_{shaft} = \bar{M}_A \cdot (\cos 75^\circ \bar{j} + \sin 75^\circ \bar{k})$

$= -11624$ N-m

Exercise 1.2

Given $\bar{a}_B = \bar{a}_A + \bar{\alpha} \times \bar{r}_{B/A} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{B/A})$, $\bar{r}_{A/O} = \bar{i} + \bar{j} + \bar{k}$ m, $\bar{r}_{B/O} = 4\bar{i} + 2\bar{j} - 3\bar{k}$ m, $\bar{a}_A = 4\bar{i} - 5\bar{j} + \bar{k}$ m/s^2, $\bar{\omega} = 5\bar{i} - 3\bar{j} + 2\bar{k}$ rad/s, $\bar{\alpha} = -20\bar{i} + 10\bar{j} - 40\bar{k}$ rad/s^2.

Find $\bar{a}_B$ manually and with software

Solution: $\bar{r}_{B/A} = \bar{r}_{B/O} - \bar{r}_{A/O} = 3\bar{i} + \bar{j} - 4\bar{k}$

$$\bar{\alpha} \times \bar{r}_{B/A} = (-40 + 40)\bar{i} + (-80 - 120)\bar{j} + (-20 - 30)\bar{k}$$

$$\bar{\omega} \times \bar{r}_{B/A} = (12 - 2)\bar{i} + (20 + 6)\bar{j} + (5 + 9)\bar{k}$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}) = (-52 - 42)\bar{i} + (-70 + 20)\bar{j} + (130 + 30)\bar{k}$$

$$\bar{a}_B = (4 + 0 - 94)\bar{i} + (-5 - 200 - 50)\bar{j} + (1 - 50 + 160)\bar{k}$$

$$= -90\bar{i} - 255\bar{j} + 111\bar{k} \text{ m/s}^2$$

Exercise 1.3

Given $S' = \frac{1}{2}\bar{\alpha} \cdot \frac{\partial \bar{H}_A}{\partial t} + \bar{\alpha} \cdot (\bar{\omega} \times \bar{H}_A)$ with expression for $\bar{H}_A$ and $\frac{\partial \bar{H}_A}{\partial t}$ in terms of I_{pq}, $\bar{\omega}$, & $\bar{\alpha}$.

Find S manually and with software

Solution: Substitute into the given formulas:

$$\bar{H}_A = [500(-50) - (-200)(-20)]\bar{i} + [300(-20) - (-200)(-50)]\bar{k}$$

$$= -29000\bar{i} - 16000\bar{k} \ \text{kg-m}^2/\text{s}$$

$$\frac{\partial \bar{H}_A}{\partial t} = [500(1500) - (-200)(1000)]\bar{i} + (800)(-500)\bar{j}$$

$$+ [300(1000) - (-200)(1500)]\bar{k}$$

$$= 9.5(10^5)\bar{i} - 4(10^5)\bar{j} + 6(10^5)\bar{k} \ \text{kg-m}^2/\text{s}^2$$

$$\bar{\omega} = -50\bar{i} - 20\bar{k}, \ \bar{\alpha} = 1500\bar{i} - 500\bar{j} + 1000\bar{k}$$

$$\bar{\omega} \times \bar{H}_A = [-(-50)(-16000) + (-20)(-29000)]\bar{j} = -2.2(10^5)\bar{j}$$

$$S = \frac{1}{2}(1500\bar{i} - 500\bar{j} + 1000\bar{k}) \cdot [9.5\bar{i} - 4\bar{j} + 6\bar{k}](10^5)$$

$$+ (1500\bar{i} - 500\bar{j} + 1000\bar{k}) \cdot (-2.2\bar{j})(10^5)$$

$$= 1.2225(10^9) \ \text{k-m}^2/\text{s}^3 \qquad \triangleleft$$

It is convenient to define a matrix $[I]$ for software, such that

$$[I] = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yy} & I_{zz} \end{bmatrix}$$

Then $\{H_A\} = [I]\{\omega\}$, $\left\{\frac{\partial H_A}{\partial t}\right\} = [I]\{\alpha\}$

Matlab

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% Exercise 1.3: Vector calculations
clear all
omega = [-50  0  -20]';
alpha = [1500  -500  1000]';
I = [[500  0  200]; [0  800  0]; [200  0  300]];
H = I * omega;
partial_H = I * alpha;
S = 0.5 * alpha' * partial_H + alpha' * (cross(omega, H));
disp(['S = ', num2str(S)])

----
S = 1222500000
```

Mathcad

$$\omega := \begin{pmatrix} -50 \\ 0 \\ -20 \end{pmatrix} \qquad \alpha := \begin{pmatrix} 1500 \\ -500 \\ 1000 \end{pmatrix}$$

$$I_{xx} := 500 \qquad I_{yy} := 800 \qquad I_{zz} := 300 \qquad I_{xz} := -200$$

$$I := \begin{pmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{pmatrix} \qquad H_A := I \cdot \omega = \begin{pmatrix} -2.900000 \times 10^{4} \\ 0.000000 \\ -1.600000 \times 10^{4} \end{pmatrix}$$

$$\text{partial\_H}_A := I \cdot \alpha = \begin{pmatrix} 9.500000 \times 10^{5} \\ -4.000000 \times 10^{5} \\ 6.000000 \times 10^{5} \end{pmatrix} \qquad \omega \times H_A = \begin{pmatrix} 0.000000 \\ -2.200000 \times 10^{5} \\ 0.000000 \end{pmatrix}$$

$$S := \frac{1}{2} \cdot \alpha^{T} \cdot \text{partial\_H}_A + \alpha^{T} \cdot \left(\omega \times H_A\right) = 1.222500 \times 10^{9}$$

Exercise 1.4

Given $\bar{\omega} = c_1 \bar{e}_1 + c_2 \bar{e}_2 + c_3 \bar{k}$ with $\bar{e}_2 \cdot \bar{e}_1 = 0$ & $\bar{e}_2 \cdot \bar{k} = 0$

Find c_1, c_2, and c_3 when $\bar{\omega} = 70\bar{i} + 110\bar{j} + 500\bar{k}$ and

$\bar{e}_1 = -0.4913\bar{i} - 0.7651\bar{j} - 0.4161\bar{k}$

Solution: $\bar{e}_2 \cdot \bar{k} = 0 \Rightarrow \bar{e}_2 = l_x \bar{i} + l_y \bar{j}$

Set $\bar{e}_2 \cdot \bar{e}_1 = 0 \Rightarrow -0.4913 l_x - 0.7651 l_y = 0$

$$l_y = -0.6421\, l_x$$

Also $|\bar{e}_2| = 1 \Rightarrow l_x^2 + l_y^2 = 1 \Rightarrow (1 + 0.6421^2) l_x^2 = 1$

$$l_x = 0.8415 \Rightarrow l_y = -0.5403$$

Then $\bar{\omega} \cdot \bar{e}_2 = c_2 = -0.5345$ rad/s ◁

$$c_1 \bar{e}_1 + c_3 \bar{k} = \bar{\omega} - c_2 \bar{e}_2 \Rightarrow c_1 \bar{e}_1 \cdot \bar{i} = (\bar{\omega} - c_2 \bar{e}_2) \cdot \bar{i}$$

$$c_1 = \frac{(\bar{\omega} - c_2 \bar{e}_2) \cdot \bar{i}}{\bar{e}_1 \cdot \bar{i}} = -143.39 \text{ rad/s}$$ ◁

so $c_3 = (\bar{\omega} - c_2 \bar{e}_2 - c_1 \bar{e}_1) \cdot \bar{k} = 440.3$ rad/s ◁

Exercise 1.5

Given $\bar{r}_{B/A} = -20\bar{i} + 30\bar{j} + 5\bar{k}$, $\bar{r}_{C/A} = 8\bar{i} + 25\bar{j} + 10\bar{k}$,

$\bar{r}_{D/A} = 4\bar{i} - 2\bar{j} + 15\bar{k}$ mm forming the edges of a nonorthogonal parallelepiped.

Find the volume.

Solution: $A = |\bar{r}_{B/A}| h = |\bar{r}_{B/A}||\bar{r}_{C/A}| \sin\theta$

$= |\bar{r}_{B/A} \times \bar{r}_{C/A}|$

$V = A\, |\bar{r}_{D/A}| \sin\phi$

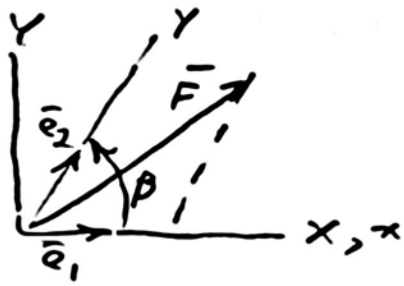
$= |r_{B/A} \times \bar{r}_{C/A}||\bar{r}_{D/A}| \sin\phi$

$= |(\bar{r}_{B/A} \times \bar{r}_{C/A}) \times \bar{r}_{D/A}|$

$V = |(175\bar{i} + 240\bar{j} - 740\bar{k}) \times \bar{r}_{D/A}|(10^{-6}) = |2.12\bar{i} - 5.585\bar{j} - 1.31\bar{k}|10^{-6}$

$= 6.116(10^{-6})\ m^3$ $\triangleleft$

Exercise 1.6



Given $\bar{F} = F_X \bar{I} + F_Y \bar{J} = F^1 \bar{e}_1 + F^2 \bar{e}_2$,

$F_X = 500\,N$, $F_Y = 350\,N$, $\beta = 65°$.

Find F^1 and F^2.

Solution: $\bar{e}_1 = \bar{I}$, $\bar{e}_2 = \cos\beta\,\bar{I} + \sin\beta\,\bar{J}$

so $\bar{F} = F^1 \bar{I} + F^2(\cos\beta\,\bar{I} + \sin\beta\,\bar{J}) = F_X \bar{I} + F_Y \bar{J}$

$\bar{F} \cdot \bar{J} = F^2 \sin\beta = F_Y \Rightarrow F^2 = \frac{350}{\sin\beta} = 386.2\,N$

$\bar{F} \cdot \bar{I} = F^1 + F^2 \cos\beta = 500 \Rightarrow F^1 = 336.8\,N$ ◁

Exercise 1,7

Given $\bar{\sigma} = \rho\bar{v}$, mass flux $= \bar{\sigma}\cdot\bar{e}$, $A = 200\,mm \times 200\,mm$,

$\bar{e} = 0.6\bar{j} + 0.8\bar{k}$, $\bar{v} = 80\cos(5\pi t)\bar{i} - 20\cos(10\pi t)\bar{j}$

$+ 40\sin(10\pi t)\bar{k}$ m/s, $\rho = 950$ kg/m3

Find mass flow for $50 < t < 100$ ms.

Solution: total mass flow rate $= \iint_A \bar{\sigma}\cdot\bar{n}\,dA$

$$= \bar{\sigma}\cdot\bar{e}A, \quad A = 0.2^2\ m^2$$

$$\text{Total mass flow} = \int_{t_0}^{t_f} (\text{Total mass flow rate})\,dt$$

$$\bar{\sigma}\cdot\bar{e} = \rho\bar{v}\cdot\bar{e} = 950\left[-12\cos(10\pi t) + 32\sin(10\pi t)\right]dt$$

$$\text{so T.M.F.} = \int_{0,05}^{0.1} (950)(0.04)\left[-12\cos(10\pi t) + 32\sin(10\pi t)\right]dt$$

$$= -456\left(\frac{\sin(10\pi t)}{10\pi}\right)\Bigg|_{0.05}^{0.10} - 1216\left(\frac{\cos(10\pi t)}{10\pi}\right)\Bigg|_{0.05}^{0.10}$$

$$= 53,22\ \text{kg}$$

Exercise 1.8

Given $\bar{r}_{P/O} = R\cos\theta\,\bar{i} + R\sin\theta\,\bar{j}$, $R = \rho + \varepsilon\sin(\alpha t)$, $\theta = \alpha t^2/2$

Find $\bar{v}_P$ & resolve into components w.r.t. $\bar{r}_{P/O}$

Solution: Chain rule: $\bar{v}_P = \dfrac{\partial \bar{r}_{P/O}}{\partial R}\dot{R} + \dfrac{\partial \bar{r}_{P/O}}{\partial \theta}\dot{\theta}$

$$\bar{v}_P = (\cos\theta\,\bar{i} + \sin\theta\,\bar{j})\varepsilon\alpha\cos(\alpha t) + (-R\sin\theta\,\bar{i} + R\cos\theta\,\bar{j})(\alpha t)$$

$$= [\varepsilon\alpha\cos\theta\cos(\alpha t) - R\alpha t\sin\theta]\,\bar{i} + [\varepsilon\alpha\sin\theta\cos(\alpha t) + R\alpha t\cos\theta]\,\bar{j} \quad \text{where } R = \rho + \varepsilon\sin(\alpha t) \qquad \triangleleft$$

Then $\bar{e}_{P/O} = \dfrac{\bar{r}_{P/O}}{|\bar{r}_{P/O}|} = \cos\theta\,\bar{i} + \sin\theta\,\bar{j}$

so $v_{parallel} = \bar{v}\cdot\bar{e}_{P/O} = \varepsilon\alpha\cos(\alpha t)[(\cos\theta)^2 + (\sin\theta)^2]$

$$= \varepsilon\alpha\cos(\alpha t) \qquad \triangleleft$$

Then $\bar{v}_{perp} = \bar{v}_P - v_{parallel}\,\bar{e}_{P/O} = R\alpha t(-\sin\theta\,\bar{i} + \cos\theta\,\bar{j})$

$$|\bar{v}_{perp}| = R\alpha t \qquad \triangleleft$$

Exercise 1.9

Given $\bar{r}_{P/O} = R\cos\theta\,\bar{i} + R\sin\theta\,\bar{j}$

Find $\bar{v}$ in term of xyz then resolve relative to $\bar{r}_{P/O}$

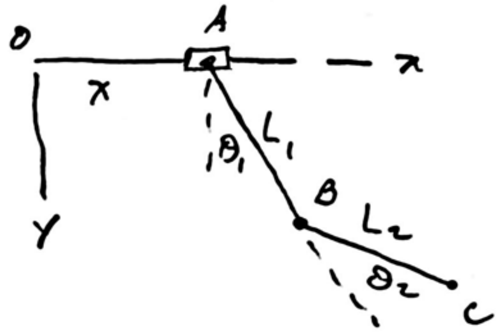
Solution;
$$\bar{v} = \frac{d}{dt}\bar{r}_{P/O} = (\dot{R}\cos\theta - R\dot{\theta}\sin\theta)\bar{i} + (\dot{R}\sin\theta + R\dot{\theta}\cos\theta)\bar{j} \qquad \triangleleft$$

Define $\bar{e}_r = \dfrac{\bar{r}_{P/O}}{|\bar{r}_{P/O}|} = \cos\theta\,\bar{i} + \sin\theta\,\bar{j}$

$$\bar{v}\cdot\bar{e}_r = (\dot{R}\cos\theta - R\dot{\theta}\sin\theta)\cos\theta + (\dot{R}\sin\theta + R\dot{\theta}\cos\theta)(-\sin\theta$$
$$= \dot{R} \qquad \triangleleft$$

$$v_\perp = |\bar{v}\times\bar{e}_r| = |(\dot{R}\cos\theta - R\dot{\theta}\sin\theta)\sin\theta - (\dot{R}\sin\theta + R\dot{\theta}\cos\theta)(\cos\theta)| = R\dot{\theta} \qquad \triangleleft$$

Exercise 1.10



Given $x = 20\sin(50t)$ mm,

$\theta_1 = 0.2\pi\cos(50t)$, $\theta_2 = 0.2\pi \times \sin(50t - \pi/3)$ rad

Find $\bar{v}_C$ from $\bar{r}_{C/O}(t)$

Solution:

$$\bar{r}_{C/O} = [x + L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)]\bar{I} + [L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)]\bar{J}$$

$$\bar{v}_C = \dot{\bar{r}}_{C/O} = [\dot{x} + L_1\dot{\theta}_1\cos\theta_1 + L_2(\dot{\theta}_1+\dot{\theta}_2)\cos(\theta_1+\theta_2)]\bar{I} + [-L_1\dot{\theta}_1\sin\theta_1 - L_2(\dot{\theta}_1+\dot{\theta}_2)\sin(\theta_1+\theta_2)]\bar{J}$$

Set $\dot{x} = 1000\cos(50t)$, $\dot{\theta}_1 = -10\pi\sin(50t)$,

$\dot{\theta}_2 = 10\pi\cos(50t - \pi/3)$ ◁